Unit - 15

A.C. CIRCUIT

Learning Objectives

After studying this chapter the students will be able to

- Describe the terms time period, frequency, instantaneous peak value and root mean square value of an alternating current and voltage.
- Represent a sinusoidally alternating current or voltage by an equation of the form $x = x_0 \sin \omega t$.
- Describe the phase of A.C and how phase lags and leads in A.C Circuits.
- Identify inductors as important components of A.C circuits termed as chokes (devices which present a high resistance to alternating current).
- Explain the flow of A.C through resistors, capacitors and inductors. Apply the knowledge to calculate the reactance's of capacitors and inductors.
- Apply the knowledge to calculate the reactances of capacitors and inductors.
- Describe impedance as vector summation of resistances and reactances.
- Construct phasor diagrams and carry out calculations on circuits including resistive and reactive components in series.
- Solve the problems using the formulae of A.C Power.
- Explain resonance in an A.C circuit and carry out calculations using the resonant frequency formulae.
- Describe that maximum power is transferred when the impedances source and lad match to each other.
- Describe the qualitative treatment of Maxwell's equations and production of electromagnetic waves.
- Become familiar with electromagnetic spectrum (ranging from radio waves to γ-rays).

CONCEPT MAP

ALTERNATING CURRENT (A.C.)

The current that is produced by a voltage source whose polarity keeps on reversing with time is called A.C.

Terms related to A.C
- Phase of A.C
  - Phase lag
  - Phase lead
- Vector representation of an alternating quantity
- Peak value
- Peak to Peak value
- Instantaneous value
  - $V = V_o \sin\omega t$
  - $V = V_o \sin 2\pi f t$
- Root mean square value (r.m.s.)
  - $V_{r.m.s} = 0.707 V_o$
  - $I_{r.m.s} = 0.707 I_o$

Impedance
$Z = V_{r.m.s}/I_{r.m.s}$ (ohm)

A.C. through
- Resistance
  - $V = V_o \sin\omega t$
  - $I = I_o \sin\omega t$
  - $P = VI = I^2R = V^2/R$
- Capacitor
  - $V = V_o \sin\omega t$
  - $q = CV_o \sin\omega t$
  - $I = \Delta V/\Delta t$
  - $X_C = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{1}{2\pi f C}$
- Inductor
  - $I = I_o \sin\omega t$
  - $V = L\frac{\Delta I}{\Delta t}$
  - $X_L = \frac{V_{r.m.s}}{I_{r.m.s}} = 2\pi f L$

R.C and R-L series circucit
$Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$, $\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$

$Z = \sqrt{R^2 + (\omega L)^2}$, $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

Series resonance circuit (Acceptor circuit)
$f = \frac{1}{2\pi\sqrt{LC}}$

Parallel resonance circuit (Rejector circuit)
$f = \frac{1}{2\pi\sqrt{LC}}$

Power in A.C. Circuit
$P = V_{r.m.s} I_{r.m.s} \cos\theta$

Metal Detector

Maxwell Equations

Electromagnetic Waves

Electrocardiogram

Alternating Current:

Alternating current is produced by Alternating Current Generators. In general AC generators, motors and other electrical equipments are simpler, cheaper and more reliable than their DC counterparts. The study of resistance, inductance and capacitance in AC circuits will help us in uses of circuit elements and AC sources.

James Clerk Maxwell in 1864 describes the electromagnetic theory which describes the relationship between electric and magnetic fields of oscillating charges. According to this theory, the accelerating electric charges radiate electromagnetic waves, which propagate at the speed of light. The frequency of the electromagnetic waves is equal to the frequency of oscillation of the charges. He formulated four equations which describe the basis all electrical and magnetic phenomena.

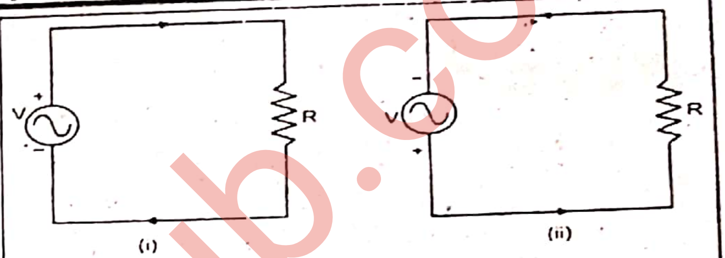
These equations unified the Optics and Electromagnetism.

For Your Information

Metal detectors are used at air ports and other sensitive areas for security purposes. Metal objects cause changes in an electromagnetic field when the pass through the door way. A circuit detects the changes and sets off an alarm.

Q.1 What is alternating voltage and current. Why they also called sinusoidal signal?

Alternating Voltage and Current:



AC circuit

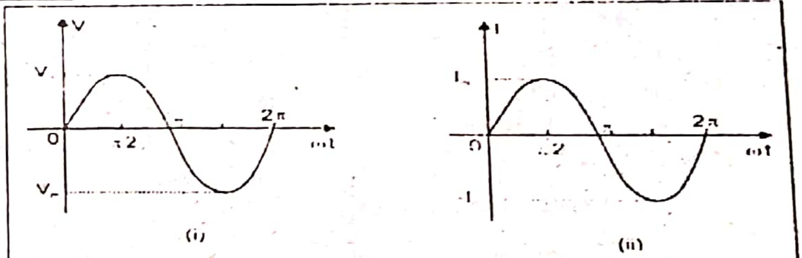
The current which changes its magnitude continuously and direction periodically is called alternating current.

In order to operate a circuit we use a D.C or an A.C source. In direct current, electrons flow continuously in one direction from the source of power through a conductor to a load and back to the source of power. The voltage in direct current remains constant. D.C power sources include batteries and D.C generators. In alternating current an A.C generator is used to make electrons flow first in one direction then in another. A source with produces potential difference of changing polarity with time is called as alternating source. A voltage which changes its polarity which causes the change in direction of current at regular interval of time is called an alternating voltage.

Figure shows an alternating voltage source connected to a resistor R. in Fig the upper terminal of alternating voltage source is positive and lower terminal negative so that current flows in the circuit as shown in Fig. (i).

After some time, the polarities of the voltage source are reversed, so that current now flows in the opposite direction. This is called alternating current because the current flows in alternate directions in the circuit.

Sinusoidal Alternating Voltage and Current:



Alternating current and voltage

The sinusoidal alternating voltage can be produced by rotating a coil with a constant angular velocity in a uniform magnetic field. A.C voltage switches polarity over time. The graph between voltage and time can be expressed by the equation:

$$V = V_m \sin\omega t \quad .... (1)$$

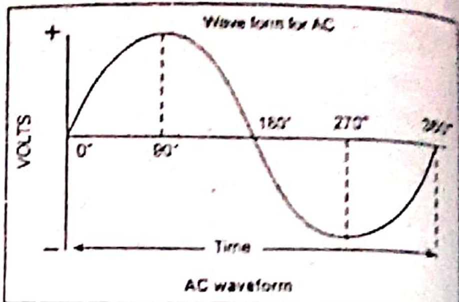
Where V is Instantaneous value of alternating voltage, V_m is maximum, value of alternating voltage and ω is angular velocity of the coil. Figures shows wave forms of voltage or current are sinusoidal and not only changes direction at regular intervals but the magnitude is also changing continuously.

Q.2 Discuss the different terms concerning to alternating voltage and current.
(i) Instantaneous value (ii) cycle (iii) time period (iv) frequency (v) peak value (vi) average value (vii) RMS value

A.C. Terminologies:

As A.C rises from zero to maximum positive value, falls to zero, increases to a maximum in the reverse direction and falls back to zero again as shown in figure. The important A.C. terminology is defined below:

1. Instantaneous Value:



The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltage and current are represented by V and I respectively. As an example, the instantaneous values of voltage at 0°, 90° and 270° are 0, + V_m, − V_m.

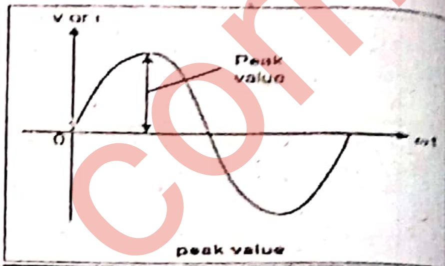
2. Cycle:

One complete set of positive and negative value of an alternating quantity is known as a cycle. A cycle can also be defined in terms of angular measure. One cycle corresponds to 360° electrical or 2π radians.

3. Time Period:

The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.

4. Frequency:



The number of cycle completed in one second is called the frequency (*f*) of the alternating quantity. It is measured in cycle/s (C/s) or Hertz (Hz). One hertz is equal to 1C/s.

The frequency of power system in Pakistan is 50 C/s or 50 Hz. It means that alternating voltage or current completes 50 cycles in one second.

5. Peak values:

The maximum value reached by an AC waveform is called its peak value. A.C reaches to its peak value twice each cycle, once at the positive maximum value and once at the negative maximum value. The peak value of a waveform is also called its amplitude, but the term "peak value" is more descriptive. The peak value is not used to specify the magnitude of alternating voltage or current. The peak of alternating voltage or current is represented by V_m and I_m.

4. Average Value:

The average value of a waveform is the average of all its values over a period of time. We consider the area above the time axis is positive and area below the time axis is negative. The algebraic signs of the areas must be taken into account when computing the total (net) area.

$$\text{Average} = \frac{\text{Total (net) area under curve for time T}}{\text{Time T}}$$

7. R.M.S. or Effective Value:

As the average value of A.C over one cycle is zero and is not suitable for power calculation. Therefore, we use RMS value to measure the effectiveness of an alternating current. The equivalent average value for an alternating current system that provides the same power to the load as a D.C equivalent circuit is called the "effective value".

This effective power in an alternating current system is therefore equal to (I^2R Average). As power is proportional to current squared, the effective current, I will be equal to $\sqrt{I^2 \text{ Ave}}$.

Therefore, the effective current in an A.C system is called the Root Mean Squared or R.M.S. value. The effective or r.m.s. value of an alternating current is that steady current (d.c.) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.

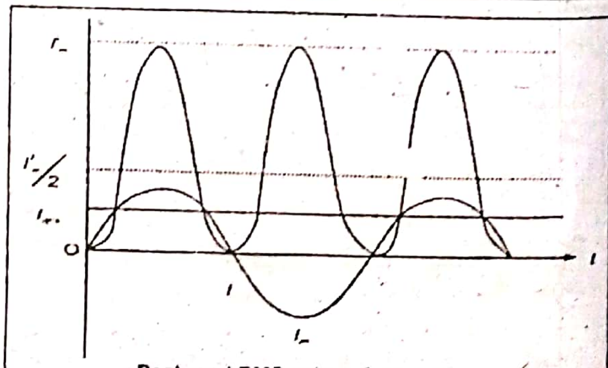
Although peak, average and peak to peak values may be important in some engineering applications, but it is the r.m.s. or effective value which is used to express the magnitude of an alternating voltage or current.

For Your Information

Importance of Sine Waveform:

Alternating voltages and currents can be produced in variety of waveforms (e.g. square waves, triangular waves, rectangular waves etc), but the engineers still choose to adopt sine waveform. It has following advantages:

1. The sine waveform produces the least disturbance in the electrical circuit and is the smoothest and efficient waveform. For example, when current in a capacitor, in an inductor or in a transformer is sinusoidal, the voltage across the element is also sinusoidal. This is not true of any other waveform.
2. The mathematical computations, connected with alternating current work, are



Peak and RMS value of current

The equation of the alternating current varying sinusoidally is given by:

$$I = I_m \sin \omega t \quad .... (2)$$

If this current is passed through a resistance R, then power delivered at any instant is

$$P = I^2R = (I_m \sin \omega t)^2R$$

$$P = I_m^2 R \sin^2 \omega t \quad .... (3)$$

Because the current is squared, power is always positive. Since the value of $\sin^2 \omega t$ varies between 0 and 1, its average value is ½

$$\therefore \text{ Average power delivered, } P = \frac{1}{2} I_m^2 R \quad .....(4)$$

If $I_{r.m.s.}$ is the r.m.s. (or effective) value of alternating current, then by definition,

$$\text{Power delivered, } P = I_{r.m.s.}^2 R \quad .... (5)$$

Comparing Eqs. (4) and (5), we have,

$$I_{r.m.s}^2 R = \frac{1}{2} I_m^2 R$$

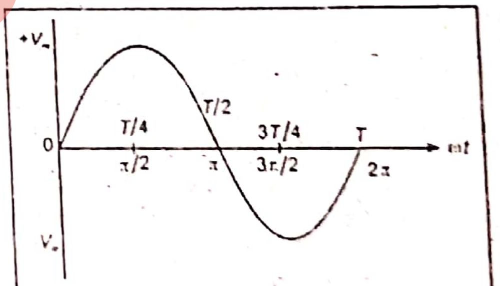
$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.7071\ I_m$$

$$I_{r.m.s} = 0.707\ I_m .... (6)$$

An alternating current can also be represented as a cosine function of time. $I = I_m \cos\omega t$. Similarly, alternating voltage can be represented as $V = V_m \cos\omega t$.

Q.2 What is meant by phase of A.C and how we describe the phase difference between two alternating quantities?

Phase of A.C.



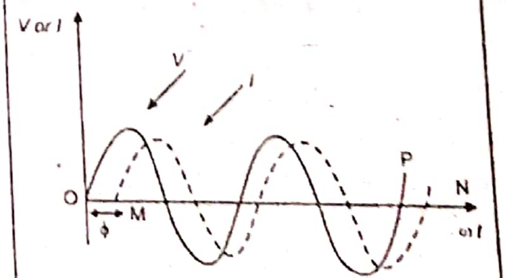
Alternating voltage wave form

In electrical engineering, we are more concerned with relative phases or phase difference between different alternating quantities rather than with their absolute values.

The word "phasor" is short for "phase vector." It is a way to represent a sine or cosine function graphically. Consider an alternating voltage wave of time period T second as shown in figure.

The maximum positive value $(+V_m)$ occurs at T/4 second or π/2 radians. Therefore phase of maximum positive value is T/4 second or π/2 radians. Similarly, the phase of negative peak $(-V_m)$ is 3T/4 second or 3π/2 radians. Phase of a particular value of an alternating quantity is the fractional part of time..

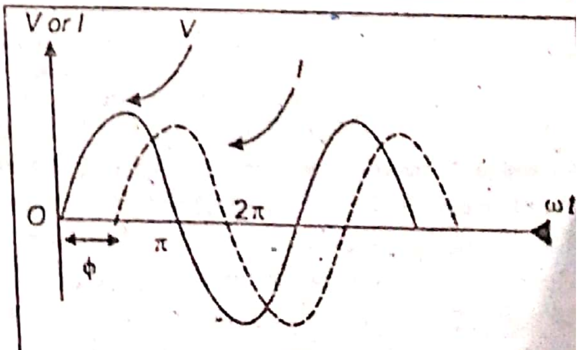
Phase Difference:



Phase difference of current and voltage

In most of practical circuits, alternating voltage and current have different phases. The alternating quantities of the same frequency have different zero point, they are said to have a phase difference.

The angle between zero points is the angle of phase difference ϕ. It is generally measured in degrees or radians. The quantity which passes through its zero point earlier is said to leading while the other is said to be lagging.



Phasor Diagram

As both alternating quantities have the same frequency, the phase difference between them remains the same. Pharos of waves can be added as vectors to produce the sum of two sine functions. Consider an A.C. circuit in which current I lags behind the voltage V by ϕ so the phase difference between voltage and current is ϕ.

This phase relationship is shown by waves in Figure. Thus in figure, voltage V is passing through its zero point 'O' and is rising in the positive direction. Similarly, current I passes through its zero point 'M' as shown in fig and is rising in the positive direction. Therefore, phase difference between voltage and current is OM (= ϕ). Similarly, difference at other points P and N is PN (= ϕ). The equations of voltage and current are:

$V = V_m \sin\omega t$ (i)

$I = I_m \sin(\omega t - \phi)$ (ii)

Alternating Quantities Representation:

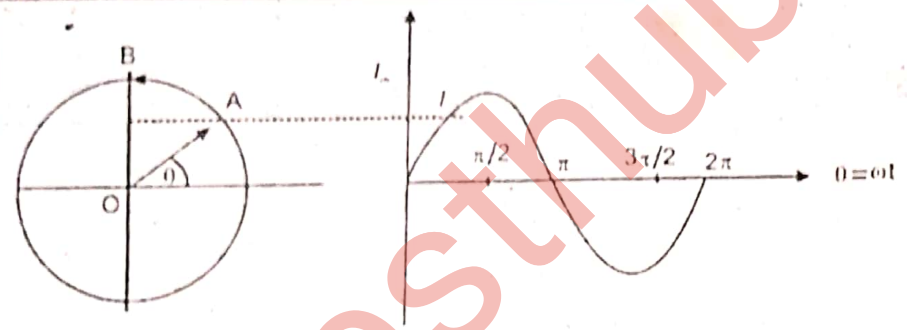
The sinusoidal alternating voltage or current is represented by a line of definite length rotating in counter clock wise direction at a constant angular velocity (ω). Such a rotating line is called a phasor. The length of the phasor is taken equal to the maximum value (on a suitable scale) of the alternating quantity, the angle with axis of reference (i.e., X-axis) indicates the phase of the alternating quantity (current in this case) and angular velocity equal to the angular velocity of the alternating quantity.

In AC circuits, currents and voltages are all sinusoidal functions. The general mathematical form of such a function is: $I = I_m \sin\omega t$. Let line OA represents the maximum value I_m on the scale. Imagine the line OA (or Phasor, as it is called) to be rotating in anticlockwise direction at an angular velocity ω rad/s about the point O. Measuring the time from the instant when OA is horizontal, let OA rotate through an angle θ in the anticlockwise direction. The projection of OA on the Y-axis is OB.

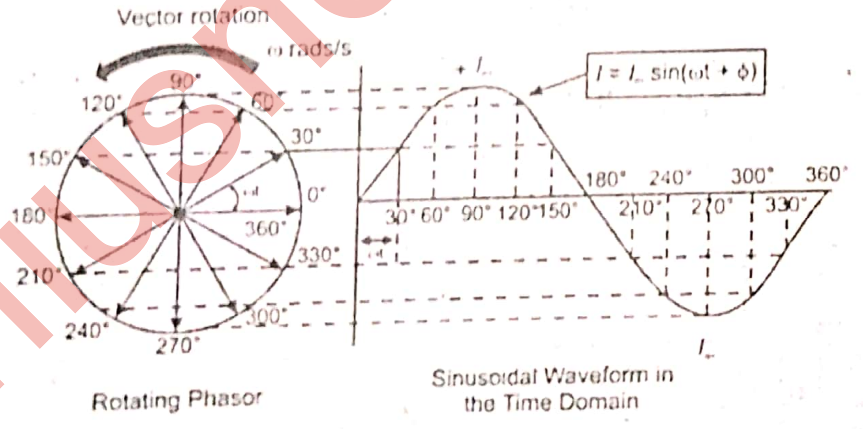
$OB = OA \sin\theta$

$I = I_m \sin\omega t$

Where I, is the value of current at that instant.



Phasor Representation



Rotating vectors at different phase angles

Hence the projection of the phasor OA and the y – axis at any instant gives the value of current at that instant. Thus when $\theta = 90°$, the projection on y – axis is OA (= I_m) itself. That the value of current at this instant (i.e. at θ or $\omega t = 90°$) is I_m can be readily established if we put $\theta = 90°$ in the current equation. If we plot the projections of the phasor on the Y – axis versus its angular position point by point, a sinusoidal alternating current wave is generated as shown in figure. So the phase represents the sine wave for every instant of time. Things start to get complicated when we need to relate two or more Ac voltage or currents that are out of step with each other. By "out of step," we mean that the two waveforms are not synchronized: that their peaks and zero points do not match up at the same points in time. If the sinusoidal voltage wave V and sinusoidal current wave I of the same frequency are out of phase such that the voltage is leading the current by ϕ^0.

Then the alternating quantities can be represented on the same phasor diagram because the phasors V_m and I_m rotate at the same angular velocity ω and hence phase difference φ between them remains the same at all time as shown in figure. When each phasor completes one revolution, it generates the corresponding cycle. The equations of the two waves can be represented as:

$$V = V_m \sin \omega t$$

$$I = I_m \sin (\omega t - \phi)$$

Since the two phasors have the same angular velocity (ω) and there is no relative motion between them, they can be displayed in a stationary diagram.

Instantaneous Power:

The instantaneous power supplied to a circuit is simply the product of the instantaneous voltage and instantaneous current. The instantaneous power is always expressed in watts, irrespective of the type of circuit used. The instantaneous power may be positive or negative. A positive value means that power flows from the source to the load. Consequently, a negative value means that power flows from the load to the source.

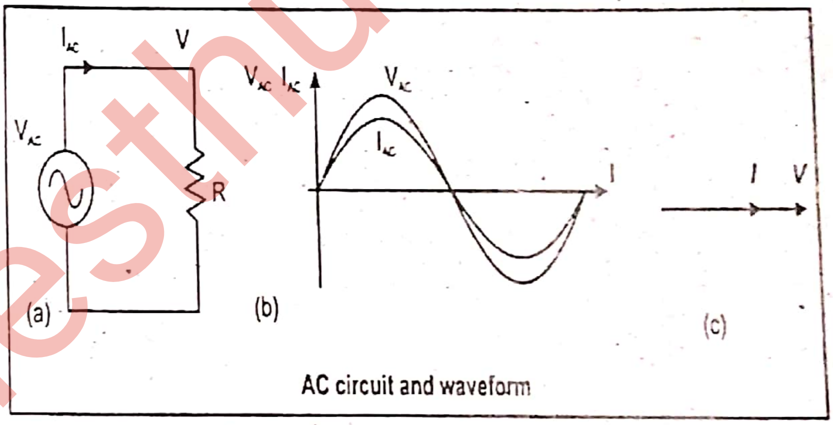
Q.3 Describe the flow of A.C through a resistor in detail, also discuss its power loss.

A.C. Through Resistor:

Consider a circuit containing a pure resistance of R connected across an alternating voltage source as shown in figure (a), then free electrons flow in one direction for the first half-cycle of the supply and then flow in the opposite direction during the next half-cycle. The applied voltage and current pass through their zero values at the same instant and attain their positive and negative peaks at the same instant such that current is in phase with the applied voltage as shown in figure(b). The alternating voltage is be given by

$$V = V_m \sin \omega t \quad \ldots(i)$$

Where, V_m is the peak value of the alternating voltage.



AC circuit and waveform

As a result of this voltage, an alternating current I will flow in the circuit. The applied voltage has to overcome the drop in the resistance only i.e., V = IR

Or $\quad I = \frac{V}{R} = \frac{V_m}{R} \sin \omega t$

$$\therefore \quad I_m = \frac{V_m}{R}$$

$$I = I_m \sin \omega t \quad \text{(ii)}$$

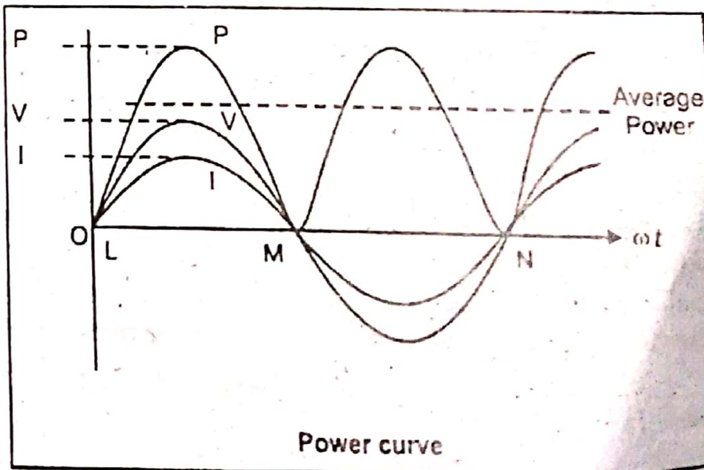
The value of I will be maximum (i.e. I_m) when sin ωt = 1. Equations. (i) and (ii) shows that the applied voltage and the circuit current are in phase with each other. This is also indicated by the phasor diagram shown in figure (c).

In terms of r.m.s. value,

$$\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} \times R$$

Or $\quad V_{rms} = I_{rms} R$

Power Loss in an Resistor:



Power curve

The power curve for a pure resistive circuit is obtained from the product of the corresponding instantaneous values of voltage and current. Figure shows that power is always positive except at points L,M and N at which it drops to zero for a moment.

This means that the voltage source is constantly delivering power to the circuit which is consumed by the circuit.

The average power dissipated in resistor R over one complete cycle of the applied is:

$$P = \langle VI \rangle = \langle V_m \sin \omega t \times I_m \sin \omega t \rangle$$

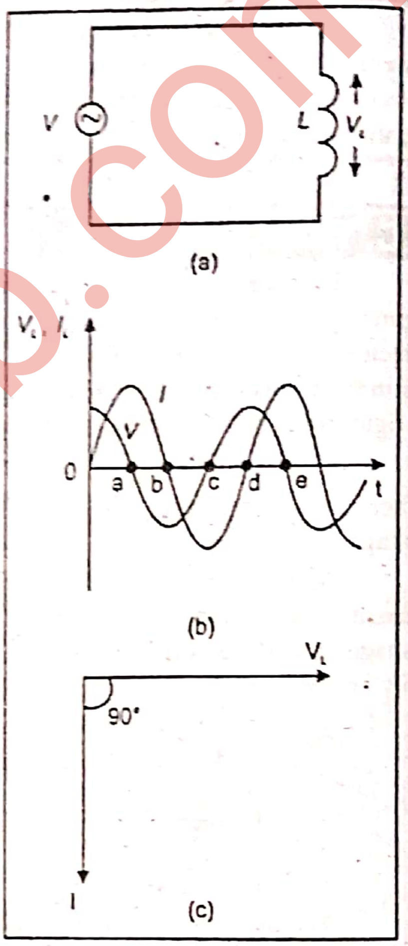
$$P = V_m I_m \langle \sin^2 \omega t \rangle$$

$$P = \frac{V_m I_m}{2} \quad \therefore \langle \sin^2 \omega t \rangle = \frac{1}{2}$$

$$P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

Q.4 Describe the flow of A.C through an inductor in detail, also discuss its power loss. Give the significance of choke coil.

A.C. Through Pure Inductor



An inductor is a two-terminal electrical component which resists changes in electric current passing through it. It consist of a conducting coil. Consider an alternating voltage applied to a pure inductance of L as shown in figure when a sinusoidal current I flow in time t then a back e.m.f. (=L ΔI/Δt) is induced due change in flux through the coil which opposes the change in flux. As there is no drop in potential, so the applied voltage has to overcome the back. emf

∴ Applied alternating voltage = Back e.m.f.

So the energy which is required in building up current in inductance L, is returned back during the decay of the current.

Let the equation for alternating current is:

$$I = I_m \sin \omega t \text{.... (i)}$$

The changing current sets up a back e.m.f. in the coil. The magnitude of back e.m.f. is

$$\varepsilon = L\frac{\Delta I}{\Delta t}$$

To maintain a constant current the applied e.m.f. must be constantly applied. The magnitude e of applied voltage is:

$$V = L\frac{\Delta I}{\Delta t}$$

$$V = L\frac{\Delta(I_m \sin \omega t)}{\Delta t}$$

$$V = LI_m\frac{\Delta(\sin \omega t)}{\Delta t} \text{.....(ii)}$$

Using the result of simple calculus:

$$\frac{\Delta(\sin \omega t)}{\Delta t} = \omega \cos \omega t \text{ .....(iii)}$$

Putting the values of Eq (iii) in (ii) we get

$$V = \omega L I_m \cos \omega t \qquad \therefore (\omega L I_m = V_m)$$

Or

$$V = V_m \cos \omega t$$

$$V = V_m \sin\left(\omega t + \frac{\pi}{2}\right) \text{ .....(iv)}$$

From equations (i) and (iv), it is clear that current lags behind the voltage by π/2 radians or 90°. Hence in a pure inductance, current lags behind the voltage by 90°.

Figure (b) also shows that current lags the voltage in an inductive coil. Inductance opposes the change in current and serves to delay the increase or decrease of current in the circuit.

This causes the current to lag behind the applied voltage which is indicated by the phasor diagram shown in figure(c)

Inductance opposes the flow of current in the circuit, so the opposition offered by an inductor to the flow of A.C. is called inductive reactive reactance X_L.

Therefore in analogy to ohm law we can write:

$$V_m = I_m X_L$$

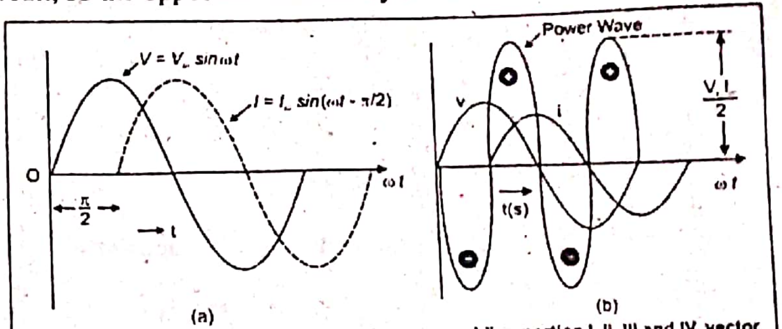
Since inductive reactance is ratio of voltage to current. So

$$X_L = \frac{V_m}{I_m}$$

$$X_L = \frac{I_m \omega L}{I_m} = \omega L$$

Or $X_L = \omega L$

$$X_L = 2\pi f L$$



The shaded portion of the graph indicates that on adding portion I, II, III and IV, vector p is equal to zero. Average power over a complete cycle of AC through an ideal inductor is zero.

The reactance of coil depends upon frequency of A.C. IN case of D.C. inductive reactance X_L is zero.

Power loss in an inductor:

During the first 90° of the cycle, the voltage is positive and the current is negative, therefore, the power supplied is negative. This means the power is flowing from the coil to the source. During the next 90° of the cycle, both voltage and current are positive and the power supplied is positive. Therefore, power flows from the source to the coil. Similarly, for the next 90° of the cycle, power flows from the coil to the source and during the last 90° of the cycle, power flow from the source to the coil. The power curve over one cycle shows that positive power is equal to the negative power. Hence the resultant power over one cycle is zero i.e. a pure inductance consumes no power. The electric power merely flows from the source to the coil and back again.

In any circuit, electric power consumed at any instant is the product of voltage and current at that instant. The average power loss in an inductive circuit is,

$$P = \langle VI \rangle$$

$$P = \langle V_m \cos \omega t \times I_m \sin \omega t \rangle$$

$$P = V_m I_m \langle \sin \omega t \rangle \langle \cos \omega t \rangle$$

$$P = 0 \qquad \therefore \langle \sin \omega t \rangle \langle \cos \omega t \rangle = 0$$

> **For Your Information**
>
> The purpose of passing current through a circuit is to transfer power from the source to the circuit. The power which is actually consumed in the circuit is called the true power or active power.
>
> We know that current and voltage are in phase in a resistance whereas they are 90°. Out of phase in L or C. Therefore, we come to the conclusion that current in phase with voltage produces true or active power whereas current 90° out of phase with voltage contributes to reactive power i.e.
>
> True Power = voltage × Current in

Choke Coil:

A choke in an inductor used in a circuit. It offers high reactance to frequencies above a certain frequency range, without appreciable limiting the flow of current. In a DC circuit, a resistor is used to restrict the current.

If I is the current and R is the resistance, the power loss in the form of heat is I^2R. In AC circuit, inductor is used. Its impedance is X_L and is large at high frequencies. In general, a choke is used to prevent electric signals along undesired paths. The choke is used as a filter in power supply to prevent ripple. It also prevents unwanted signals to enter other parts of the circuits, e.g. radio frequency choke (RFC) prevents radio frequency signals from entering audio frequency circuit. Thus, undesired signals and noise can be attenuated.

Q.5 Describe the flow of A.C through an capacitor in detail, also discuss its power loss.

A.C. Through Capacitor

Consider an alternating voltage applied to a capacitor of capacitance C as shown in figure. When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electron move to and fro around the circuit, connecting the plates, thus constituting alternating current. The basic relation between the charge q on the capacitor and voltage V across its plates i.e. q = CV holds at every instant. Let the applied alternating voltage is:

$$V = V_m \sin \omega t \quad \dots (i)$$

Then at any instant I be the current and q be the charge on the plates. Charge on capacitor,

$Q = CV$

$Q = C V_m \sin \omega t$

And $I = \dfrac{\Delta q}{\Delta t}$

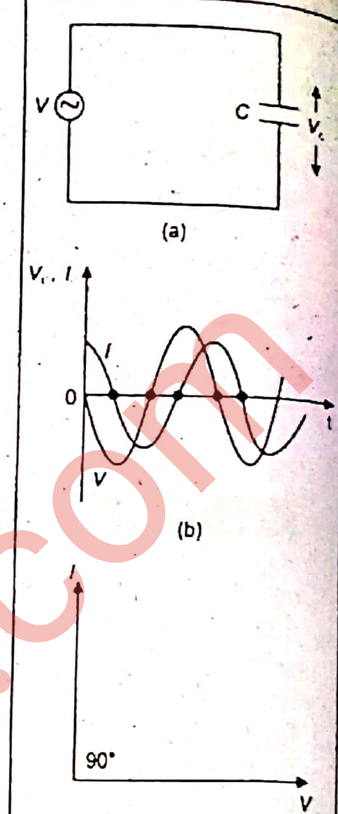
$I = \dfrac{\Delta(CV_m \sin(\omega t))}{\Delta t}$

By using mathematical formulae $\Delta \sin(\omega t) = \omega\cos(\omega t)$

$I = CV_m\omega\cos(\omega t)$

$I = CVm\ w \sin\left(\omega t + \dfrac{\pi}{2}\right)$

$I = I_m \sin\left(\omega t + \dfrac{\pi}{2}\right)$ .... (ii) $\quad \because CV_m\omega = I_m$



(c) Vector diagram

Capacitor in AC circuit

Equations. (i) and (ii) shows that current leads the voltage by π/2radians or 90°. Hence in a pure capacitance, current leads the voltage by 90°. Capacitance opposes the change in voltage and serves to delay the increase or decrease of voltage across the capacitor.

This causes the voltage to lag behind the current. This is also illustrated in the phasor diagram shown in Fig: 15.16 (b): Like inductance which opposes the flow of A.C., capacitance also opposes the flow of AC current in the circuit. From the above:

$I_m = \omega C V_m$

Or $\dfrac{V_m}{I_m} = \dfrac{1}{C\omega}$

Just like ohm law the ratio of V/I is the measure of opposition offered by a resistor to the flow of current. In case of capacitor this opposition is capacitive reactance which opposes the flow of current.

$\dfrac{V_m}{I_m} = \dfrac{V_c}{I} = \dfrac{1}{C\omega}$

Clearly, the opposition offered by capacitance to current flow is $1/\omega C$. This quantity $1/\omega C$. This quantity $1/\omega C$ is called the capacitive reactance X_c of the capacitor. It has the same dimensions as resistance and is, therefore, measured in Ω.

$I = V_c/X_c$

Where capacitive reactance is

$X_c = \dfrac{1}{\omega C}$

$X_c = \dfrac{1}{2\pi fC}$

The capacitive reactance depends upon frequency of A.C. In case of D.C., X_c has infinite value.

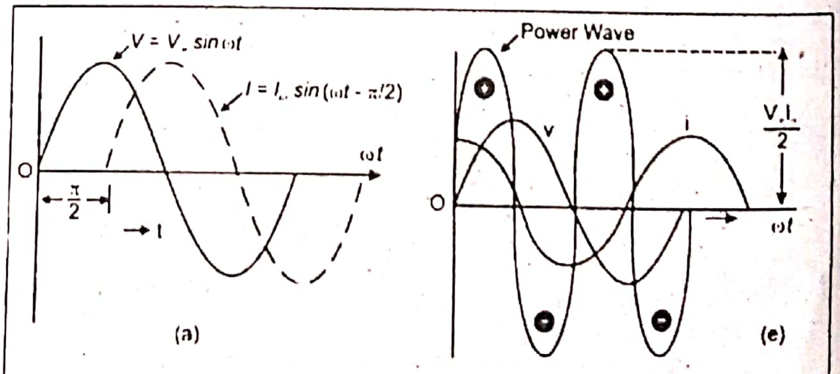
Power Loss in a Capacitive Circuit:

In pure capacitive circuit the current lead the voltage by 90° in phase therefore; the power curve for capacitor and inductor is same. The average power loss in capacitive circuit is,

$P = \langle VI \rangle = \langle V_m \sin \omega t \times I_m \cos \omega t \rangle$

$P = V_m I_m \langle \sin \omega t \rangle \langle \cos \omega t \rangle$

$P = 0 \qquad \therefore \langle \sin \omega t \rangle \langle \cos \omega t \rangle = 0$



Phase diagram for current and voltage and power curve for pure capacitive circuit.

This fact is also illustrated in the wave diagram shown in Fig: 15.16: Which shows that in one cycle the positive power is equal to negative power so power absorbed by capacitor in one cycle is zero.

MCQ's From Past Board Papers

1. Power dissipated in pure inductor is:
(A) Large (B) Small (C) Infinite (D) Zero

2. Direct current cannot flow through:
(A) Resistor (B) Capacitor (C) Inductor (D) Voltmeter

3. Pure choke consumes:
(A) Minimum power (B) Maximum power (C) No power (D) Average power

4. Capacitive reactance X_c =
(A) $2\pi fC$ (B) $\frac{1}{2\pi fC}$ (C) $4\pi fC$ (D) $\frac{1}{4\pi fC}$

5. Phase difference between V and I of an A. C through resistor is:
(A) Zero degree (B) 90° (C) 180 (D) 270°

6. Choke consumes extremely small:
(A) Current (B) Charge (C) Power (D) Potential

7. At what frequency will an inductor of 1.0 H have a reactance of 500 Ω?
(A) 50 Hz (B) 80 Hz (C) 500 Hz (D) 1000 Hz

8. The device which allows only the continuous flow of an A.C. through a circuit is:
(A) Capacitor (B) Inductor (C) D.C. motor (D) Battery

9. At high frequency, the current through a capacitor of A.C. circuit will be:
(A) Large (B) Small (C) infinite (D) Zero

10. The basic circuit element in DC circuit which controls current:
(A) Resistor only (B) Capacitor only (C) Inductor only (D) All of these

11. The capacitive reactance to pure D.C. is:
(A) Zero (B) Infinite (C) Variable (D) Equal to inductive reactance

12. Resistance of pure choke is:
(A) Zero (B) Large (C) Very small (D) infinite

13. Inducitive reactance of an inductor is:
(A) $X_L = \pi f L$ (B) $X_L = 4\pi f L$ (C) $X_L = 2\pi f L$ (D) $X_L = 2 \pi L$

14. Capacitor will have a large reactance at:
(A) Low frequency (B) High frequency (C) Zero frequency (D) Negative frequency

15. In capacitor:
(A) Current leads voltage by $\frac{\pi}{2}$ (B) Voltage leads voltage by $\frac{\pi}{2}$
(C) Current leads the voltage by π (D) Both are in phase

16. Power dissipation in a pure inductive or in a pure capacitance circuit is:
(A) Infinite (B) Zero (C) Minimum (D) Maximum

17. In case of capacitor, the unit of reactanc is:
(A) ohm (B) mho (C) farad (D) henry

18. The phase difference between current and voltage in an Inductive circuit is
(A) Zero (B) 90° (C) 180° (D) 45°

19. 100 μF capacitor is connected to an AC-voltage of 24 V and frequency 50 Hz. The reactance of the capacitor is
(A) 30.8 Ω (B) 31.8 Ω (C) 34.8 Ω (D) 40 Ω

20. The slope of q – t curve at any instant of time gives:
(A) Current (B) Voltage (C) Charge (D) Both A and B

21. In alternating current, Inductors behave like
(A) Semi conductors (B) Inductors (C) Resistors (D) Insulators

22. The reactance of an Inductor at 50 Hz is 10Ω Its reactance at 100 Hz becomes
(A) 20 Ω (B) 5 Ω (C) 2.5 Ω (D) 1 Ω

23. Which consumes small power?
(A) Inductor (B) Resistor (C) Motor (D) All of them

24. A device which opposes the flow of A.C only is
(A) resistor (B) capacitor (C) inductor (D) None

25. In pure capacitor AC circuit, the current I and q are
(A) In phase (B) Out of phase (C) Parallel to each other (D) None of above

26. X_L is low for low frequency f, but X_C is
(A) Zero (B) low (C) High (D) Same as X_L

27.	If the frequency of AC supplied is doubled then the capacitor reactance becomes			
	(A) half	(B) two times	(C) four times	(D) one fourth
28.	If the capacitive reactance of AC circuit is made four times then the frequency of the circuit becomes:			
	(A) Twice	(B) One half	(C) Four times	(D) One forth
29.	The device which allows only the flow of D.C. is:			
	(A) Capacitor	(B) Transformer	(C) Inductor	(D) Generator
30.	The inductive reactance of a coil is directly proportional to:			
	(A) Inductance	(B) Resistance	(C) Frequency of A.C.	(D) Both Frequency of A.C
31.	Choke consumes extremely small			
	(A) Current	(B) Charge	(C) Power	(D) Potential

Answers Key

1. D	2. B	3. C	4. B	5. A	6. C	7. B	8. A	9. A	10. A	11. B	12. A
13. C	14. A	15. A	16. B	17. A	18. B	19. B	20. A	21. C	22. A	23. A	24. C
25. B	26. C	27. A	28. D	29. C	30. C	31. C					

Q.6 Discuss the flow of A.C through RL series circuit. Also discuss the power losses, impedance triangle and Q-factor for a given RL circuit.

R.L Series A.C. Circuit:

In RL-series circuit, the voltage V will be the phasor sum of the two component voltage V_R and V_L. This means that the current flowing through the coil will lag the voltage by an amount less than 90°. The phase lagging ϕ depends upon the values of V_R and V_L. Figure (a) shows a pure resistance R connected in series with a coil of pure inductance L.

Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in figure (c).

The voltage drop V_R (= I R) is in phase with current and is represented in magnitude and direction by the phasor OP.

The voltage drop V_L (= I X_L) leads the current by 90° and is represented in magnitude and direction by the phasor PM.

The applied voltage V is the phasor sum of these two drops i.e.

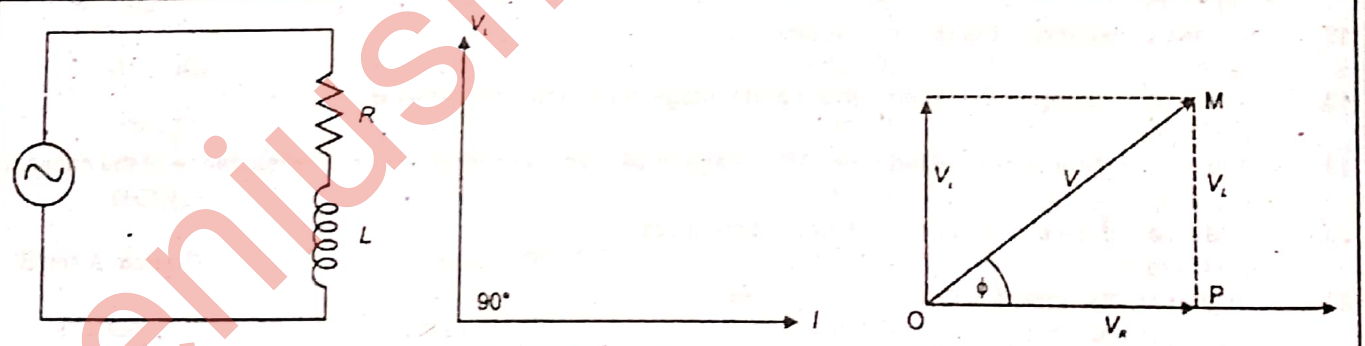
$$V^2 = V_R^2 + V_L^2$$

or $$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(I R)^2 + (I X_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

Or $$I = \frac{V}{\sqrt{R^2 + V_L^2}}$$



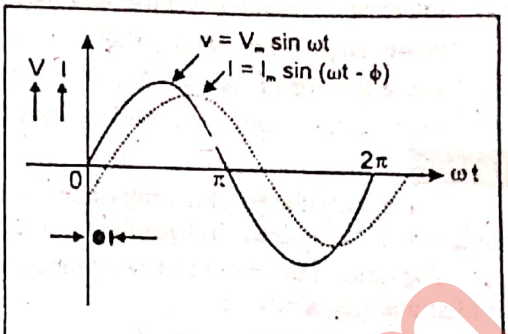
(a) : RL series circuit (b) Phasor diagram (c) Phasor diagram of RL series circuit

The quantity $\sqrt{R^2 + X_L^2}$ is the opposition offered to current flow and is called impedance of the circuit. It is represented by Z and is measured in ohms (Ω)

$$I = V/Z$$

where $$Z = \sqrt{R^2 + X_L^2}$$

The phasor diagram shows that circuit current I lags behind the applied voltage V by $\phi°$. This fact is also illustrated in the wave diagram shown in figure (b). The value of phase angle ϕ can be determined from the phasor diagram.



Wave diagram

$$\tan\phi = \frac{V_L}{V_R}$$

$$\tan\phi = \frac{IX_L}{IR}$$

$$\tan\phi = \frac{X_L}{R}$$

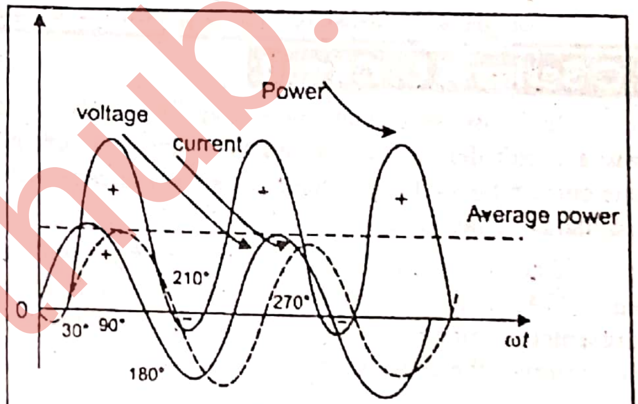
Since X_L and R are known, ϕ can be calculated. If the applied voltage is $V = V_m \sin \omega t$, then equation for the circuit current will be:

$I = I_m \sin(\omega t - \phi)$

where $I_m = V_m/Z$

Fig 15.17 (d): shows that in an inductive circuit current lags behind the applied voltage. The angle (i.e. ϕ) of lagging is greater than 0° but less than 90°. It is determined by the ratio of inductive reactance to resistance ($\tan\phi = X_L$ I R) in the circuit.

The greater the value of this ratio, the greater will be the phase angle ϕ.



Power curve

Power in RL circuit:

Average Power, $P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$

Or $P = V I \cos\phi$

Where V and I are the r.m.s. values of voltage and current. The term $\cos\phi$ is called power factor of the circuit and its value is given by (from phasor diagram):

Power factor, $\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$

Or Power factor = $\cos\phi$ cosin of angle between V and I.

Or $P = VI\cos\phi = (IZ)\, I\, (R/Z) = I^2 R$ [$\because \cos\phi = R/Z$ and $V = IZ$]

In a resistor, the current and voltage are in phase i.e. $\phi = 0°$.

Therefore, power factor of a pure resistive circuit is cos 0° = 1. Similarly, phase difference between voltage and current in a pure inductance or capacitance is 90°. Hence power factor of pure L or C is zero.

This is the reason that power consumed by pure L or C is zero. For a circuit having R, L and C in varying proportions, the value of power factor will lie between 0 and 1.

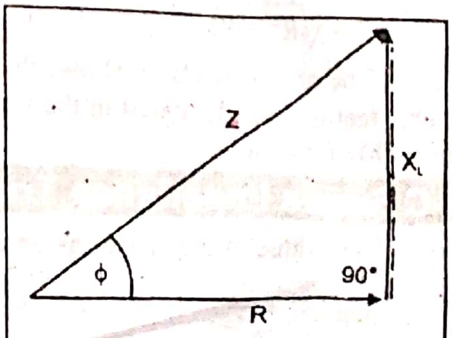
Figure shows that power is negative between 0° and 30° and between 180° and 210°. The negative area means that the inductance of the circuit returns the power to the source. Conversely power is positive between 30° and 180° and so on. But as the area of positive curve is greater than negative area of curve. So net power over one cycle is positive. This shows that power is consumed in R-L series circuit.

R-L Series Impedance Triangle:

In a DC circuit, the ratio of voltage to current is called resistance. However, in an AC circuit this ratio is known as Impedance, Z. Impedance is the total resistance to current flow in an "AC circuit" containing both resistance and inductive reactance. In R.L. series circuit,

$$Z = \sqrt{R^2 + X_L^2} \quad \text{where } X_L = 2\pi f L$$

The magnitude of impedance in R.L. series circuit depends upon the values of R.L and the supply frequency *f*. The R-L series circuit is shown in Fig 15.17(a). The phasor diagram is a triangle whose sides represent R, X_L and Z. This triangle is called an "Impedance Triangle".



Impedance triangle

Impendence triangle is a useful concept in A.C. circuit as it enables us to calculate:

1. Power angle ϕ i.e. $\cos\phi = R/Z$
2. Phase angle ϕ i.e. $\tan\phi = X_L/R$

Therefore, it is always useful to draw the impedance triangle while analyzing an a.c. circuit.

Q-factor:

The quality factor of a component is its energy storing ability. The Q-factor of a circuit is a ratio of energy stored in the circuit to the energy dissipated in one cycle.

The ratio of the inductive reactance (X_L) of a coil to its resistance (R) at a given frequency is known as Q-factor of the coil at that frequency i.e.,

$$Q - \text{factor} = \frac{X_L}{R} = \frac{\omega L}{R}$$

Also,

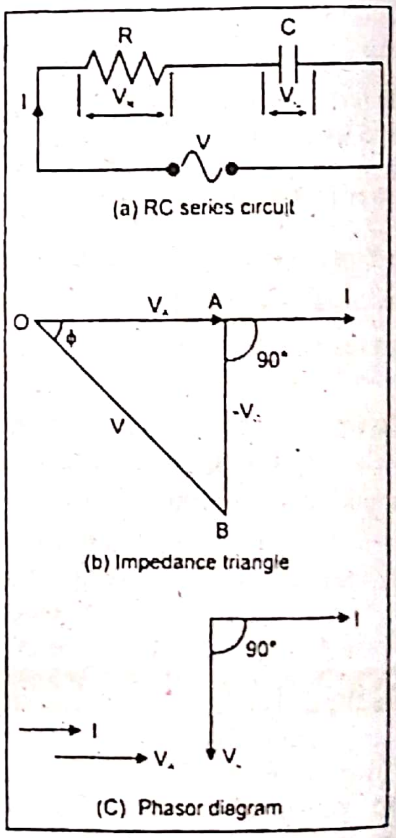
$$Q - \text{factor} = 2\pi \times \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

The Q-factor is used to describe the quality or effectiveness of a coil. A coil is usually designed to have high value of L compared to its resistance R. The greater the value of Q-factor of a coil, the greater is its inductance (L) as compared to its resistance (R).

Q.7 Discuss the flow of A.C through RC series circuit. Also discuss the power losses, impedance triangle and Q-factor for a given RL circuit.

R.C Series A.C. Circuit:

In RL-series circuit, the voltage V will be the phasor sum of the two component voltage V_R and V_L. In order to draw a vector diagram we found the current is common and can therefore be used as the reference source because the same current flows through the resistance and capacitance. The individual vector diagrams for a pure resistance and a pure capacitance is shown in figure.



(a) RC series circuit
(b) Impedance triangle
(C) Phasor diagram

The voltage drop V_R (= IR) is in phase with current and is represented by the phasor OA. The voltage drop V_C (= IX_C) lags behind the current by 90° and is represented in magnitude and direction by the phasor AB. The applied voltage V is the phasor sum of these two drop i.e.

$$V^2 = V_R^2 + V_C^2$$

$$V = \sqrt{V_R^2 + V_C^2}$$

$$V = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{R^2 + X_C^2}$ offer opposition to current flow and is called impedance of the circuit.

$$I = V/Z$$

Where

$$Z = \sqrt{R^2 + X_C^2}$$

The phasor diagram shows that circuit current I leads the applied voltage V by ϕ. This fact is also illustrated in the wave diagram and impedance triangle (as shown in figure (b) of the circuit.

R-C Series Impedance Triangle:

The value of the phase can be determined as under:

$$\tan\phi = -\frac{V_C}{V_R}$$

$$\tan\phi = -\frac{IX_C}{IR}$$

$$\tan\phi = -\frac{X_C}{R}$$

Power in R.C. Circuit:

The equation for voltage and current are:

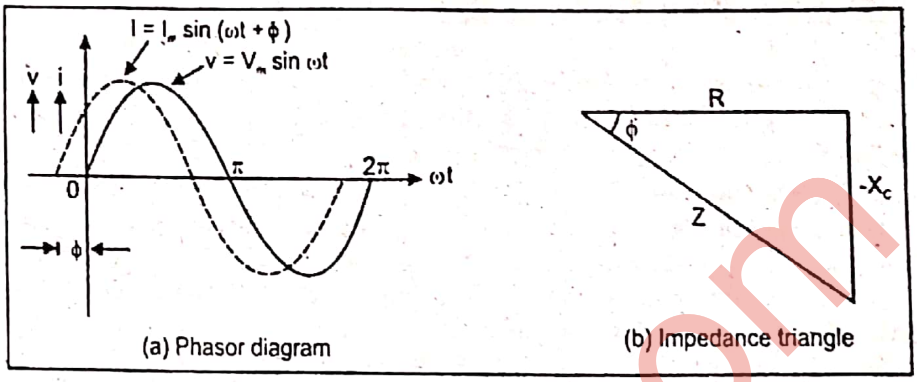
$V = V_m \sin\omega t$;

And $I_m \sin(\omega t + \phi)$

So the average power,

$\langle P\rangle = \langle V\rangle\langle I\rangle$

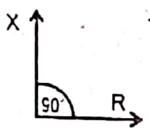
$\langle P\rangle = VI\cos\phi$

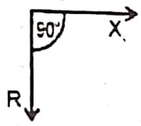


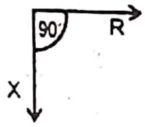
(a) Phasor diagram (b) Impedance triangle

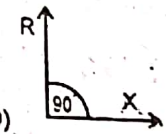
MCQ's From Past Board Papers

1. The combination effect of resistance and reactance is known as:
   (A) Inductance (B) Conductance (C) Resistance (D) Impedance
2. S.I. unit of impedance is:
   (A) Henry (B) Hertz (C) Ampere (D) Ohm
3. The impedance of R–C series A.C. circuit is given by Z = ______
   (A) $\sqrt{R^2-(\omega C)^2}$ (B) $R^2 + (\omega C)^2$ (C) $\sqrt{R^2+(\omega C)^2}$ (D) $\sqrt{R^2+\frac{1}{(\omega C)^2}}$
4. The A.C. circuit in which current and voltage are is phase the power factor is
   (A) Zero (B) 1 (C) 0.5 (D) infinity
5. Which phase diagram is true of RL-Series circuit? **(Federal 2013, 15)**

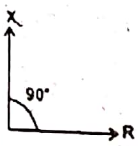
(A) 

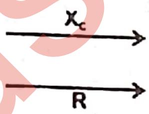
(B) 

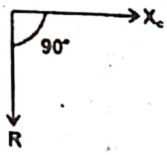
(C) 

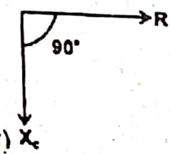
(D) 

6. Impendance is denoted by
   (A) A (B) Z (C) P (D) Q
7. Which phase diagram is true for RC-series circuit? **(Fed 2014)**

(A) 

(B) 

(C) 

(D) 

8. For R – L series circuit, the voltage leads the current by phase angle of:
   (A) $\tan^{-1}(\omega L/R)$ (B) $\tan^{-1}(\omega C/R)$ (C) $\tan^{-1}(1/\omega CR)$ (D) $\tan^{-1}(\omega/RC)$
9. The power factor of RL-series circuit is:
   (A) 0 (B) 1 (C) Less than 1 (D) More than 1
10. The impedance Z can be expressed as:-
    (A) $Z = \frac{V_{rms}}{I_{rms}}$ (B) $Z = \frac{I_{rms}}{V_{rms}}$ (C) Z = I + V (D) Z = I – V
11. When 10V are applied to an A.C. circuit, the current flowing in it is 100 mA, its impedance is:
    (A) 50Ω (B) 75Ω (C) 100Ω (D) 90Ω
12. If L and R represent inductance and resistance respectively, then the dimensions of $\frac{L}{R}$ will be: **(Fed 2017)**
    (A) $[M^0L^0T^{-1}]$ (B) $[M^0L^0T^{-2}]$ (C) $[M^0L^{-1}T^{-2}]$ (D) $[M^0L^0T]$

Answers Key

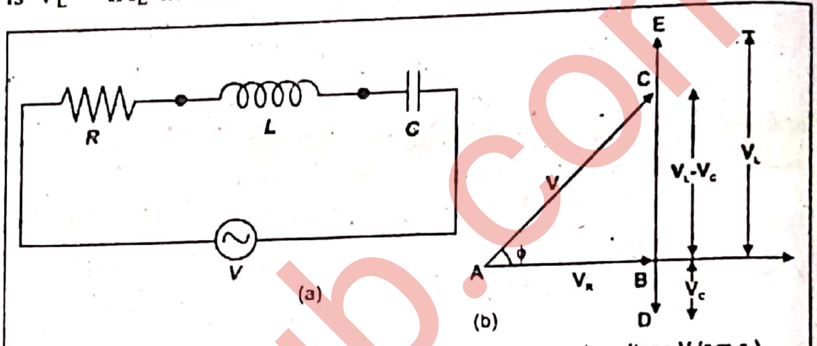
1. D	2. D	3. D	4. B	5. A	6. B	7. D	8. A	9. C	10. A	11. C	12. D

Q.8 Discuss the Series resonance circuit detail also write down its characteristics.

R-L-C series A.C. Circuit:

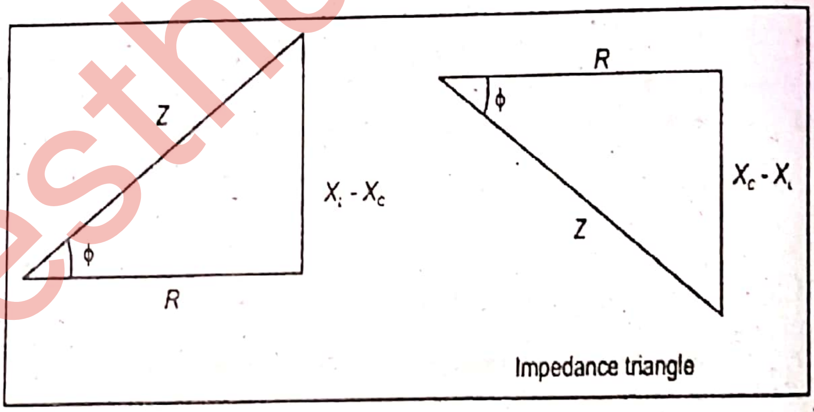
Many AC circuit are very useful for us, which include resistance, inductive reactance, inductive reactance and capacitive reactance. In this section, we will look at some implications of connecting a resistor (R), an inductor (L), and a capacitor (C) together in what is called a series RLC circuit. The simplest and most important AC circuit we can analyze is the series LRC circuit, illustrated in figure.

The analysis of this circuit is quite easy since all the circuit elements share the same current. We can draw a phasor diagram for the current and voltages across the inductor, capacitor, and resistor. The P.D. across R, is $V_R = IR$ in this case V_R is in phase with I. The P.D. across L, is $V_L = IX_L$ in this case V_L leads I by 90°. The P.D. across C, is $V_C = IX_C$ in this case where V_C lags I by 90°. V_L and V_C are thus 180° out of phase. In phasor diagram figure (b), AB represents V_R, BE represents V_L and BD represents V_C. It may be seen that V_L is in phase opposition to V_C.



AC circuit of R, L and C connected in series across a supply voltage V (r.m.s.). The resulting circuit current is I (r.m.s). (b) phasor sum

It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. If $V_L > V_C$ then the net voltage drop across L-C combination is $V_L - V_C$ and their resultant is in the direction V_L represented by BC. Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and their result is in the direction V_L represented by BC. Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and represented by AC.



$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$

The quantity $(X_L - X_C)$ is called the reactance of the circuit, denoted by X

$$X^2 = (X_L - X_C)^2$$

So, we can write

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + X^2}$$

Where Z is the opposition offered to current flow and is called impedance of the circuit.

Circuit power factor,

$$\cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Since X_L, X_C and R, are known, phase angle ϕ of the circuit can be determined.

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

So if the current is represented by a cosine function, $I = I_m \cos\omega t$

The source voltage leads the current by an angle and its equation is-

$$I = I_m\cos(\omega t + \phi)$$

Power consumed, $P = V I \cos\phi$

We have seen that the impedance of a R-L-C series circuit is given by:

$$Z = \sqrt{R_2 + (X_L - X_C)^2}$$

i. When $X_L - X_C$ positive (i.e. $X_L > X_C$), phase angle ϕ is positive and the circuit will be inductive. In other words, in such a case, the circuit current I will lag behind the applied voltage V by ϕ.

ii. When $X_L - X_C$ is negative (i.e. $X_C > X_L$), phase angle ϕ is negative and the circuit is capacitive. That is to say the circuit current I leads the applied voltage V by ϕ ; the value of ϕ being given by Eq. (15.31) above.

iii. When $X_L - X_C = 0$ (i.e. $X_L = X_C$), the circuit is purely resistive. In other words, circuit current I and applied voltage V will be in phase i.e.

$\phi = 0°$ the circuit will then have unity power factor.

If the equation for the applied voltage is $V = V_m \sin \omega t$, then equation for the circuit current will be

$I = I_m \text{Sin} (w t \pm \phi)$ where $I_m = V_m / Z$

The value of ϕ will be positive or negative depending upon which reactant (X_L or X_C) predominates.

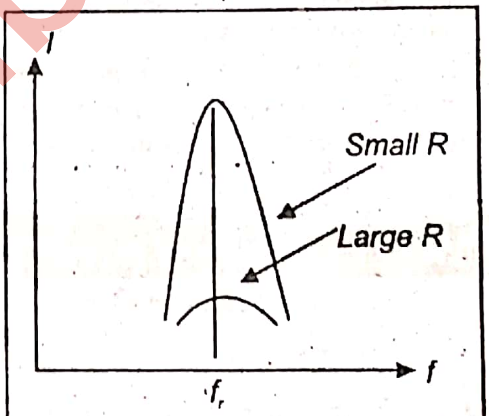
Figure (c) shows the impedance triangle of the circuit for the case when $X_L > X_C$ whereas impendence triangle in Fig.(d) is for the case when $X_C > X_L$.

In the impedance equation along with the equations for the inductive and capacitive reactance, we see that impedance has a rather complicated dependence on the frequency of the oscillator.

$$Z = \sqrt{R_2 + (X_L - X_C)^2}$$

$$X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}$$

When the frequency is very small, the capacitive reactance is large and $XC \approx Z$. When the frequency is very large, the inductive reactance is large and $X_L \approx Z$.Z is a minimum when $X_L = X_C$, and Z is a minimum, the current in the circuit is a maximum. When this happens, the resistance provides the only impedance in the circuit. Z = R. This condition is called resonance and is electrical analog to resonance in harmonic oscillators such as a swinging pendulum or a mas on the end of a spring.



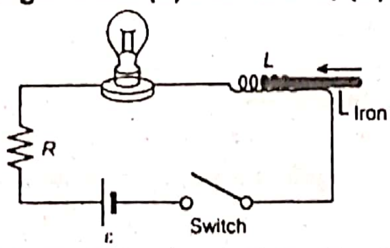
Resonance curve

Resonance means to be in step with. When applied voltage and circuit current in an A.C. circuit in step with (i.e. phase angle is zero or power factor is unity), the circuit is said to be in electrical resonance. If this condition exists in a series A.C. circuit, it is called series resonance. The frequency at which resonance occurs is called resonant frequency (fr).

An A.C. circuit containing reactive elements (L and C) is said to be in resonance when the circuit power factor is unity.

QUIZ

Q. The switch in the circuit shown in Figure is closed and the light bulb glows steadily. The inductor is a sample air-core solenoid. As an iron rod is being inserted into the interior of the solenoid, the brightness of the light bulb (a) increases, (b) decreases, or (c) remains the same.



A. When we are inserting the iron core into the inducting coil, the flux is changing (increasing). So emf induced in the coil which decreases the net emf of the circuit. Hence induced current decreases. So the bulb become dim by following the given equation:

$$I = \frac{V - \varepsilon}{R}$$

Resonance in R-L-C Series Circuits:

R-L-C series circuit is said to be in resonance when the circuit power factor is unity i.e. $X_L = X_C$. The frequency f_r at which it occurs is called resonant frequency. The resonance (i.e. $X_L = X_C$) in an R-L-C series circuit can be achieved by changing the supply frequency because X_L and X_C are frequency dependent. At a certain frequency f_r, X_L becomes equal to X_C and resonance takes place.

At series resonance, $X_L = X_C$

Or $2\pi f_r L = \frac{1}{2\pi f_r C}$

∴ Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$

The above equation shows that on increasing either the inductance or the capacitance causes the resonant frequency to decrease. For a given value of inductance and capacitance, there is only one resonant frequency.

Resonance Curve:

The curve between current and frequency is known as resonance curve of a typical R-L-C series circuit. Current reaches its maximum value at the resonant frequency (f_r), falling off rapidly on either side at that point.

It is because if the frequency is below f_r, $X_C > X_L$ and the net reactance is no longer zero. If the frequency is above f_r, then $X_L > X_C$ and the net reactance is again not zero. In both cases, the circuit impedance will be more than the impedance ($Z_r = R$) at resonance. The result is that the magnitude of circuit current decreases rapidly as the frequency changes from the resonant frequency.

The Q-factor of a series circuit circle indicates how many times the P.D. across L or C is greater than the applied voltage at resonance. For example, when R-L-C series circuit is connected to a 220V source having a Q-factor of the coil as 20, then voltage across the coil or capacitor will be:

$V_C = V_L = QV_R = 20 \times 220 = 440$ V at resonance.

MCQ's From Past Board Papers

1. Resonating frequency of RLC series circuit is f_r = ______.
   (A) $\frac{2\pi}{\sqrt{LC}}$ (B) $\frac{1}{2\pi}\sqrt{LC}$ (C) $\frac{1}{2\pi\sqrt{LC}}$ (D) $2\pi\sqrt{LC}$
2. At high frequency, RLC series circuit shows that behavior of:
   (A) RC circuit (B) RL circuit (C) Pure capacitive circuit (D) Pure RLC circuit
3. At resonance RLC series circuit shows the behavior of:
   (A) Pure resistive circuit (B) pure capacitive circuit (C) pure inductive circuit (D) pure RLC circuit
4. At resonance frequency, the impedance of RLC series circuit is:
   (A) Zero (B) Minimum (C) Maximum (D) Moderate
5. The impedance of RLC series circuit at resonance is given by:
   (A) $Z = \sqrt{R^2 + (X_L - X_C)^2}$ (B) $Z = \sqrt{R^2 + X_L^2}$ (C) $Z = R$ (D) $Z = \sqrt{R^2 + X_C^2}$
6. In RLC series circuit at resonance the phase difference between capacitor and inductor reactances is:
   (A) 90° (B) 270° (C) 0° (D) 180°
7. In R-L-C Circuit, the energy is dissipated in:-
   (A) R only (B) R and L (C) R and C (D) R, L and C
8. In RLC series circuit, the condition for resonance is
   (A) $X_L < X_C$ (B) $X_L > X_C$ (C) $Z < X_C$ (D) $X_L = X_C$
9. At resonance in RLC series circuit, phase difference between voltage and current in:
   (A) 0° (B) 90° (C) 120° (D) 180°
10. The unit of $\sqrt{LC}$ is:
   (A) Second (B) Ampere (C) Hertz (D) Farad

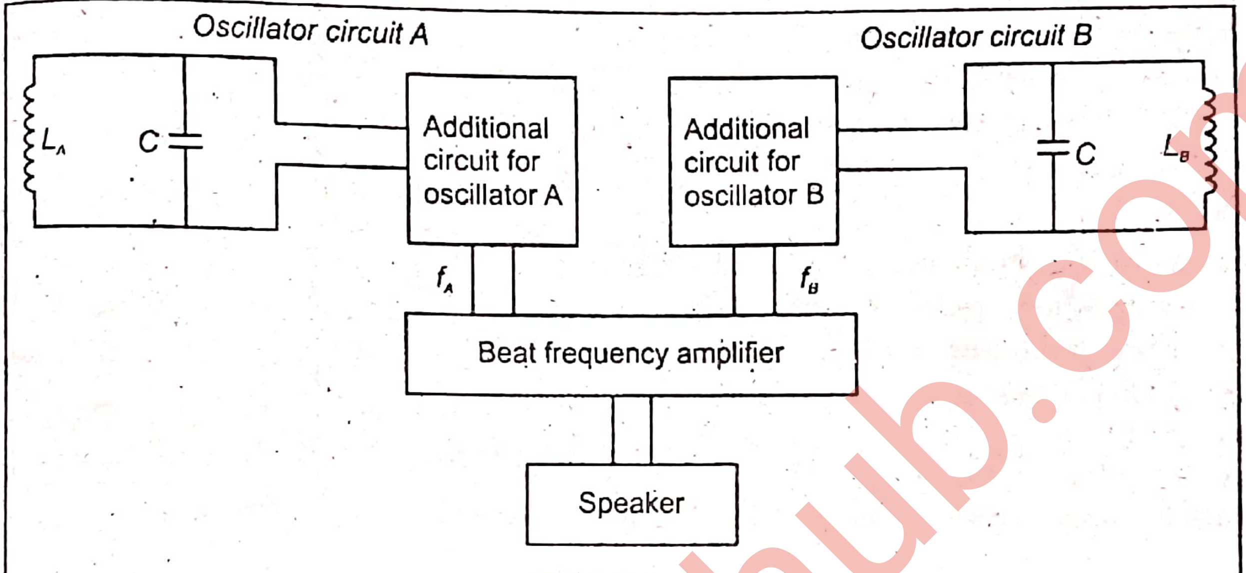
Answers Key

1. C	2. B	3. A	4. B	5. C	6. D	7. D	8. D	9. A	10. A

Q.9 Write down a not on principle of metal detector.

Principle of Metal Detectors:

A coil and capacitor are electrical components, which together can produce oscillations of current. An L-C circuit behaves just like an oscillating mass-spring system. In this case energy oscillates between a capacitor and an inductor. The circuit is called an electrical oscillator. Two such oscillators A and B are used for the operation of common type of metal detector. In the absence of any nearby metal object, the inductances L_A and L_B are the same and hence the resonance frequency of the two circuits is also same.



Metal detectors

When inductor B, called the search coil comes near a metal object the inductance LB decreases and corresponding oscillator frequency increases and thus a beat note is heard in the attached speaker. Such detectors are extensively used not only for various security checks but also to locate buried metal objects.

MCQ's From Past Board Papers

1. **In tuning circuit if capacitance is doubled and inductance is halved then its resonance frequency**
   (A) Doubled (B) Halved (C) Remain the same (D) Increases to 4 times
2. **In three phase voltage across any two live lines is about:**
   (A) 220 V (B) 230 V (C) 400 V (D) 430 V
3. **In three phase A.C supply coils are inclined at an angle of**
   (A) 0° (B) 90° (C) 120° (D) 180°
4. **Power factor is equal to:**
   (A) sin θ (B) tan θ (C) sec θ (D) cos θ
5. **Choke consumes extremely small:**
   (A) Current (B) Charge (C) Power (D) Potential
6. **The expression P = VI holds only when current and voltage are:-**
   (A) In phase (B) Out of phase (C) At right angle to each other (D) At angle of 120°
7. **When an inductor comes close to a metallic object, its inductance is:**
   (A) Decreased (B) Increased (C) Becomes half (D) Becomes 4 times
8. **Metal detector consist of**
   (A) L.C circuit (B) R.L circuit (C) R.C circuit (D) RLC series circuit
9. **In three phase A.C. supply, if first coil has phase 0°, then the other two coils will have phases:**
   (A) 0° and 120° (B) 120° and 240° (C) 240° and 360° (D) 0° and 360°
10. **Metal detector consists of**
    (A) L.C circuit (B) R.L circuit (C) R.C circuit (D) R.L.C series circuit
11. **In resonance circuit at resonance, the phase difference between current and voltage is ____.**
    (A) 90° (B) 180° (C) 0° (D) 360°

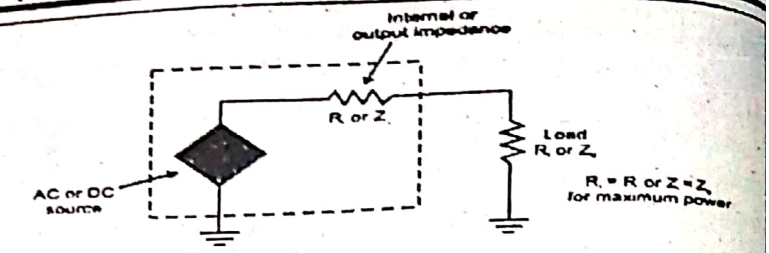
Answers Key

1. C	2. C	3. C	4. D	5. C	6. A	7. A	8. A	9. B	10. A	11. C

Q.10 Describe the maximum power transfer principle.

Maximum Power Transfer:

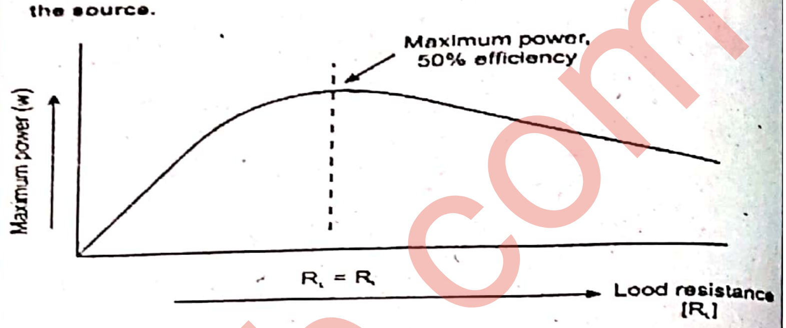
The maximum power transfer theorem says that to transfer the maximum amount of power from a source to a load, the load impedance should match the source impedance.



Maximum power is transferred from a source to a load when the load when the load resistance equals the internal resistance of the source.

In the basic circuit, a source may be AC or DC, and its internal resistance (R_i) or generator output impedance (Z_g) drives a load resistance (R_L) or impedance (Z_L) (figure (a)).

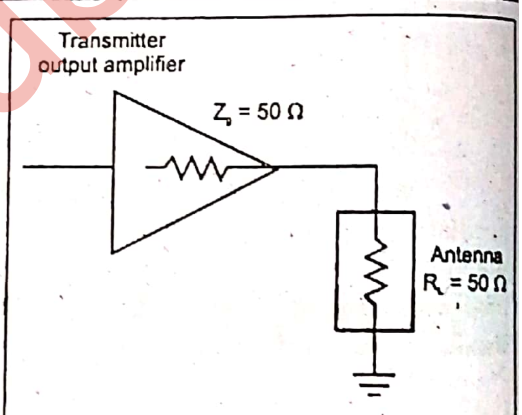
$$Z_L = Z_g$$



Varying the load resistance on a source shows that maximum power to the load is achieved by matching load and source impedances. At this time, efficiency is 50%.

A plot of load power versus load resistance reveals that matching load and source impedances will achieve maximum power (figure (b)).

A key factor of this theorem is that when the load matches the source, the amount of power delivered to the load is the same as the power dissipated in the source. Therefore, transfer of maximum power is only 50% efficient.



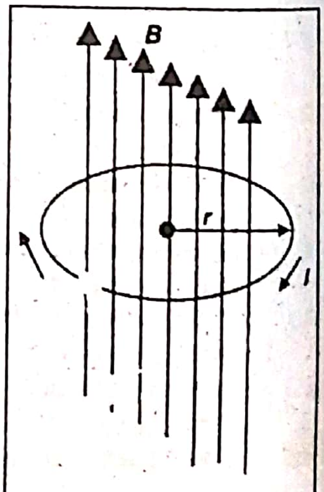
Antenna impedance must equal the transmitter output impedance to receive maximum power.

The source must be able to dissipate this power. To deliver maximum power to the load, the generator has to develop twice the desired output power. One of the important application s of maximum power is the delivery of maximum power to an antenna (figure(c)).

Q.11 Discuss the Maxwell's equations in detail.

Maxwell's Equations:

In the early days of 19^{th} century, two different units of electric charge were used, one for electrostatics and the other for magnetic phenomena involving currents. These two units of charge had different physical dimensions. Their ratio has units of velocity and measurements showed that the ratio had a numerical value that was precisely equal to the speed of light. It was regarded as an extraordinary coincidence that had no explanation. Maxwell in a search for an explanation of the coincidence found that all the basic principles of electromagnetism can be formulated in terms of four fundamental equations, now called Maxwell's equations. These euqations exists as experimental laws in the form of Gauss' law, Faraday's law and Ampere's law. These equations predict the existence of electromagnetic wave and that such waves are radiated by accelerating charges. For simplicity, we present Maxwell's equations as applied to free space. We know that changing magnetic flux density B through a certain region of space produces an induced emf in the region.



Changing magnetic field

So an induced current will flow in a closed loop of wire in the region as shown in Fig(a). According to Faraday's law the induced emf or the induced potential difference V is given by:

$$\varepsilon = \frac{\Delta \phi}{\Delta t}$$

$$\varepsilon = \frac{\Delta}{\Delta t}(B.A)$$

As potential difference is due to an electric field, it means an electric field will be generated at each point of the loop. BY symmetry E is circular in direction and constant in magnitude E at each point of the loop. If a unit positive charge is circulated once round the circular loop of radius r, the work done will be

$$W = 2\pi r F_e = 2\pi r(qE)$$

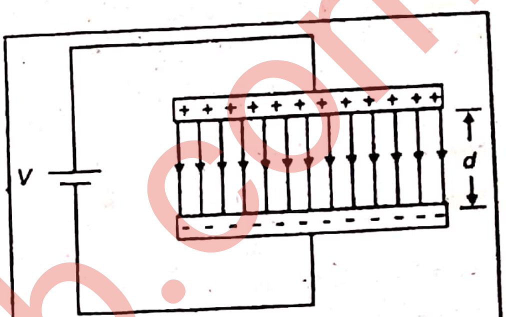
$$\varepsilon = \frac{W}{q} = 2\pi r E$$

By definition W will equal to the emf or V in the loop:

$$\varepsilon = \frac{\Delta\phi}{\Delta t} = 2\pi r E$$

$$E = \frac{1}{2\pi r}\frac{\Delta\phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t}$$

This equation shows that a changing magnetic flux gives rise to an electric field. Experiments have shown that the electric field produced by changing magnetic field is present even if the conducting loop is absent.



Uniform electric between a pair of charged plate.

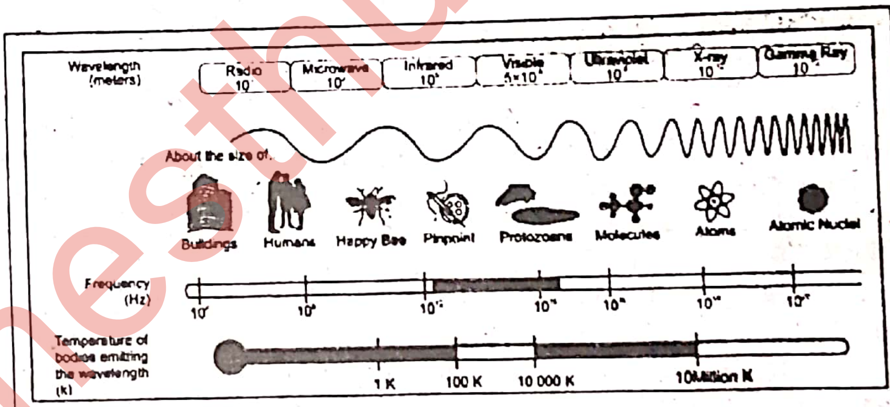
Analogues to changing magnetic flux it is found that a changing electric flux gives rise to a magnetic field. In order to arrive at this statement, let a capacitor be connected to a battery as arranged in figure. Current starts growing in the circuit but very quickly decreases to zero when the capacitor is fully charged. An electric is established between the plates of the capacitor.

The charge Q on an air filled capacitor of capacitance C, plate area A, plate separation d and potential difference V is given by

$$Q = CV$$

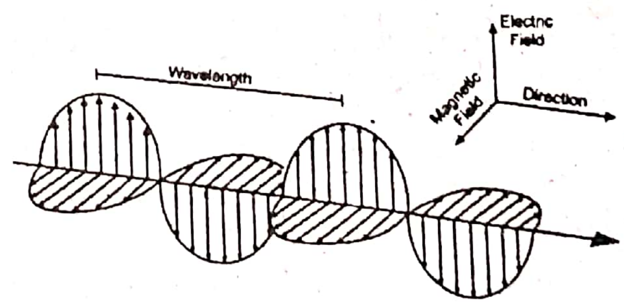
$$Q = \frac{\varepsilon_0 A}{d} V$$

$$Q = \varepsilon_0 AE \qquad \therefore \frac{V}{d} = E$$



Electromagnetic spectrum

In figure the current through the dielectric of the capacitor is due to changing electric field. Suppose the capacitor is now connected to an alternating emf source. It is observed that the current flows continuously in the circuit. It does not stop. Why is to so different than the case of the battery! The reason is that outside the capacitor the current in the wires is due to the conduction electrons but what is the entity that drives that current in the dielectric between the plates of the capacitor. We note that in the present case the E-Field between the plates of the capacitor changes with time. There exists $\Delta E/\Delta t$. It was first conceived by Maxwell that the change in the electric field is the cause of current in the capacitor.



EM waves

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta(\varepsilon_0 AE)}{\Delta t}$$

$$\frac{\Delta Q}{\Delta t} = \frac{\varepsilon_0 \Delta(AE)}{\Delta t}$$

$$I = \frac{\varepsilon_0 \Delta(\phi)}{\Delta t}$$

Where I is the current and ϕ is the electric flux through the area A. Changing electric flux is equivalent to a current. This type of current, which is due to changing electric flux is called displacement current.

We must extend our concept about current. A current can arise due to the flow of charges and also due to changing electric flux. The former is called conduction current and the later is called displacement current. According to Ampere's law, each type of current produces magnetic field around itself. We have thus shown that a changing E-field creates a B-field.

A changing E-field creates a B-field, which in turn creates an E-field, an electromagnetic disturbance or waves are generated. The fundamental requirement for generation of electromagnetic waves is an electric charge with changing velocity (acceleration) since it will create changing electric flux. The velocity of an oscillating charge as it moves to and fro along a wire is always changing.

Light is a type of electromagnetic waves. Maxwell predicted theoretically that the velocity of electromagnetic waves in free space is given by

$$c=\frac{1}{\sqrt{\varepsilon_o\mu_o}}=\frac{1}{\sqrt{8.85\times10^{-12}\times4\pi\times10^{-7}}}=3\times10^{8}ms^{-1}$$

Where $\varepsilon_o=8.85\times10^{-12}\ C^2N^{-1}m^{-2}$ is the permittivity of free space and $\mu_o=4\pi\times10^{-7}\ Wb\ A^{-1}m^{-1}$ is the permeability of free space. Put these values in the above relation we find that the velocity of lights is given by

Faraday's law shows that a changing magnetic field gives rise to an electric field. Ampere-Maxwell law shows that a changing electric field gives rise to a magnetic field. It follows that when either electric or magnetic field varies with time, the other field is induced in space. The net effect is that an electromagnetic disturbance is generated due to changing electric and magnetic fields. The disturbance propagates in the form of an electromagnetic waves.

Q.12 Describe the electromagnetic spectrum in detail.

Electromagnetic Wave:

Electromagnetic waves have some common properties such as electric field and magnetic field. Therefore it can be described in terms of electric and magnetic fields and they all travel through vacuum with the same speed equal to speed of light. These radiations are fundamentally differentiated by their wavelength or frequency. The names given to the in figure show the different regions of the spectrum along with given names. There are no gaps in the spectrum, nor their sharp boundaries between the different regions.

1. Light:

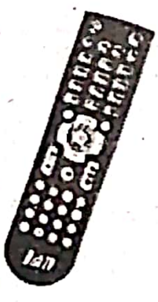
The visible region of the spectrum is most familiar to us, is the electromagnetic radiation emitted by the Sun. The wavelength of the visible region ranges from about 400 nm (violet) to about 700nm (red).

Light is often emitted when the outer electrons in atoms de-excites, such transitions are called optical transitions. The color of the light tells us about nature of the atoms from which it was emitted, so the study of the light emitted from the Sun and from distant stars gives information about their compositions.

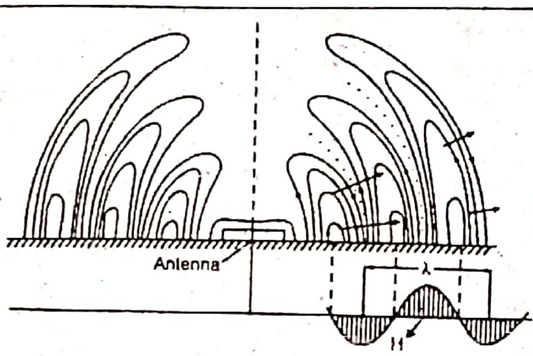
2. Infrared:

Infrared radiations have wavelength stronger than the visible (from 0.7 μm to about 1 mm). They are commonly emitted by atoms or molecules when they change their rotational or vibrational motion. Infrared radiation is an important means of heat transfer and is sometimes called heat radiation. The warmth you feel when you place your hand near a glowing light bulb is basically the result of the infrared radiation emitted from the bulb.

All objects emit thermal radiation because of their temperature. Objects of temperatures ranges form, 3 K to 3000 K emit their most intense thermal radiation in the infrared region of the spectrum.



Remote

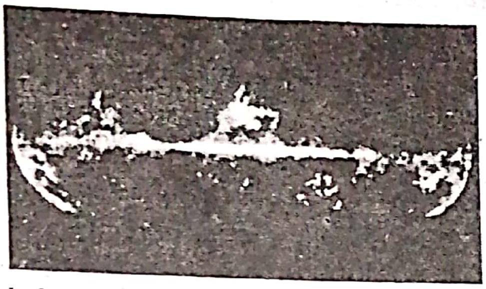


Radio waves propagation

The main technology used in home remote controls in infrared light. The signal between a remote control handset and the device it controls consists of pulses of infrared light, which is invisible to the human eye. Infrared radiation is also used for cooking the surface of food (the interior is then heated by convection and conduction).

3. Microwaves:

Microwaves can be regarded as short radio waves, with typical wavelengths in the range 1 mm to 1 m. They are commonly produced by electromagnetic oscillators in electric circuits, as in the case of microwave ovens. Microwaves are often used to transmit telephone conversations: figure shows a Microwaves station that serves to relay telephones calls. Microwaves also reach us from extraterrestrial sources.



Infrared image of Milky Way galaxy

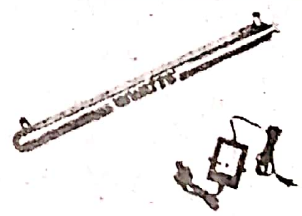
Microwave relay station

Neutral hydrogen atoms, which populate the regions between the stars in our galaxy, are common extraterrestrial source of microwaves emitting radiation with a wavelength of 21 cm.

4. Radio waves:

Radio waves have wavelengths longer than 1 m. They are produced form terrestrial sources through electrons oscillating in wires of electric circuits. By carefully choosing the geometry of these circuits, as in an antenna, we can control the distribution in space of the emitted radiation (if the antenna acts as a transmitter) or the sensitivity of the detector (if the antenna acts as a receiver). Traveling outward at the speed of light, the expanding of TV signals transmitted on Earth.

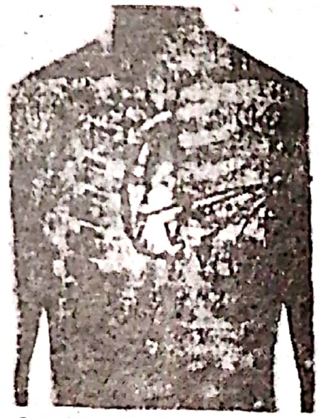
Radio waves reach us form extraterrestrial sources, the sun being a major source that often interferes with radio or TV reception on Earth. Mapping the radio emissions from extraterrestrial sources, known as radio astronomy, has provided information about the universe that is often not obtainable using optical telescopes.



ultraviolet lamp

5. Ultraviolet:

The radiations of wavelengths shorter than the visible being with the ultraviolet (1nm to 400nm), which can be produced in atomic transitions of the outer-electrons as well as in radiation from thermal sources such as the Sun. Because our atmosphere absorbs strongly at ultraviolet wavelengths, little of this radiation from the Sun reaches the ground. Which is mostly done by ozone, which has been depleted in recent years as a result of chemical reactions with fluorocarbons released from aerosol sprays, refrigeration equipment, and other sources, Brief exposure to ultraviolet radiation cause common sun burn but long-term exposure can lead to more serious effects, including skin cancer.



Sensitive electrodes are place on certain parts of the body.

A lamp producing Ultraviolet (UV) radiation is emitted through clear, pre-filtered, particle free water. This UV light is extremely effective in killing and eliminating bacteria, yeast's, viruses, molds and other harmful organisms known to man. Used in industry and hospitals to treat water. Many times used as a post disinfecting method for residential water treatment.

6. X-rays:

X rays (typical wavelengths 0.01 nm to 10 nm) can be produced with discrete wavelengths in individual transitions among the inner (most tightly bound) electrons of an atom, and they can also be produced when charged particle (such as electrons) are decelerated. X rays can easily penetrate soft tissue but are stopped by bone and other solid matter: for this reason they have found wide use in medical diagnosis.

7. Gamma rays:

Gamma rays are electromagnetic radiations with the shortest wavelengths (less than 10 pm). They are the most penetrating of electromagnetic radiations, and exposure to intense gamma radiation can have a harmful effect on the human body. These radiations can be emitted in transitions of an atomic nucleus from one state to another and can also occur in the decays of certain elementary particles: for example, a neutral pion can decay into two gamma rays according to $\pi^0 \rightarrow \gamma + \gamma$

Q.13 What is electrocardiography? Describe is working in detail.

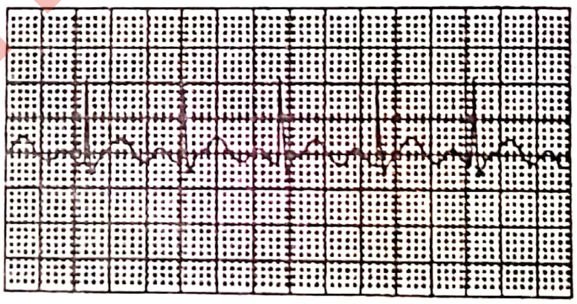
Electrocardiogram (E.C.G):

The electrocardiogram or ECG is worldwide used for diagnosing heart conditions. An electrocardiogram is a recording of the small electric waves being generated during heart activity.

The electric activity starts at the top of the heart and spreads down. A normal heart beat is initiated by a small pulse of electric current. This tiny electric "shock" spreads rapidly in the heart and makes the heart muscle contract.

If the whole heart muscle contracted at the same time, there would be no pumping effect. Therefore the electric activity starts at the top of the heart and spreads down, and then up again, causing the heart muscle to contract in an optimal way for pumping blood. The electric waves in the heart are recorded in milli-volts by the electrocardiograph.

Our heart produces time-varying voltages as it beats. These heart voltages produce small voltage difference between points on your skin that can be measured and used to diagnose the condition of your heart. The waves are registered by electrodes placed on certain parts of the body. Which are then printed on paper in form of a curve as shown in fig:(b).

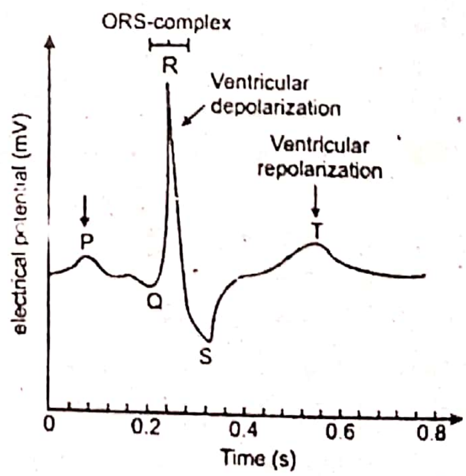


Some of the characteristics of ECG curve are shown in fig(b): When the curve falls below the base line it shows a negative deflection and when it rises above the base line it shows a positive deflection.

A negative deflection indicates that recorded wave has traveled away from the electrode and a positive deflection means it has traveled towards it.

The small rise and fall in the voltage between two electrodes placed either side of the heart which is displayed as a wavy line either on a screen or on paper.

A typical plot shown in fig(b), the P deflection corresponds to the contraction of the atria at the start of the heartbeat. The QRS group corresponds to the contraction of the ventricles. The T deflection corresponds to a re-polarization or recovery of the heart cells in preparation for the next beat.



An ECG curve reflects the perspective of the electrode recording it.

MCQ's From Past Board Papers

1. When we accelerate the charges, which type of waves are produced
(A) Mechanical waves (B) Travelling waves (C) Stationary waves (D) electromagnetic wave

2. The velocity of an oscillating charge as it moves to and fro along a wire is:
(A) Changing (B) Constant (C) Infinite (D) Zero

3. Electromagnetic waves do not transport ________.
(A) Energy (B) Momentum (C) Charge (D) Information

4. The process of combining the low frequency signal with high frequency carrier waves is called ____.
(A) Wave transmission (B) Modulation (C) Resonance (D) Beats

5. In modulation, low frequency signal is known as:
(A) Loaded signal (B) Fluctuated signal (C) Harmonic signal (D) Modulation signal

6. The A.M. transmission frequencies range from:
(A) 540 KHz to 1000 kHz (B) 540 KHz to 1600 kHz (C) 520 KHz to 1600 kHz (D) 540 KHz to 1400 kHz

7. Which one of the following is the velocity of carrier wave:
(A) $3 \times 10^6 ms^{-1}$ (B) $3 \times 10^8 ms^{-1}$ (C) $3 \times 10^9 ms^{-1}$ (D) $3 \times 10^{10} ms^{-1}$

8. Which one is in the order of decreasing frequency?
(A) X-rays, radio waves, infrared rays (B) Ultraviolet rays, visible light, radio waves
(C) Infrared rays, visible light, x-rays (D) Yellow, green, red

9. The net reactance of a circuit is zero. The circuit may consist of
(A) An inductor only (B) A capacitor only (C) Both inductor and capacitor (D) None of these

10. Which one of the following requires a material medium for their propagation?
(A) Heat waves (B) X–rays (C) Sound waves (D) Ultraviolet rays

11. The range of F.M transmission frequencies is from:-
(A) 540 KHz to 1600 Khz (B) 88 KHz to 108 KHz (C) 88MHz to 108 MHz (D) 540 MHz to 1600 MHz

12. High frequency radio wave is called as:-
(A) Fluctuative wave (B) Carrier wave (C) Matter wave (D) Mechanical wave

13. Electromagnetic waves emitted from radio antenna are:
(A) Stationary (B) Longitudnal (C) Transverse (D) Both A & B

14. Electrons vibrating 94.000 times each second will produce radio waves of frequency
(A) 94 Hz (B) 940Hz (C) 940KHz (D) 94KHz

15. In frequency modulation, which factor is changed?
(A) Amplitude of carrier waves (B) Frequency of carrier wave
(C) Amplitude f signal (D) Frequency of signal

16. A changing electric flux creates
(A) Electric field (B) gravitational field (C) magnetic field (D) electric charge

Answers Key

1. D	2. A	3. C	4. B	5. D	6. B	7. B	8. B	9. C	10. C	11. C	12. B
13. C	14. A	15. A	16. C								

FORMULAE

1	Relation between frequency and time period of AC	$f = \frac{1}{T}$	$f \times T = 1$	
2	Instantaneous value of AC voltage	$V = V_o \sin\frac{2\pi}{T}t$	$V = V_o \sin\omega t$	$V = V_o \sin 2\pi ft$
3	Instantaneous value of AC current	$I = I_o \sin\frac{2\pi}{T}t$	$I = I_o \sin\omega t$	$I = I_o \sin 2\pi ft$
4	Root mean square value of AC voltage	$V_{rms} = \frac{V_o}{\sqrt{2}}$	$V_{rms} = 0.707V_o$	
5	Root mean square value of AC current	$I_{rms} = \frac{I_o}{\sqrt{2}}$	$I_{rms} = 0.707I_o$	
6	Instantaneous power in pure resistive circuit	$P = VI$	$P = I^2R$	$P = \frac{V^2}{R}$
7	Charge on capacitor on any instant	$q = CV$	$q = CV_o \sin\omega t$	
8	Reactance of a capacitor	$X_C = \frac{V_{rms}}{I_{rms}}$	$X_C = \frac{1}{\omega C}$	$X_C = \frac{1}{2\pi fC}$
9	Reactance of an inductor	$X_L = \frac{V_{rms}}{I_{rms}}$	$X_L = \omega L$	$X_L = 2\pi fL$
10	Impedance	$Z = \frac{V_{rms}}{I_{rms}}$		
11	Impedance of RC-circuit	$Z = \sqrt{R^2 + X_C^2}$	$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$	$Z = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$
12	Impedance of RL-circuit	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + (\omega L)^2}$	$Z = \sqrt{R^2 + (2\pi fL)^2}$
13	Phase difference (θ) between voltage and current for RC-circuit	$\theta = \tan^{-1}\frac{X_C}{R}$	$\theta = \tan^{-1}\frac{1}{\omega CR}$	$\theta = \tan^{-1}\frac{1}{2\pi fCR}$

14	Phase difference (θ) between voltage and current for RL-circuit	$\theta = \tan^{-1}\frac{X_L}{R}$	$\theta = \tan^{-1}\frac{\omega L}{R}$	$\theta = \tan^{-1}\frac{2\pi f L}{R}$
15	Power in AC circuits, when current and voltage are in phase	$P = V_{rms}I_{rms}$		
16	Power dissipated in AC circuits	$P = V_{rms}I_{rms}\cos\theta$		
17	Resonance frequency in RLC-series AC- circuit	$\omega_r = \frac{1}{\sqrt{LC}}$	$f_r = \frac{1}{2\pi\sqrt{LC}}$	
18	Resonance frequency in LC-parallel AC-circuit	$\omega_r = \frac{1}{\sqrt{LC}}$	$f_r = \frac{1}{2\pi\sqrt{LC}}$	

UNITS

1	Reactance	VA^{-1}	Ω	
2	Impedance	VA^{-1}	Ω	

Key Points

- *A voltage which changes its polarity at regular interval of time is called an alternating voltage.*
- The sinusoidal alternating voltage can be expressed by the equation:

  $V = Vm\ sin\omega t$

- The shape of the curve obtained by plotting the instantaneous value of voltage of current as ordinate against time as abscissa is called is waveform or wave shape.
- One complete set of positive and negative values of an alternating quantity is known as a cycle.
- The time taken in second to compete one cycle of alternating quantity is called its time period. It is generally represented by T.
- The number of cycle that occurs in one second is called the frequency (*f*) of the alternating quantity.
- *The average value of a waveform is the average of all its values over a period of time.*
- *The effective or r.m.s. value of a n alternating current is that steady current (d.c) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.*
- The equation of the alternating current varying sinusoidally is given by:

  $I = I_m\ sin\omega t$

- When two alternating quantities of the same frequency have different zero point, they are said to have a phase difference.
- Sinusoidal alternating voltage or current is represented by a line of definite length rotating in counter clock wise direction at a constant angular velocity (ω). Such a rotating line is called a phasor.
- The phasor representation enables us to quickly obtain the numerical value and at the same time as the events taking place in the circuit.
- The applied voltage and current across resistor and phase with each other. As they pass through their zero values at the same instant and attain their positive and negative peaks at the same instant.
- When an alternating current flows through a pure inductive coil then the current lags behind the voltage by $\pi/2$ radians or 90°. The opposition offered by an inductor to the flow of charges is called inductive reactance X_L.
- When an alternating voltage is applied across the plates of a capacitor then the current is leading the voltage by $\pi/2$ radians or 90°.

Solved Examples

Example 15.1:

An A.C. circuit consists of a pure resistance of 20 Ω and is connected across A.C. supply or 220V, 50Hz. Calculate (a) current (b) power consumed and (c) equation for voltage and current.

Solution:

Resistance $R = 20\Omega$

Voltage $V = 220V$

Frequency $f = 50\ Hz$

Maximum value of an alternating voltage is $V_m = \sqrt{2}V = \sqrt{2} \times 220 = 311.1V$

(a) Current, I = V/R = 220/20 = 11A

(b) Average power P dissipated in the resistor is

$$P = VI = 220 \times 11 = 2420 \text{ w}$$

Maximum value of an alternating current $I_m = \sqrt{2}I = \sqrt{2} \times 11 = 15.55\,A$

$$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ rad s}^{-1}$$

(c) Equations for voltage and current is:

$V = V_m \sin(\omega t)$ & $I = I_m \sin(\omega t)$

Putting values

V = 311.1 sin (314t) & I = 15.55 sin (314t)

Example 15.2:

A pure inductive coil allow a current of 20A to flow from a 220 V, 50Hz supply Find (1) inductive reactance (2) inductance of the coil (3) power absorbed. Write down the equation for voltage an current.

Solution:

Current $I = 20$A, Voltage $V = 220$ V

Frequency f = 50 Hz

1. Circuit current $I = V/X_L$

∴ Inductive reactance, $X_L = V/I = 220/20 = 11\,\Omega$

2. Now, $X_L = 2\pi f L$

$$\therefore L = \frac{X_L}{2\pi f} = \frac{11}{2\pi \times 50} = 0.035\text{H}$$

3. Power absorbed = Zero

$V_M = 220 \times \sqrt{2} = 311.1\text{V}$ $I_m = 20 \times \sqrt{2} = 28.28\,\text{A};$

$\omega = 2\pi \times 50 = 314\,rad\,s^{-1}$

Example 15.3:

The current through an 60 mH inductor is 0.2 sin (377t –25°)A. Write the mathematical espression for the voltage across it.

Solution:

Inductance L = 60 m H,

Current I = 0.2 sin (377t –25°)A

Mathematical expression for the voltage V = ?

Inductive reactance, $X_L = 2\pi fL = 377 \times 6010^{-3} = 22.62\Omega$

Maximum value of an alternating $V_m = I_m X_L = 0.2 \times 22.62 = 4.5\text{V}$

Since the voltage leads the current by 90° therefore 90° is added to the phase angle of voltage.

$V = V_m \sin(377t - -25° + 90°)$ $V = 4.5\sin(377t + 65°)\text{V}$

Example 15.4:

A318 μF capacitor is connected across a 220V, 50Hz system. Determine (a) the capacitive reactance (b) RMS value of current and (c) equations for voltage and current.

Solution:

Capacitance, C=318 μF=318×10^{-6}F

Voltage V=220V

Frequency f = 50 Hz

(a) Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 318 \times 10^{-6}} = 10.26\Omega$

(b) RMS value of current, $I = V / X_C = 220 / 10.26 = 21.44A$

(c) maximum value of an alternating voltage & current is

$V_m = 220 + \sqrt{2} = 311.1V$

$I_m = I \times \sqrt{2} = \sqrt{2} \times 21.44 = 30.32A,$

Frequency $\omega = 2\pi \times 50 = 314Hz$

∴ Equation for voltage and current are:

$$V = V_m \sin(\omega t)$$

$$\& \quad I = I_m \sin(\omega t + \pi / 2)$$

Putting values

$V = 311.1 \sin 314t,$

$I = 30.32 \sin(314t + \pi / 2),$

Example 15.5:

A coil having a resistance of 7 Ω and an inductance of 31.8 mH is connected to 220V, 50Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor and (d) power consumed.

Solution:

Inductance, $L = 31.8\,mH = 31.8 \times 10^{-3} H$

Voltage V = 220 V

Coil resistance $R = 7\Omega$

Frequency f = 50 Hz

(a) Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\Omega$

Coil impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\Omega$

Circuit current, $I = V/Z$

$= 220 / 12.2 = 18.03A$

(b) $\tan\phi = X_L / R = 10/7$

∴ Phase angle, $\phi = \tan^{-1}(10/7) = 55^0$ lag

(c) Power factor $= \cos\phi = \cos 55^0 = .573$ lag

(d) Power consumed $P = VI\cos\phi$

$= 220 \times 18.03 \times 0.573 = 2272.8W$

Example 15.6:

A 100V, 50Hz a.c. supply is applied to a capacitor of capacitance 79.5 μF connected in series with a non-inductive resistance of 30 Ω. Find (1) impedance (2) current (3) phase angle and (4) equation for the instantaneous value of current.

Solution:

Voltage V = 100 V

Resistance R = 30 Ω

Frequency $f = 50$ Hz
Capacitance $C = 79.5\,\mu$ F

(1) Capacitive reactance, $X_C \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 5079.5} = 40\Omega$

Circuit impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50\Omega$

(2) Circuit current, $I = V/Z = 100/50 = 2$ A

(3) $\tan\phi = Xc / R = 40/30 = 1.33$

$\therefore$ Phase angle $\phi = \tan^{-1} 1.33 = 53°$ *lead*

$I_m = I \times \sqrt{2} \Rightarrow I_m = 2\sqrt{2} = 2.828\text{A}$

$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ rad s}^{-1}$

(4) equation for current is $I = 2.828 \sin(314t + 53°)$

xample 15.7:

A 220V, 50Hz A.C. supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected ı series with a 6.8 μ F capacitor. Calculate (a) impedance (b) current (c) phase angle between current nd voltage (d) power factor and Є power consumed.

olution:

$X_L = 2\pi f L = 2\pi \times 50 \times .06 = 18.85\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\Omega$

ı) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(2.5)^2 + (18.85 - 468)^2} = 449.2\Omega$

) Circuit Current, $I = V/Z = 220/449.2 = 0.4897\,\text{A}$

) $\tan\phi = \frac{X_L - X_C}{R} = \frac{18.85 - 468}{2.5} = -179.66$

Phase angle, $\phi = \tan^{-1}(-179.66) = -89.7°$

The negative sign with ϕ shows that current is leading the voltage

) Power factor, $\cos\phi = \frac{R}{Z} = \frac{2.5}{449.2} = 0.00557$

) Power consumed, $P = VI \cos\phi = 220 \times 0.4897 \times 0.00557 = 0.60007\text{w}$

Text Book Exercises

Q.1 Select the correct answer of the following questions.

(i) The AC system is preferred to DC system because:

(a) AC voltage can be easily changed in magnitude
(b) DC motor angular velocity is effected badly
(c) High voltage AC transmission is less efficient
(d) Domestic appliances require AC voltage for their operation

(ii) A capacitor is perfectly insulator for:

(a) direct current (b) alternating current
(c) direct as well as alternating current (d) none of these

(iii) The peak value of alternating current is $5\sqrt{2}$ A. The mean square value of current will be:

(a) 5A, (b) 2.5A, (c) $5\sqrt{2}$A (d) 5_2 A

(iv) In choke coil the reactance X_L and resistance R are:

(a) $X_L = R$ (b) $X_L = << R$ (c) $X_L >> R$ (d) $X_L = \infty$

(v) In an LRC circuit, the capacitance is made one-fourth, when in resonance. Then what should be change in inductance, so that the circuit remains in resonance?

(a) 4 times (b) 1/4 times (c) 8 times (d) 2 times

(vi) In AC system we generate sine wave form because:

(a) It can be easily draw (b) It produces least disturbance in electrical circuit
(b) It is nature standard (d) Other waves cannot be produced easily

(vii) The phase difference between the current and voltage at resonance is:

(a) 0 (b) π (c) $-\pi$ (d) $\pi/2$

(viii) An alternating voltage is given by 20 sin 157 t. The frequency of alternating voltage is:

(a) 50Hz (b) 25Hz (c) 100Hz (d)75Hz

(ix) In LR current which one of the following statement is correct?

(a) L and R oppose each other
(b) R value increases with frequency
(c) The inductive reactance decreases with frequency
(b) The inductive reactance decreases with frequency

(x) An alternating quantity (voltage or current) is completely known if we know it's:

(a) maximum value (b) frequency and phase (c) effective value (d) both (a) & (b)

(xi) For electromagnetic waves, Maxwell generalized:

(a) Gausses law for magnetism (b) gausses law for electricity
(c) faradays law (d) amperes law

(xii) An electromagnetic wave goes from air to glass which of the following does not change?

(a) Radio waves (b) X-rays (c) Ultra violet radiation (d) Ultra sound waves

(xiii) The circuit in which current and voltage are in phase, the power factor is:

(a) Zero (b) 1 (c) –1 (d) 2

No.	Option	ANSWER	EXPLANATION
(i)	(a)	AC voltage can be easily changed in magnitude	A.C can be step up or step down by mean of transformer
(ii)	(a)	direct current.	For D.C f = 0 So $X_c = \frac{1}{2\pi fc} = \frac{1}{0} = \infty$
(iii)	(d)	5^2 A	$I_o = 5\sqrt{2}$ And $I_{rms} = \sqrt{<I^2>} = \frac{I_0}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}}$ $<I^2> = 5^2$
(iv)	(c)	$X_L >> R$	Thick copper wire coil so it have large reactance but small resistance
(v)	(a)	4 times	By $f_r = \frac{1}{2\pi\sqrt{LC}}$ As capacitance become $\frac{1}{4}$th so in order to make f_r constant, the inductance must be four times
(vi)	(b)	It produces least disturbance in electrical circuits	Because their effective value (rms value) of A.C is equal to same amount of DC.
(vii)	(a)	0	V and I are in phase at resonance
(viii)	(b)	25Hz	V = 20 sin 157 t As V = Vo sin ω t So $\omega t = 157t$ $2\pi f = 157 \Rightarrow 2 \times 3.14 \times f = 157$ $f = 25$ Hz
(ix)	(c)	The inductive reactance decreases with frequency	$X_L = 2\pi fL$ $X_L \propto f$
(x)	(d)	both (a) & (b)	Because effective value may also be calculated by it maximum value
(xi)	(d)	amperes law	Maxwell is responsible for important generalization of Ampere's law.
(xii)	(d)	Ultra sound waves	No option but (d) can be marked among given, as it is not electromagnetic wave
(xiii)	(b)	1	As V and I are in phase so $\theta = 0^\circ$ power factor = cos θ = $\cos 0^\circ = 1$

Comprehensive Questions

Q.2 Write short answers of the following questions.

1. Explain with diagrams sinusoidal alternating voltage and sinusoidal alternating current.

Ans: See Theory Question No. 1

2. Define mean, peak and rms value of sinusoidal current and sinusoidal voltage. Obtain mathematical expression for the rms value of current.

Ans: See Theory Question No. 2

3. A sinusoidal alternating voltage of angular frequency ω is connected across a resistor R. F mathematical expression for instantaneous voltage instantaneous current and the aver dissipated per cycle of the applied voltage.

Ans: See Theory

4. A sinusoidal alternating voltage of angular frequency to ω is connected across a capacito Find mathematical expression for instantaneous voltage instantaneous current and the aver power dissipated per cycle of the applied voltage.

Ans: See Theory

5. A sinusoidal alternating voltage of angular frequency ω is connected across an inductor inductance *L* and resistance *R*. Find mathematical expression for instantaneous volta instantaneous current, average power dissipated per cycle of the applied voltage and dr power curve.

Ans: See Theory

6. Explain the term impedance of an AC circuit. Find expression for the of the *RLC* series circuit

Ans: See Theory Question No. 6

7. Describe that maximum power is transferred when the impedances of source and load match each other.

Ans: See Theory Question No. 10

8. In an *RL* series circuit will the current lag or lead the applied alternating voltage? Explain t answer with a phasor diagram.

Ans: See Theory Question No. 6

9. In an *RC* series circuit will the current lag or lead the applied alternating voltage? Explain th answer with a phasor diagram.

Ans: See Theory Question No. 7

10. What do you mean by the term phasor diagram? Why we use it for a sinusoidal current an voltage?

Ans: See Theory Question No. 1

11. Explain the resonance of a series *RLC* circuit. Show that resonance occurs at a frequenc determined by: $f_r = \frac{1}{2\pi\sqrt{LC}}$

ns: See Theory Question No. 8

2. Describe that maximum power is transferred when the impedances of source and load match t each other.

ns: See Theory Question No. 10

3. Describe the statement of four Maxwell's Equations. Use Maxwell's theory to show tha $E = \frac{1}{2\pi r}\frac{\Delta\Phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t}$ and $I = \frac{\varepsilon_o(\Phi)}{\Delta t}$

ns: See Theory Question No. 10 & 11

4. What is the principle of ECG? Sketch the wave curve of heart beats and explain the term positive deflection and negative deflection.

ns: See Theory Question No. 13

5. Describe the principle of metal detectors with suitable diagram.

ns: See Theory Question No. 9

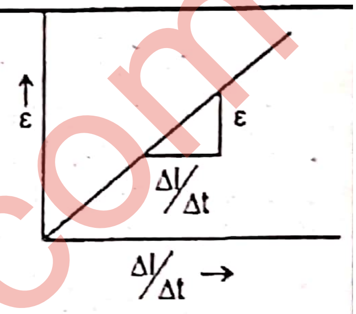
16. What is ECG and its principle, by sketching its curve identify the terms positive deflection, negative deflection, P deflection, QRS and T deflection?

Ans: See Theory Question No: 13

Conceptual Questions

1. (a) Sketch a graph of e.m.f. induced in an inductive coil rate of change of current. What is the significance of the gradient?

Ans: The graph will be straight line



Explanation:

As the self induced emf can be mathematically expressed as

$$\varepsilon = L\frac{\Delta I}{\Delta t}$$

Or $\varepsilon \propto \frac{\Delta I}{\Delta t}$

As induced emf and the time rate of change in current are directly proportional. So the graph between them is a inclined straight line

Signification:

The slope of graph between ε and $\frac{\Delta I}{\Delta t}$ gives the value of self inductance L as

$$\text{Slope} = \frac{\varepsilon}{\Delta I/\Delta t}$$

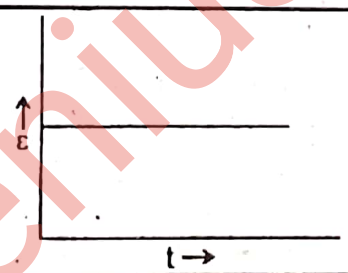
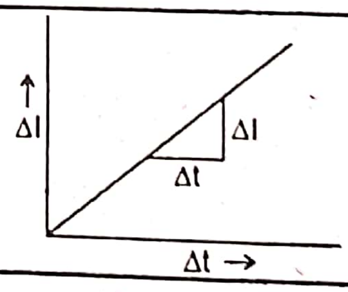
i.e. $L = \frac{\varepsilon}{\Delta I/\Delta t}$ (VsA^{-1})

1. (b) Explain why it is difficult to measure the rate of change of current?

Ans: Current is sometimes a little harder to measure then voltage because an ammeter usually needs to be inserted into a break in the line carrying the current i.e. in series rather then just connected across two point in a circuit as just done for voltage. But it can be easily measured by its magnetic field around the current carrying conductor.

1. (c) How do graphs of e.m.f. against time and current against time make it possible to measure self-inductance?

Ans:

	
ε = constant	$\frac{\Delta I}{\Delta t}$ = constant

So the ratio of these two constants is also a constant which is known is self inductance

Of $$L = \frac{\varepsilon}{\frac{\Delta I}{\Delta t}}$$

2. (a) Current and voltage provided by an AC generator are sometimes negative and sometimes positive. Explain why for, and AC generator connected to a resistor, power can never be negative? (b) Explain, using sketch graphs, why the frequency of variation of power in an AC generator is twice as that of the current and voltage.

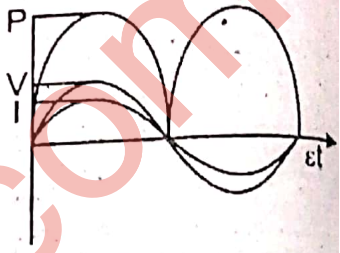
Ans: a) An A.C source is connected with a resistor, then the expression for average power at any instant is equal to the product of I_{rms} and V_{rms}. Therefore the power dissipated in a resistor can be express as (V_{rms} and I_{rms} are in phase so power factor $\cos\theta = \cos 0 = 1$)

$$<P> = V_{rms} I_{rms}$$

$$<P> = I_{rms}^2 R = \frac{V_{rms}^2}{R} \quad (\because V_{rms} = I_{rms} R)$$

Thus from above expression, it is clear that power dissipation in a resistor can never be negative.

b) The mean power can be written as



$$<P> = V_{rms} I_{rms}$$

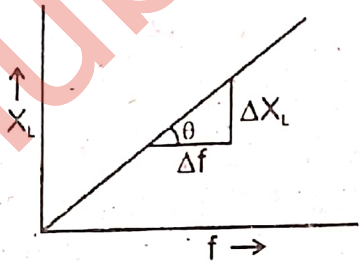
$$<P> = \frac{V}{\sqrt{2}} \times \frac{I}{\sqrt{2}}$$

$$<P> = \frac{VI}{2}$$

$$2<P> = VI$$

3. What determines the gradient of a graph of inductive reactance against frequency?

Ans: As inductive reactance can be expressed as



$$X_L = \omega L$$

Or $X_L = 2\pi f L$ $\quad (\omega = 2\pi f)$

So $X_L \propto f$ (linearly proportional)

Hence the graph between X_L and f is a straight line.

The gradient or slope of graph is expressed as

$$\text{Slope} = \frac{\Delta X_L}{\Delta f} \longrightarrow (1)$$

As $X_L = 2\pi f L$

Or $\frac{X_L}{f} = 2\pi L$

So equ (1) becomes

$$2\pi L = \frac{\Delta X_L}{\Delta f}$$

Or $L = \frac{1}{2\pi} \frac{\Delta X_L}{\Delta f}$

Or $L = \frac{\text{gradient}}{2\pi}$ using its gradient, we can measure self inductance of coil.

4. How does doubling the frequency affect the reactance of (a) an inductor (b) a capacitor?

Ans: a) As Inductive reactance can be expressed as

$$X_L = 2\pi f L$$

And $X_L' = 2\pi f' L \quad \because f' = 2f$

$$X_L' = 2\pi(2f) L$$

$$X_L' = 2(2\pi f L)$$

$$X_L' = 2X_L$$

So inductive reactance become doubled

b) As Capacitive reactance can be expressed as

$$X_C = \frac{1}{2\pi fC}$$

And $X_C' = \frac{1}{2\pi f'C}$ $\because f' = 2f$

$$X_C' = \frac{1}{2\pi(2f)C}$$

$$X_C' = \frac{1}{2}\left(\frac{1}{2\pi fC}\right)$$

$$X_C' = \frac{1}{2}X_C$$

So capacitive reactance become halved.

5. If the peak value of a sine wave is 1,000 volts, what is the effective (E_{eff}) value?

Ans: Given data: $V_o = 1000$ volts

To find: $V_{rms} = ?$

Solution: $V_{rms} = \frac{V_o}{\sqrt{2}}$

$= 0.707\ V_0$

$= 0.707 \times 1000$

$= 707$ volts

6. Show that reactance is measured in ohms for both inductors and capacitors.

Ans: a) The opposition provided by the inductor in the flow of A.C is called induction reactance

$$X_L = \frac{V_{rms}}{I_{rms}}$$

$$X_L = \frac{V_0 \sin wt}{I_0 \sin wt}$$

So its unit is:

$$\text{Unit of } X_L = \left(\frac{\text{volt}}{\text{ampere}}\right)$$

$$\text{Unit of } X_L = \frac{V_0}{I_0}\ \text{(ohms)}$$

So it is measured in ohms

b) The opposition provided by the capacitor in the flow of A.C is called capacitive reactance

$$X_C = \frac{V_{rms}}{I_{rms}}$$

$$X_C = \frac{V_0 \sin wt}{I_0 \sin wt}$$

$$\text{Unit of } X_C = \left(\frac{\text{volt}}{\text{ampere}}\right)$$

$$X_C = \frac{V_0}{I_0}\ \text{(ohms)}$$

So it is also measured in ohms.

7. Describe the principle of ECG.

Ans: The electrocardiogram or ECG is worldwide used for diagnosing heart condition. An electrocardiogram is a recording of the small electric waves being generated during heart activity.
The electric activity starts at the top of the heart and spreads down. A normal heart beat is initiated by a small pulse of electric current. This tiny electric 'shock' spreads rapidly in the heart and makes the heart muscle contract.
If the whole heart muscle contracted at the same time, there would be no pumping effect.
Therefore the electric activity starts at the top of the heart and spreads down, and then up again, causing the heart muscle to contract in an optimal way for pumping blood. The electric waves in the heart are recorded in millivolts by the electrocardiograph.

Numerical Problems

1. The peak voltage of an ac supply is 300 V. What is the rms voltage?

Give:

Maximum voltage = V_{max} = 300V

To find:

Root mean square voltage (V_{rms}) = ?

Solution:

The rms voltage is defined as

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = .707\ V_{max}$$

Putting values:

$$V_{rms} = 0.707 \times 300V$$

$$\boxed{V_{rms} = 212.1V}$$

2. The rms value of current in an ac circuit is 10 A. What is the peak current?

Given:

Root mean square current I_{rms} = 10A

To find:

Maximum current I_{max} = ?

Solution:

The rms current is defined as

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$I_{max} = \sqrt{2} \times I_{rms}$$

Putting values:

$$I_{max} = \sqrt{2} \times 10A$$

$$I_{max} = 1.41 \times 10A$$

Or $\boxed{I_{max} - 14.1A}$

3. The a.c. voltage across a 0.5 μF capacitor is 16 sin ($2\times10^3 t$) V. Find (a) the capacitive reactance (b) the peak value of current through the capacitor.

Given:

Capacitance C = 0.5 μF = 0.5×10^{-6} F
Instantaneous voltage Vc = 16 sin (2×10^3 t) v

Required:

- Capacitive reactance $X_c = ?$
- Peak current $I_{max} = ?$

Solution:

Few more data points can be calculated by comparing the given instantaneous voltage equation with the general instantaneous voltage equation for capacitor.

The given equation is $V_c = 16\sin(2\times10^3 t) \longrightarrow$ (i)

The general equation is $V_c = V_{max}\sin\omega t \longrightarrow$ (ii)

Comparing equation (i) and (ii), we get

The peak voltage is $V_{max} = 16V$

The angular frequency $= \omega = 2\times10^3\ rad/s$

(a) The capacitive reactance x_c is given by the relation

$$X_c = \frac{1}{\omega_c}$$

Putting values:

$$X_c = \frac{1}{2\times10^3\ rad/s \times 0.5\times10^{-6}F}$$

$$X_c = \frac{1}{2\times10^3\ rad/s\times0.5\times10^{-6}C/v} = X_C = 1\times10^3\ \Omega = 1k\Omega$$

(b) By Ohm's aw the peak current be written as

$$I_{max} = \frac{V_{max}}{X_c}$$

Putting values: $I_{max} = \dfrac{16}{10^3} \Rightarrow 16\times10^{-3}\ A \Rightarrow 16\ mA$

$$\boxed{I_{max} = 16\times10^{-3}\ A = 16mA}$$

4. The voltage across a 0.01 μF capacitor is 240 sin ($1.25\times10^4 t - 30°$), write the mathematical expression for current through it.

Given: Capacitance $C = 0.01\mu F = 0.01\times10^{-6}\ F$

Instantaneous voltage $V_c = 240\sin(1.25\times10^4 t - 30°)$

To find: Instantaneous current $I_c = ?$

Solution:

Few more data points can be calculated by comparing the given instantaneous voltage equation with the general instantaneous.

The given equation is $Vc = 240\sin(1.25\times10^{-4}t - 30°) \longrightarrow$ (i)

The general equation is $V_c = V_{max}\sin\omega t \longrightarrow$ (ii)

Comparing equation (i) and equation (ii), we get

The peak voltage is $V_{max} = 240V$

The angular frequency is $\omega t = 1.25\times10^4$ rad/s

Instantaneous current for capacitor i_c can be written as $I_c = I_{max}\sin(\omega t - \phi + 90^0) \longrightarrow$ (iii)

By Ohm's law the peak current can be written as

$$I_{max} = \frac{V_{max}}{X_c}$$

$$I_{max} = V_{max}\,\omega C \longrightarrow (iv) \text{ since } X_c = \frac{1}{\omega C}$$

Putting equation (iv) in equation (iii), we get

$$I_c = V_{max} \times \omega_c \times \sin(\omega t - \phi + 90°)$$

Putting

$$I_c = 240v \times 1.25 \times 10^4 \text{ rad/s} \times 0.01 \times 10^{-6} F \times \sin(1.25\ 10^4 t - 30° + 90°)$$

Or $I_c = 3 \times 10^{-2}\ V/s \times c/V \times \sin(1.25\ 10^4 t + 60°)$

Hence $I_c = 3 \times 10^{-2} A \times \sin(1.25 \times 10^4 + 60°)$

$$\boxed{I_c = 3 \times 10^{-2} \sin(1.25 \times 10^4 t + 60°)}$$

5. An inductor with an inductance of $100\mu H$ passes a current of 10mA when its terminal voltage is 6.3V. Calculate the frequency of A.C supply.

Given:

Inductance $L = 100\mu H = 100 \times 10^{-6} H$

Current $I = 10mA = 10 \times 10^{-3} A$

Voltage $v = 6.3V$

Required:

Ac frequency $f = ?$

Solution:

The inductive reactance X_L is

$$X_L = \omega_L = 2\pi f \times L$$

Or $f = \frac{X_L}{2\pi L} \longrightarrow (i)$

The inductive reactance X_L can also be written as

$$X_L = \frac{V}{I} = \frac{6.3V}{10 \times 10^{-3}A}$$

or $X_L = 0.63 \times 10^3\ \Omega \longrightarrow (ii)$

Putting values in equation (i)

$$f = \frac{0.63 \times 10^3\ \Omega}{2 \times 3.14 \times 100 \times 10^{-6} H}$$

$$\boxed{f = 1.003 \times 10^6 Hz}$$

6. (a) Calculate the inductive reactance of a 3.00 mH inductor, when 60.0 Hz and 10.0 kHz AC voltage are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120 V?

Given:

Inductance $L = 3.00mH = 3.00 \times 10^{-3}H$

AC frequency $f_1 = 60Hz$

AC frequency $f_2 = 10kHz = 10 \times 10^3 Hz$

Root mean square voltage $v_{rms} = 120V$

Required:

(a) Inductive reactance $X_{L_1} = ?$

(b) Inductive reactance $X_{L_2} = ?$

(c) I_{rms1} at frequency $f_1 = ?$

(d) I_{rms2} at frequency $f_2 = ?$

Solution:

(a) The inductive reactance at frequency f_1 is

$$X_{L1} = 2\pi f_2 \times L$$

Putting values: $X_{L1} = 2 \times 3.14 \times 60\ \text{Hz} \times 3.00 \times 10^{-3}\ \text{H}$

or $X_{L1} = 1.13041\ /\not{s} \times V\not{s}/A$

or $X_{L_1} = 1.1304\ \text{ohm}$

The inductive reactance at frequency f_1 is

$$X_{L2} = 2\pi f_2 \times L$$

Putting values: $X_{L2} = 2 \times 3.14 \times 10 \times 10^3\ \text{Hz} \times 3.00 \times 10^{-3}\ \text{H}$

or $X_{L2} = 188.41\ /\not{s} \times V\not{s}/A$

or $X_{L_2} = 188.4\ \text{ohm}$

(b) The rms current at frequency f_1 is $I_{rms_1} = \frac{V_{rms}}{X_{L_1}}$.

Putting values: $I_{rms_1} = \frac{120}{1.1304\Omega}$

or $\boxed{I_{rms_1} = 106.16A}$

The rms current at frequency f_2 is I_{rms_1} is

$$I_{rms_2} = \frac{V_{rms}}{X_{L_2}}$$

Putting values: $I_{rms_2} = \frac{120}{188.4\Omega}$

or $\boxed{I_{rms_2} = 0.637A}$

7. For the same RLC series circuit having a 40.0 Ω resistor, a 3.00 mH inductor, and a 5.00 μF capacitor: (a) Find the resonant frequency. (b) Calculate I_{rms} at resonance if V_{rms} is 120 V.

Given:

Resistance $R = 40.0\Omega$

Inductance $L = 3.00\text{mH} = 3.00 \times 10^{-3}\text{H}$

Capacitance $C = 5.00\mu\text{F} = 5.00 \times 10^{-6}\text{F}$

Root mean square voltage $V_{rms} = 120\text{V}$

Required:

(a) Resonant frequency $f_o = ?$

(b) I_{rms} at frequency $f_o = ?$

Solution:

The resonant frequency is $f_o = \frac{1}{2\pi\sqrt{L \times C}}$

Putting values: $f_o = \dfrac{1}{6.28\sqrt{15.00 \times 10^{-9}\ vs/A \times C/v}}$

Hence, $f_o = \dfrac{1}{6.28\sqrt{1.500 \times 10^{-8}\ v\cancel{s}/A \times A\cancel{s}/v}}$

Hence, $f_o = \dfrac{1}{7.691 \times 10\text{-}4s}$

Therefore, $\boxed{f_o = 1.300 \times 10^3\ Hz = 1.3Hz}$

Since at resonance the series RLC circuit behaves as purely resistive circuit and the impedance Z=R, therefore the rms current at resonance is

$$I_{rms} = V_{rms} / R$$

putting values:

$$I_{rms} = \frac{120v}{40\Omega}$$

or $\boxed{I_{rms} = 3.00A}$

8. A coil of pure inductance 318mH is connected in series with a pure resistance of 75Ω. The voltage across resistor is 150V and the frequency of power supply is 50Hz. Calculate the voltage of power supply and the phase angle.

Given:

Inductance $L = 318mH = 318 \times 10^{-3}H$

Resistance $R = 75\Omega$

AC frequency $f = 50Hz$

Voltage across resistor $V_R = 150V$

Required:

Supply voltage v = ?, Phase angle θ = ?

Solution:

The inductive reactance X_L is $X_L = \omega_L = 2\pi f \times L$

or $X_L = 2 \times 3.14 \times 50Hz \times 318 \times 10^{-3}H$

or $X_L = 99852 \times 10^{-3}\ V\cancel{s}/A \times 1/\cancel{s}$

Hence, $X_L = 99.852V/A = 99.852\Omega$

The supply voltage relation is $V = I \times Z \longrightarrow$ (i)

For series AC circuit the current is same through all components therefore total current 'I' can be calculated by calculating the current across the resistor I_R. Such that $I_R = I$

$$I = \frac{V_R}{R} = \frac{150V}{75\Omega} = 2A \longrightarrow \text{(ii)}$$

For series R_L circuit the impedance Z is given by the relation]

$$Z = \sqrt{R^2 + X_L^2}$$

Putting values: $Z = \sqrt{(75\Omega)^2 + (99.85\Omega)^2}$

$$Z = 1.24.88\Omega \longrightarrow \text{(iii)}$$

Putting equation (ii) and equation (iii) in equation (i), we get

$$V = 2A \times 124.88\Omega$$

$$\boxed{V = 249.76V = 250V}$$

The Phase angel 'θ' is $\theta = \tan^{-1}\frac{X_L}{R}$

Putting values $\theta = \tan^{-1}\frac{99.852\Omega}{75\Omega}$

$\boxed{\theta = 53.06°}$

9. A resistor of resistance 30 Ω is connected in series with a capacitor of capacitance 79.5μF across a power supply of 50Hz and 100V. Find (a) impedance (b) current (c) phase angle and (c) equation for the instantaneous value of current.

Given:

Capacitance $C = 79.5\mu F = 79.5 \times 10^{-6} F$

Resistance $R = 30\Omega$

AC frequency $f = 50Hz$

Toot mean square voltage $V_{rms} = 100V$

Required: (a) The impedance Z =?

(b) The current I = ?

(c) Phase angle θ = ?

(d) Instantaneous current I = ?

Solution:

The capacitive reactance X_c is

$$X_c = \frac{1}{\omega_c} = \frac{1}{2\pi f \times C}$$

$$Xc = \frac{1}{2.314 \times 50Hz \times 79.5 \times 10^{-6}F}$$

$$Xc = \frac{1}{24.963 \times 10^{-3}\ I/s \times C/V}$$

$$Xc = \frac{1}{24.963 \times 10^{-3}\ A/V} = 0.0400 \times 10^{3}\ V/A = 40\Omega$$

(a)

The impedance Z is $Z = \sqrt{R^2 + X_c^2}$

$Z = \sqrt{(30\Omega)^2 + (40\Omega)^2}$

$\boxed{Z = 50\Omega}$

(b) The current 'I' is given as $I_{rms} = \frac{V_{rms}}{Z}$

Putting values: $I_{rms} = \frac{100V}{50\Omega}$

$\boxed{I_{rms} = 2A}$

(c) The Phase angle θ is $\theta = \tan^{-1}\frac{Xc}{R}$

Putting values: $\theta = r\tan^{-1}\frac{40\Omega}{30\Omega}$

$\boxed{\theta = 53.12°}$

(d) The instantaneous current I is

$I_{inst} = I_{max}\sin(2\pi ft + \theta)$ since

$I_{max} = \sqrt{2}\, I_{rms}$ therefore

$I_{inst} = \sqrt{2}\, I_{rms} \sin(2\pi ft + \theta)$

Putting values: $I_{inst} = \sqrt{2} \times 2A \times \sin\{2 \times 3.14 \times 50) + 53.12°\}$

$\boxed{I_{inst} = 2.8284 \times \sin(314t + 53.12°)}$

10. A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor .(d) power consumed.

Given:

Inductance $L = 3.18mH = 31.8 \times 10^{-3}$ A

Resistance $R = 7\Omega$

AC frequency $f = 50Hz$

Root mean voltage $V_{rms} = 230V$

Required:

(a) Current $I = ?$

(b) Phase angle $\theta = ?$

(c) Power factr $\cos\theta = ?$

(d) Power consumed $<\rho> = ?$

Solution:

The inductive reactance X_L is $X_L = \omega_L = 2\pi f \times L$

Putting values : $X_L = 2 \times 3.14 \times 50Hz \times 31.8 \times 10^{-3}H$

Or $X_L = 9985.2 \times 10^{-3}\, V\cancel{s}/A \times 1/\cancel{s}$ hence, $X_L = 9.9852V/A = 9.9852\Omega$

The impedance is $Z = \sqrt{R^2 + X_L^2}$

Putting values: $Z = \sqrt{(7\Omega)^2 + (9.9852\Omega)^2}$

Hence, $Z = 12.194\Omega$

(a)

The current I is given as $I_{rms} = \dfrac{V_{rms}}{Z}$

Putting values: $I_{max} = \dfrac{230V}{12.194\Omega}$

$\boxed{I_{max} = 18.86A}$

(b) The phase angle θ is $\theta = \tan^{-1}\dfrac{X_L}{R}$

Putting values: $\theta = \tan^{-1}\dfrac{9.9852\Omega}{7\Omega}$

$\boxed{\theta = 54.95° = 55°}$

(c) The power factor cosθ is

$\boxed{\cos\theta = \cos 50° = 0.573}$

(d) The power consumed is

$<\rho> = I_{rms} V_{rms} \cos\theta$

Putting values $<\rho> = 18.86A \times 230V \times 0.573$

Or $\boxed{<\rho> = 2484.24\ W}$

Additional Conceptual Short Questions With Answers

1. Show that $\frac{L}{R}$ has the unit of time (Sec).

Ans. As $\varepsilon = L\frac{\Delta I}{\Delta t}$

$$\Rightarrow \quad L = \frac{\epsilon\,\Delta t}{\Delta I}$$

$$\text{Unit of } L = \frac{Vs}{A}$$

$$R = \frac{V}{I}$$

$$\text{Unit of } R = \frac{V}{A}$$

$$\text{Unit of } \frac{L}{R} = \frac{VS/A}{V/A}$$

$$= \frac{VSA}{AV} = s$$

$$= \text{Sec}$$

2. Why A.C is preferred on D.C for long distance transmission?

Ans. A.C is preferred, as it can be stepped up or down. Power loss is reduced if A.C is used.

3. 230 V A.C is more dangerous than 230 V D.C Why?

Ans. For 230 V A.C the peak value is 325.22V but 230V D.C has same maximum value, So A.C is more dangerous than D.C of the same voltage.

4. Microwaves are better carrier of signals than radio waves why?

Ans. The frequency of microwaves is higher than that of radiowaves, therefore microwaves can propagate through a longer distance without much loss of intensity.

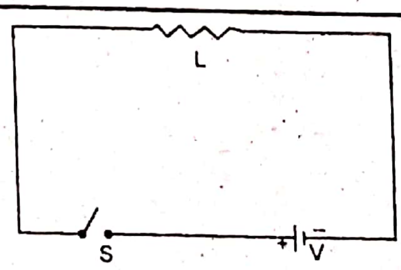
5. Name the device that will (a) permit flow of direct current but oppose the flow of alternating current (b) permit flow of alternating current but not the direct current.

Ans. (a) An inductor is a device which will permit flow of direct current (D.C), but oppose the flow of alternating current.

(b) A capacitor is a device which will permit the flow of alternating current but oppose the flow of direct current.

6. A circuit contains an iron-cored inductor, a switch and a D.C. source arranged in series The switch is closed and after an interval reopened. Explain why a spark jumps across the switch contacts?

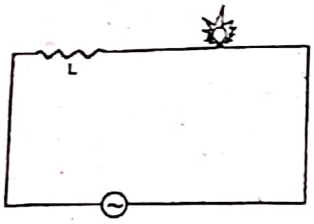
Ans. When the switch is closed, current grows from zero to maximum value. This changing current produces change in the magnetic flux due to which an emf is induced in the coil.

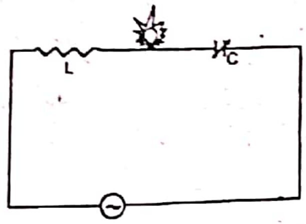


This induced emf becomes zero when current become steady. When the switch is reopened again induced emf (back emf) is developed across the inductor (coil). When this back emf has a sufficiently large value, it produces a spark at the ends of the switch contacts.

7. A choke coil placed in series with an electric lamp in an A.C. circuit causes the lamp to become dim. Why is it so? A variable capacitor added in series in this circuit may be adjusted until the lamp glows with normal brilliance. Explain, how this is possible?

Ans. When a choke (inductor) is placed in series with A.C. lamp, thus lamp glows dim, because choke is offering more reactance (X_L) and hence current through the lamp is small.





When a variable capacitor is added in series with the choke, then the impedance of the circuit becomes minimum, because at a resonance condition the reactance of choke and reactance of capacitor are equal in magnitude and directed opposite, so they cancel each other effect and the impedance of the circuit becomes minimum and current becomes maximum. Thus lamp starts glowing with normal brilliance.

Self-Assessment Paper 1

Q.No.1 Encircle the correct option.

1. An inductive coil has a resistance of 100 Ω. When an AC signal of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45°. What is the inductance of the coil?
(A) 10 mH (B) 12 mH (C) 16 mH (D) 20 mH
2. When a voltage $V = V_o \cos\omega t$ is applied across a resistor of resistance R, the average power dissipated per cycle in the resistor is given by ________
(A) zero (B) $\frac{V_o}{\sqrt{2\omega R}}$ (C) $\frac{V_o^2}{2R}$ (D) $\frac{V_o^2}{2\omega R}$
3. Maxwell derived mathematically that the speed of the electromagnetic waves is ________
(A) $\frac{1}{\varepsilon_o}$ (B) $\frac{1}{\sqrt{\mu_o/\varepsilon_o}}$ (C) $\sqrt{\mu_o\varepsilon_o}$ (D) $\frac{1}{\sqrt{\mu_o\varepsilon_o}}$
4. A wire of resistance R is coiled inductively so that its inductance is L. The impedance of the coil at a frequency of f is ________
(A) $(R+2\pi fL)^{1/2}$ (B) $\frac{R+1}{2\pi fL}$ (C) $(R^2+f^2L^2)^{1/2}$ (D) $(R^2+4\pi^2f^2L^2)^{1/2}$
5. If a glass plate is inserted in between the plate of a capacitor in series with a lighted bulb, the brightness of the bulb ________
(A) remains same (B) increases (C) brightness decreases (D) no light
6. If I_o is the peak value of AC supply, then its rms value is given as I_{rms} = ________
(A) $\frac{I_o}{\sqrt{2}}$ (B) $\frac{I_o}{0.707}$ (C) $\sqrt{2I_o}$ (D) $\sqrt{\frac{I_o}{2}}$
7. The range of F.M transmission frequencies is from
(A) 540 kHz to 1600 kHz (B) 88 kHz to 108 kHz (C) 88 MHz to 108 MHz (D) 540 MHz to 1600 MHz
8. The slope of (q – t) curve at any instant of time gives
(A) current (B) voltage (C) charge (D) both A and B
9. At resonance frequency, the impedance of RLC series circuit is
(A) maximum (B) minimum (C) zero (D) infinite
10. In three phase AC supply, the phase difference in voltage of any two phases is
(A) 90° (B) 180° (C) 120° (D) 360°

Q.No.2 Write Short Answers any SIX of the following questions. (6 × 2 = 12)

1. What determines the gradient of the graph of inductive reactance against frequency?
2. Why the average value of a.c current is zero, but root mean square value of voltage is not zero?
3. Name the device which will permit the flow of A.C. but opposes the flow of D.C.
4. In R–L circuit, will the current lag or lead the voltage? Illustrate y our answer by a vector diagram.
5. Write down the principle of generation of electromagnetic waves.
6. Differentiate between resistance and the reactance.
7. How will doubling the frequency of a.c effects the capacitive and inductive reactances?
8. A choke coil placed in series with an electric lamp in an a.c circuit causes the lamp to become dim. Why is it so? A variable capacitor added in series in this circuit may be adjusted until the lamp glows with normal brilliance. Explain how this is possible?

Q.No.3 Extensive Question. (5 + 3 = 8)

Q. (a) Describe in detail the effect of inductance in an ac circuit.
(b) A coil having a resistance of 318 mH is connected in series with a pure resistance of 75 ohms. The voltage across resistor is 150V and frequency of power supply is 50Hz. Calculate the voltage of power supply and the phase angle.

Self-Assessment Paper 2

Q.No.1 Encircle the correct option.

1. **In RLC series circuit, at low frequency**
(A) $X_C < X_L$ (B) $X_C > X_L$ (C) $X_C = X_L$ (D) none of these
2. **During frequency modulation when amplitude of signal is zero, the frequency of carrier waver is**
(A) zero (B) maximum (C) minimum (D) normal
3. **The reaction of an inductor at 50 Hz is 10 Ω its reactance at 100 Hz becomes**
(A) 20 Ω (B) 5 Ω (C) 2.5 Ω (D) 1 Ω
4. **In RLC AC circuit at resonance, the phase difference between current and voltage is**
(A) 90° (B) 180° (C) 0° (D) 360°
5. **In modulation, low frequency signal is known as**
(A) loaded signal (B) fluctuated signal (C) harmonic signal (D) modulation signal
6. **Which one of the following requires a material medium for their propagation?**
(A) heat waves (B) x - rays (C) sound waves (D) ultraviolet rays
7. **At high frequency, RLC series circuit shows that behavior of**
(A) RC series circuit (B) RL series circuit (C) LC series circuit (D) none of these
8. **An ac emf given by $V = V_o \sin \omega t$ has a maximum value of 10 volt and frequency of 50Hz, then the instantaneous emf at $t = \frac{1}{600}$ s is**
(a) 10 volt (b) 5 volt (c) $5\sqrt{3}$ volt (d) 1 volt
9. **In which of the following sequence are the electromagnetic radiations in decreasing order of wavelength?**
(a) infrared, radio, x-rays, visible (b) radio, infrared, visible, x-rays
(c) radio, visible, infrared, x- rays (d) x-rays, visible, infrared, radio
10. **In frequency modulation the amount of frequency deviation depends on the:**
(a) frequency of audio signal (b) amplitude of audio signal
(c) both frequency and amplitude of audio signal (d) none of these

Q.No.2 Write Short Answers any SIX of the following questions.

1. What is meant by root mean square value of voltage?
2. How many times does the lamp reach to its maximum brilliance when frequency of a.c is 50 Hz.
3. What is meant by A.M and FM?
4. In R-C circuit, will the current lag or lead the voltage? Illustrate y our answer by a vector diagram.
5. How the reception of a particular radio station is selected on your radio set?
6. The peak value of ac is 10A, find its rms value.
7. What is an electrical oscillator and how it is used in metal detector?

Q.No.3 Extensive Questions.

Q. (a) Discuss RLC series resonance circuit in detail.

(b) The a.c voltage across a 0.5 micro farad capacitor is 16 sin $(1.25 \times 10^4 t - 30°)$V. Write the mathematical expression for the current through it.

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