1.2 Definition of Plasma

Any ionized gas cannot be called a plasma, of course; there is always some small degree of ionization in any gas. A useful definition is as follows:

A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.

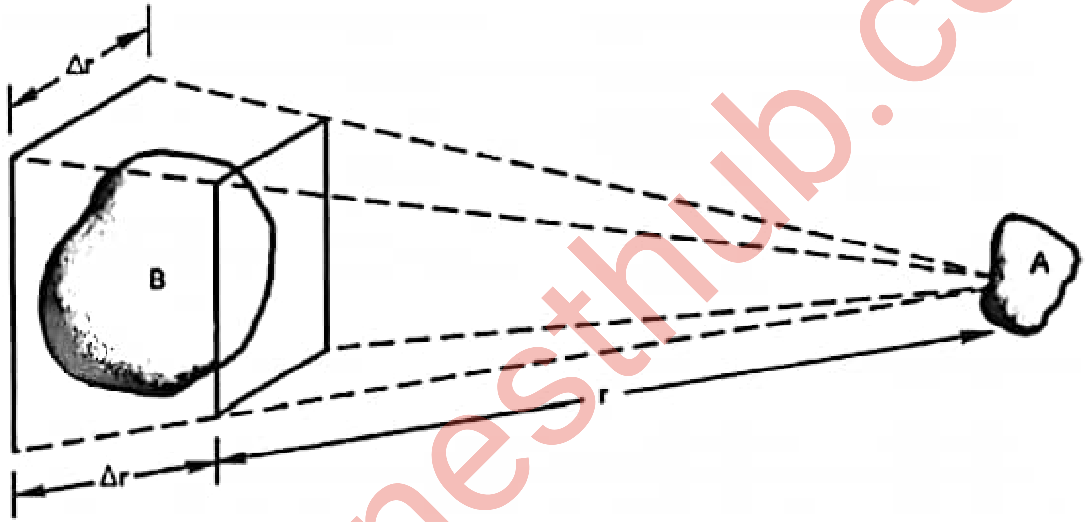

Fig. 1.1 Illustrating the long range of electrostatic forces in a plasma

We must now define "quasineutral" and "collective behavior." The meaning of quasineutrality will be made clear in Sect. 1.4. What is meant by "collective behavior" is as follows.

Consider the forces acting on a molecule of, say, ordinary air. Since the molecule is neutral, there is no net electromagnetic force on it, and the force of gravity is negligible. The molecule moves undisturbed until it makes a collision with another molecule, and these collisions control the particle's motion. A macroscopic force applied to a neutral gas, such as from a loudspeaker generating sound waves, is transmitted to the individual atoms by collisions. The situation is totally different in a plasma, which has *charged* particles. As these charges move around, they can generate local concentrations of positive or negative charge, which give rise to electric fields. Motion of charges also generates currents, and hence magnetic fields. These fields affect the motion of other charged particles far away.

Let us consider the effect on each other of two slightly charged regions of plasma separated by a distance r (Fig. 1.1). The Coulomb force between A and B diminishes as $1/r^2$. However, for a given solid angle (that is, $\Delta r/r =$ constant), the volume of plasma in B that can affect A increases as r^3. Therefore, elements of plasma exert a force on one another even at large distances. It is this long-ranged Coulomb force that gives the plasma a large repertoire of possible motions and enriches the field of study known as plasma physics. In fact, the most interesting

1.3 Concept of Temperature

Before proceeding further, it is well to review and extend our physical notions of "temperature." A gas in thermal equilibrium has particles of all velocities, and the most probable distribution of these velocities is known as the Maxwellian distribution. For simplicity, consider a gas in which the particles can move only in one dimension. (This is not entirely frivolous; a strong magnetic field, for instance, can constrain electrons to move only along the field lines.) The one-dimensional Maxwellian distribution is given by

$$f(u) = A\exp\left(-\tfrac{1}{2}mu^2/KT\right) \tag{1.2}$$

where $f\,du$ is the number of particles per m^3 with velocity between u and $u+du$, $\frac{1}{2}mu^2$ is the kinetic energy, and K is Boltzmann's constant,

$$K = 1.38 \times 10^{-23}\,\mathrm{J/^\circ K}$$

Note that a capital K is used here, since lower-case k is reserved for the propagation constant of waves. The density n, or number of particles per m^3, is given by (see Fig. 1.2)

$$n = \int_{-\infty}^{\infty} f(u)\,du \tag{1.3}$$

The constant A is related to the density n by (see Problem 1.2)

$$A = n\left(\frac{m}{2\pi KT}\right)^{1/2} \tag{1.4}$$

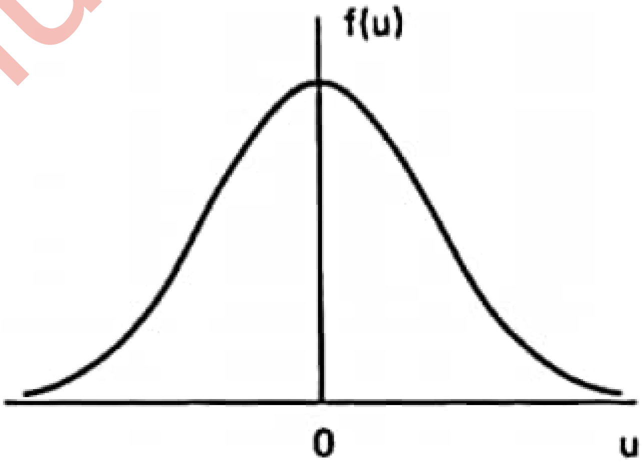

Fig. 1.2 A Maxwellian velocity distribution

The width of the distribution is characterized by the constant T, which we call the temperature. To see the exact meaning of T, we can compute the average kinetic energy of particles in this distribution:

$$E_{av} = \frac{\int_{-\infty}^{\infty} \frac{1}{2}mu^2 f(u)du}{\int_{-\infty}^{\infty} f(u)du} \tag{1.5}$$

Defining

$$v_{th} = (2KT/m)^{1/2} \quad \text{and} \quad y = u/v_{th} \tag{1.6}$$

we can write Eq. (1.2) as

$$f(u) = A\exp(-u^2/v_{th}^2)$$

and Eq. (1.5) as

$$E_{av} = \frac{\frac{1}{2}mAv_{th}^3 \int_{-\infty}^{\infty} [\exp(-y^2)]y^2 dy}{Av_{th}\int_{-\infty}^{\infty} \exp(-y^2)dy}$$

The integral in the numerator is integrable by parts:

$$\int_{-\infty}^{\infty} y \cdot [\exp(-y^2)]y dy = [-\tfrac{1}{2}[\exp(-y^2)]y]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -\tfrac{1}{2}\exp(-y^2)dy$$
$$= \tfrac{1}{2}\int_{-\infty}^{\infty} \exp(-y^2)dy$$

Canceling the integrals, we have

$$E_{av} = \frac{\frac{1}{2}mAv_{th}^3 \frac{1}{2}}{Av_{th}} = \tfrac{1}{4}mv_{th}^2 = \tfrac{1}{2}KT \tag{1.7}$$

Thus the average kinetic energy is $\frac{1}{2}KT$.

It is easy to extend this result to three dimensions. Maxwell's distribution is then

$$f(u, v, w) = A_3 \exp[-\tfrac{1}{2}m(u^2 + v^2 + w^2)/KT] \tag{1.8}$$

where

$$A_3 = n\left(\frac{m}{2\pi KT}\right)^{3/2} \tag{1.9}$$

The average kinetic energy is

$$E_{av} = \frac{\iiint_{-\infty}^{\infty} A_3 \frac{1}{2}m(u^2+v^2+w^2)\exp\left[-\frac{1}{2}m(u^2+v^2+w^2)/KT\right]du\,dv\,dw}{\iiint_{-\infty}^{\infty} A_3 \exp\left[-\frac{1}{2}m(u^2+v^2+w^2)/KT\right]du\,dv\,dw}$$

We note that this expression is symmetric in u, v, and w, since a Maxwellian distribution is isotropic. Consequently, each of the three terms in the numerator is the same as the others. We need only to evaluate the first term and multiply by three:

$$E_{av} = \frac{3A_3 \int \frac{1}{2}mu^2 \exp\left(-\frac{1}{2}mu^2/KT\right)du \iint \exp\left[-\frac{1}{2}m(v^2+w^2)/KT\right]dv\,dw}{A_3 \int \exp\left(-\frac{1}{2}mu^2/KT\right)du \iint \exp\left[-\frac{1}{2}m(v^2+w^2)/KT\right]dv\,dw}$$

Using our previous result, we have

$$E_{av} = \tfrac{3}{2}KT \tag{1.10}$$

The general result is that E_{av} equals $\frac{1}{2}KT$ per degree of freedom.

Since T and E_{av} are so closely related, it is customary in plasma physics to give temperatures in units of energy. To avoid confusion on the number of dimensions involved, it is not E_{av} but the energy corresponding to KT that is used to denote the temperature. For $KT = 1$ eV $= 1.6 \times 10^{-19}$ J, we have

$$T = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 11,600$$

Thus the conversion factor is

$$1\,\text{eV} = 11,600\,°\text{K} \tag{1.11}$$

By a 2-eV plasma we mean that $KT = 2$ eV, or $E_{av} = 3$ eV in three dimensions.

It is interesting that a plasma can have several temperatures at the same time. It often happens that the ions and the electrons have separate Maxwellian distributions with different temperatures T_i and T_e. This can come about because the collision rate among ions or among electrons themselves is larger than the rate of collisions between an ion and an electron. Then each species can be in its own thermal equilibrium, but the plasma may not last long enough for the two temperatures to equalize. When there is a magnetic field **B**, even a single species, say ions, can have two temperatures. This is because the forces acting on an ion along **B** are different from those acting perpendicular to **B** (due to the Lorentz force). The components of velocity perpendicular to **B** and parallel to **B** may then belong to different Maxwellian distributions with temperatures $T_\perp$ and $T_\parallel$.

Before leaving our review of the notion of temperature, we should dispel the popular misconception that high temperature necessarily means a lot of heat. People are usually amazed to learn that the electron temperature inside a fluorescent light bulb is about 20,000° K. "My, it doesn't feel that hot!" Of course, the heat capacity must also be taken into account. The density of electrons inside a fluorescent tube is much less than that of a gas at atmospheric pressure, and the total amount of heat transferred to the wall by electrons striking it at their thermal velocities is not that great. Everyone has had the experience of a cigarette ash dropped innocuously on his hand. Although the temperature is high enough to cause a burn, the total amount of heat involved is not. Many laboratory plasmas have temperatures of the order of 1,000,000° K (100 eV), but at densities of only 10^{18}–10^{19} per m^3, the heating of the walls is not a serious consideration.

Problems

1.1. Compute the density (in units of m^{-3}) of an ideal gas under the following conditions:

(a) At 0 °C and 760 Torr pressure (1 Torr = 1 mmHg). This is called the Loschmidt number.

(b) In a vacuum of 10^{-3} Torr at room temperature (20 °C). This number is a useful one for the experimentalist to know by heart (10^{-3} Torr = 1 μ).

1.2. Derive the constant A for a normalized one-dimensional Maxwellian distribution

$$\hat{f}(u) = A\exp\left(-mu^2/2KT\right)$$

such that

$$\int_{-\infty}^{\infty} \hat{f}(u)du = 1$$

Hint: To save writing, replace $(2KT/m)^{1/2}$ by v_{th} (Eq. 1.6).

1.2a. (Advanced problem). Find A for a two-dimensional distribution which integrates to unity. Extra credit for a solution in cylindrical coordinates.

$$\hat{f}(u, v) = A\exp\left[-m\left(u^2 + v^2\right)/2KT\right]$$

1.4 Debye Shielding

A fundamental characteristic of the behavior of plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside a plasma by inserting two charged balls connected to a battery (Fig. 1.3). The balls would attract particles of the opposite charge, and almost immediately a cloud of ions would surround the negative ball and a cloud of electrons would surround

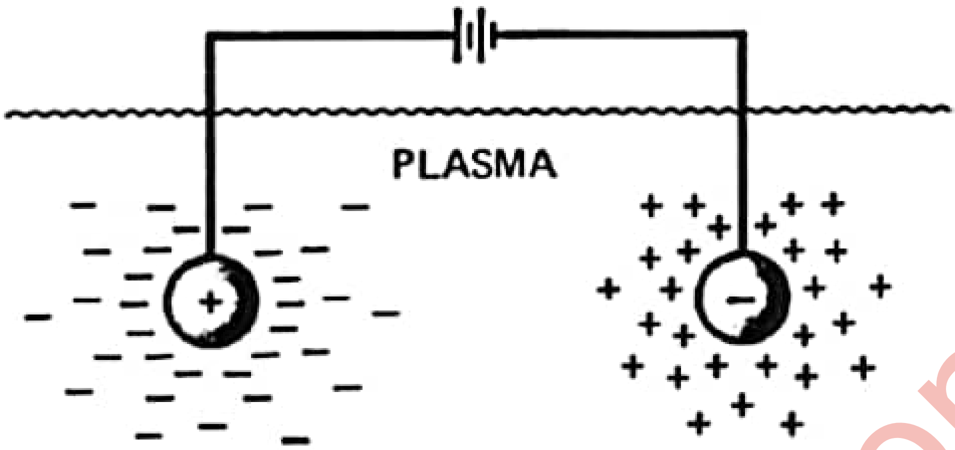

Fig. 1.3 Debye shielding

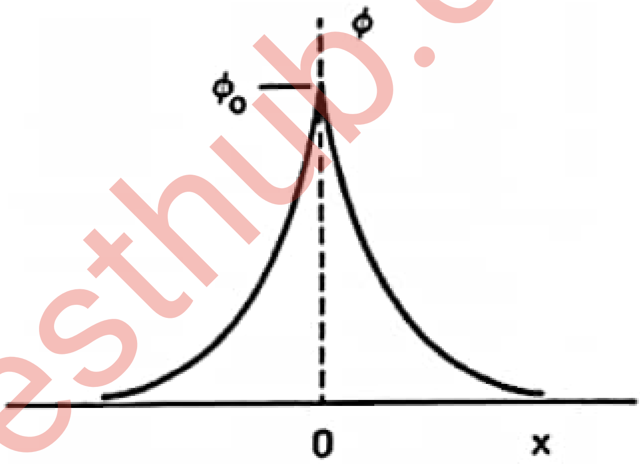

Fig. 1.4 Potential distribution near a grid in a plasma

the positive ball. (We assume that a layer of dielectric keeps the plasma from actually recombining on the surface, or that the battery is large enough to maintain the potential in spite of this.) If the plasma were cold and there were no thermal motions, there would be just as many charges in the cloud as in the ball, the shielding would be perfect, and no electric field would be present in the body of the plasma outside of the clouds. On the other hand, if the temperature is finite, those particles that are at the edge of the cloud, where the electric field is weak, have enough thermal energy to escape from the electrostatic potential well. The "edge" of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy KT of the particles, and the shielding is not complete. Potentials of the order of KT/e can leak into the plasma and cause finite electric fields to exist there.

Let us compute the approximate thickness of such a charge cloud. Imagine that the potential ϕ on the plane $x = 0$ is held at a value ϕ_0 by a perfectly transparent grid (Fig. 1.4). We wish to compute $\phi(x)$. For simplicity, we assume that the ion–electron mass ratio M/m is infinite, so that the ions do not move but form a uniform background of positive charge. To be more precise, we can say that M/m is large enough that the inertia of the ions prevents them from moving significantly on the time scale of the experiment. Poisson's equation in one dimension is

$$\varepsilon_0 \nabla^2 \phi = \varepsilon_0 \frac{d^2\phi}{dx^2} = -e(n_i - n_e) \quad (Z = 1) \tag{1.12}$$

If the density far away is n_∞, we have

$$n_i = n_\infty$$

In the presence of a potential energy $q\phi$, the electron distribution function is

$$f(u) = A \exp\left[-\left(\tfrac{1}{2}mu^2 + q\phi\right)/KT_e\right] \tag{1.13}$$

It would not be worthwhile to prove this here. What this equation says is intuitively obvious: There are fewer particles at places where the potential energy is large, since not all particles have enough energy to get there. Integrating $f(u)$ over u, setting $q = -e$, and noting that $n_e(\phi \to 0) = n_\infty$, we find

$$n_e = n_\infty \exp(e\phi/KT_e)$$

This equation will be derived with more physical insight in Sect. 3.5. Substituting for n_i and n_e in Eq. (1.12), we have

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = en_\infty\left(e^{e\phi/KT_e} - 1\right)$$

In the region where $|e\phi/KT_e| \ll 1$, we can expand the exponential in a Taylor series:

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = en_\infty\left[\frac{e\phi}{KT_e} + \frac{1}{2}\left(\frac{e\phi}{KT_e}\right)^2 + \cdots\right] \tag{1.14}$$

No simplification is possible for the region near the grid, where $|e\phi/KT_e|$ may be large. Fortunately, this region does not contribute much to the thickness of the cloud (called a sheath), because the potential falls very rapidly there. Keeping only the linear terms in Eq. (1.13), we have

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{KT_e}\phi \tag{1.15}$$

Defining

$$\boxed{\lambda_D \equiv \left(\frac{\varepsilon_0 KT_e}{ne^2}\right)^{1/2}} \tag{1.16}$$

where n stands for n_∞, and KT_e is in joules. KT_e is often given in eV, in which case, we will write it also as T_{eV}.

1.5 The Plasma Parameter

The picture of Debye shielding that we have given above is valid only if there are enough particles in the charge cloud. Clearly, if there are only one or two particles in the sheath region, Debye shielding would not be a statistically valid concept. Using Eq. (1.17), we can compute the number N_D of particles in a "Debye sphere":

$$N_D = n\frac{4}{3}\pi\lambda_D^3 = 1.38 \times 10^6 T^{3/2}/n^{1/2} \quad (T \text{ in } ^\circ\text{K}) \tag{1.19}$$

In addition to $\lambda_D \ll L$, "collective behavior" requires

$$N_D \ggg 1 \tag{1.20}$$

1.6 Criteria for Plasmas

We have given two conditions that an ionized gas must satisfy to be called a plasma. A third condition has to do with collisions. The weakly ionized gas in an airplane's jet exhaust, for example, does not qualify as a plasma because the charged particles collide so frequently with neutral atoms that their motion is controlled by ordinary hydrodynamic forces rather than by electromagnetic forces. If ω is the frequency of typical plasma oscillations and τ is the mean time between collisions with neutral atoms, we require $\omega\tau > 1$ for the gas to behave like a plasma rather than a neutral gas.

The three conditions a plasma must satisfy are therefore:

1. $\lambda_D \ll L$.
2. $N_D \ggg 1$.
3. $\omega\tau > 1$.

Problems

1.3. Calculate n vs. KT_e curves for five values of λ_D from 10^{-8} to 1, and three values of N_D from 10^3 to 10^9. On a log-log plot of n_e vs. KT_e with n_e from 10^6 to 10^{28} m^{-3} and KT_e from 10^{-2} to 10^5 eV, draw lines of constant λ_D (solid) and N_D (dashed). On this graph, place the following points (n in m^{-3}, KT in eV):

1. Typical fusion reactor: $n = 10^{20}$, $KT = 30{,}000$.
2. Typical fusion experiments: $n = 10^{19}$, $KT = 100$ (torus); $n = 10^{23}$, $KT = 1000$ (pinch).
3. Typical ionosphere: $n = 10^{11}$, $KT = 0.05$.
4. Typical radiofrequency plasma: $n = 10^{17}$, $KT = 1.5$
5. Typical flame: $n = 10^{14}$, $KT = 0.1$.
6. Typical laser plasma; $n = 10^{25}$, $KT = 100$.
7. Interplanetary space: $n = 10^6$, $KT = 0.01$.

differ in density by only 10^3, while water and white dwarf stars are separated by only a factor of 10^5. Even neutron stars are only 10^{15} times denser than water. Yet gaseous plasmas in the entire density range of 10^{28} can be described by the same set of equations, since only the classical (non-quantum mechanical) laws of physics are needed.

1.7.1 Gas Discharges (Gaseous Electronics)

The earliest work with plasmas was that of Langmuir, Tonks, and their collaborators in the 1920s. This research was inspired by the need to develop vacuum tubes that could carry large currents, and therefore had to be filled with ionized gases. The research was done with weakly ionized glow discharges and positive columns typically with $KT_e \simeq 2$ eV and $10^{14} < n < 10^{18}$ m^{-3}. It was here that the shielding phenomenon was discovered; the sheath surrounding an electrode could be seen visually as a dark layer. Before semiconductors, gas discharges were encountered only in mercury rectifiers, hydrogen thyratrons, ignitrons, spark gaps, welding arcs, neon and fluorescent lights, and lightning discharges. The semiconductor industry's rapid growth in the last two decades has brought gas discharges from a small academic discipline to an economic giant. Chips for computers and the ubiquitous handheld devices cannot be made without plasmas. Usually driven by radiofrequency power, partially ionized plasmas (gas discharges) are used for etching and deposition in the manufacture of semiconductors.

1.7.2 Controlled Thermonuclear Fusion

Modern plasma physics had its beginnings around 1952, when it was proposed that the hydrogen bomb fusion reaction be controlled to make a reactor. A seminal conference was held in Geneva in 1958 at which each nation revealed its classified