Ch#2 Numericals:

(16) The larmor freq of electron is

a) $\omega_e = eB/m = \frac{1.6 \times 10^{-19} \times 1}{9.1 \times 10^{-31}} = 1.76 \times 10^{11}$ rad/

b) For ion

$\omega_i = eB/m = \frac{1.6 \times 10^{-19} \times 1}{1.67 \times 10^{-27}} = 9.59 \times 10^{7}$ rad/sec

Since $\omega_0 = 10^{9}$ rad/sec $\omega_e >> \omega_0 >> \omega_i$

The motion of electron is adiabatic, while that of ion is not

4, 12, 14, 16, 8, 7, 2, 3, 4, 5, 6, 10 Compute $r_L = ?$

a)
1. A 10keV e^- in the earth's magnetic feild of 5×10^{-5} T.

Sol: $B = 5\times10^{-5}$ T

$E = 10keV = 10^4 eV$.

$r_L = ?$

$r_L = V_\perp / \omega_c \rightarrow (1) \quad = \frac{V_\perp m}{qB}$

$\frac{1}{2} m V_\perp^2 = E$

$V_\perp^2 = 2E/m$.

$V_\perp = \sqrt{2E/m} = \sqrt{\frac{2\times10^4 \times 1.6\times10^{-19}}{9.1\times10^{-31}}} = 0.59\times10^8 ms^{-1}$

$V_\perp = 5.9\times10^7 ms^{-1}$.

$r_L = \frac{(5.9\times10^7)(9.1\times10^{-31})}{(1.6\times10^{-19})(5\times10^{-5})}$

$\boxed{r_L = 6.7 m}$

b) A solar wind proton with streaming velocity 300km/s $B = 5\times10^{-9}$T?

$V = 300 km/s = 3\times10^5 m/s$.

$r_L = \frac{V_\perp m}{qB} = \frac{(3\times10^5)(1.67\times10^{-27})}{(1.6\times10^{-19})(5\times10^{-9})}$

$'' = 0.626\times10^6 m$.

$\boxed{'' = 6.26 km}$

c) 1keV He^+ ion in solar atom near a sunspot where $B = 5\times10^{-2}$T

$\Rightarrow {}_2He^4$. $4m_p$ $E = 1\times10^3 eV$.

$B = 5\times10^{-2}$

$V_\perp = \sqrt{2E/m} = \sqrt{\frac{1\times10^3 \times 1.6\times10^{-19}}{4(1.6\times10^{-27})}}$

$V_\perp = 2.19\times10^5 ms^{-1}$

$$r_\perp = \frac{4\times1.67\times10^{-27}\times2.19\times10^{5}}{2\times1.6\times10^{-19}\times5\times10^{-2}}$$

$r_\perp = 0.091\,m.$

$\boxed{r_\perp = 9.1\,cm}$

d) A 3.5MeV He^{++} ash particles in an 8T DT fusion reactor.

$r_L = ?$

$E = 3.5\times10^{6}\,eV.$

$B = 8T$

$$r_\perp = V_\perp/\omega_c = \frac{V_\perp 4m_p}{2qB}$$

$$V_\perp = \sqrt{2E/4m_p} = 1.29\times10^{7}\,ms^{-1}$$

$$\frac{4\times1.67\times10^{-27}\times1.29\times10^{7}}{2\times1.6\times10^{-19}\times8}$$

$r_L = 0.0336\,m.$

$\boxed{r_L = 3.36\,cm}.$

(2) In TFTR (Tokamak Fusion Test Reactor) at Pricenton the plasma will be heated by injection of 200keV neutral deutrium atoms, which after entering the M.F, are converted to 200keV D ions (A=2) by charge exchange)

are confined $r_L >> a$ $a = 0.6m$ is the

their minor radius of the toroidal plasma. Compute the max Larmour radius in a 5-T feild and see if as this is satisfied?

Sol:

$E = 200 keV$

$A = 2$

$a = 0.6 m.$

$B = 5 T$

For Deutrium ion $= 2m_p = 2 \times 1.67 \times 10^{-27}$

$= 3.34 \times 10^{-27} kg$

$q = e = 1.6 \times 10^{-19}.$

→ Assume that energy can be entirely converted to K.E. The momentum derived as.

$E_K = \frac{1}{2} m v_\perp^2$

$E_K = \frac{m^2 v_\perp^2}{2m}$

$" = p^2/2m.$

$p = \sqrt{2Em}$

$p = \sqrt{2 \times (3.34 \times 10^{-27}) \times 200 \times 10^3 \times 1.6 \times 10^{-19}}.$

$p = 1.46 \times 10^{-20} kg m s^{-1}.$

$r_\perp = v_\perp / \omega_c$

$" = \frac{v_\perp m}{qB} = \frac{p}{qB} = \frac{1.46 \times 10^{-20}}{1.6 \times 10^{-19} \times (5)}$

$r_\perp = 0.018 m.$

$r_\perp << \boxed{a = 0.6 m.}$

So Larmor radius satisfies the confined ion condition.

③ An ion engine has a 1T M.F and a hydrogen plasma is to be shot out at an $E \times B$ velocity of 1000km/sec. How much internal E-feild present in plasma?

Sol:

$$F_e = qE$$

$$F_L = qvB.$$

$$qE = qvB.$$

$$E = vB$$

$$E = (1 \times 10^6)(1)$$

$$\boxed{E = 1 \times 10^6 \, V/m.}$$

$$v_E = \frac{E \times B}{B^2}$$

$$v_E = E/B$$

$$v_E \times B = E$$

⑤ Suppose e^- obeys the Boltzman 1.5 in a cylindrically symmetric plasma column in which $n(r)$ varies with scale length λ that is $\frac{\partial n}{\partial r} = -n/\lambda$

a) using $E = \nabla\phi$, find radial for given λ

Sol: We simply for ϕ from the Boltzman's relation for e^-

$$n = n_o e^{e\phi/k_B T_e}$$

or $$\frac{n}{n_o} = e^{e\phi/k_B T_e}.$$

Take log on both sides

$$\ln(n/n_o) = \ln e^{\,}\left(e\phi/k_B T_e\right)$$

$$\phi = \frac{k_B T_e}{e} \ln(n/n_o) \rightarrow (A)$$

Also: $E - \nabla\phi$

where $-\partial/\partial r \, \phi \hat{r}$

where $\frac{\partial n}{\partial r} = -n/\lambda$.

$$\vec{E} = -\frac{kT_e}{e}\left(1/n \cdot \frac{\partial n}{\partial r}\hat{r}\right)$$

$$'' = -\frac{kT_e}{e}\left(1/n \cdot (-n/\lambda)\right)\hat{r}$$

$$\boxed{'' = \frac{kT_e}{e\lambda}\hat{r}}$$

b) For e^- show that finite Larmor radius effects are large if V_E is as large as V_{th}, specifically show that $r_L = 2A$ if $V_e = V_{th}$.

Sol:

$$V_E = -\frac{E_r}{B}\hat{\theta}$$

$$V_E = -\frac{kT_e}{e\lambda \cdot B}\hat{\theta}.$$

Consider electron.

$$v_{th} = \sqrt{\frac{2kT_e}{m}} \quad \text{or} \quad v_{th}^2 = \frac{2kT_e}{m}$$

The magnitude of V_E is given by.

$$V_E = \frac{kT_e}{eB\lambda}$$

$$'' = \frac{kT_e}{m} \cdot \frac{m}{eB} \cdot 1/\lambda.$$

X and ÷ by 2

$$'' = \frac{1}{2}\frac{2kT_e}{m} \cdot \frac{m}{eB} \cdot 1/\lambda.$$

$$V_E = \frac{v_{th}^2}{2} \times \frac{1}{\omega_c} \times 1/\lambda$$

$$r_L = V_\perp/\omega_c$$

So $V_E = \frac{V_{th}}{2}\frac{r_\perp}{\lambda}$

Now $r_L = \frac{mv_\perp}{eB}$ for a distribution vector ~~degree~~ must find r_L
~~distributions~~
Since $v_\perp$ contains the degree of freedom we have

$$\frac{1}{2} m v_\perp^2 = 2 \times \frac{1}{2} kT_e$$

most convient average

$$(v_\perp)_{rms} = \sqrt{\frac{2kT_e}{m}} = v_{th}.$$

So v_E we have

$$v_E = \frac{v_{th}}{2\lambda} \times r_L$$

As given $v_E = v_{th}$.

$$\cancel{v_{th}} = \frac{\cancel{v_{th}}}{2\lambda} \cdot r_L$$

$$\boxed{r_L = 2\lambda}$$

c) Is B also for ions?

Sol: if ions instead of electron.

$$v_{th} = \sqrt{\frac{2kT_i}{m}} = v_\perp$$

$$r_{Li} = \frac{v_{\perp i}}{\omega_{ci}}$$

$$v_E = \frac{1}{2\lambda}\left(\frac{2kT_e}{m}\right)\left(\frac{m}{eB}\right).$$

$$v_E = \frac{1}{2\lambda} \frac{T_e}{T_i} \frac{v_{mi} v_{\perp i}}{\omega_{ci}}$$

$$\text{"} = \frac{1}{2\lambda} \frac{T_e}{T_i} v_{thi} r_{Li}$$

$v_E = v_{thi}$

$r_{Li} = 2\lambda$ provided $T_i = T_e$

6) Suppose that a so-called quantum machine has uniform feild of 0.2T ... ?

a) Calculate max V_E if $a = 1cm$.

given: $B = 0.2T$.

$kT_e = kT_i = 0.2eV = 0.2 \times 1.6 \times 10^{-19} J$

$= 0.32 \times 10^{-19} J$.

Given:– $n = n_0 \exp\left[\exp\left(\frac{-r^2}{a^2} - 1\right)\right] \rightarrow (a)$

$n = n_0 \exp(e\phi/k_B T_e) \rightarrow (b)$

Comparing we get:–

$\exp\left(\frac{-r^2}{a^2} - 1\right) = e\phi/k_B T_e \rightarrow (c)$

$\phi = \frac{k_B T_e}{e} \exp\left(\frac{-r^2}{a^2} - 1\right) \rightarrow (1)$

As E.f is

$E = -\partial\phi/\partial r \, \hat{r}$

$E = -\frac{\partial}{\partial \phi}\left[\frac{kT_e}{e} \exp\left(\frac{-r^2}{a^2} - 1\right)\right]\hat{r}$

$E = -\frac{kT_e}{e} \exp\left(\frac{-r^2}{a^2}\right) \times (-1/a^2)(2r)\hat{r}$.

$E = \frac{kT_e}{e} \times \frac{2r}{a^2} \exp\left(\frac{-r^2}{a^2}\right)\hat{r}$.

and max V_E is given by $V_E = E/B \rightarrow (D)$

$V_E = \frac{2kT_e}{e a^2 B} r \exp\left(\frac{-r^2}{a^2}\right)\hat{r}$.

Product rule used.

So $\frac{\partial V_E}{\partial r} = \frac{2kTe}{ea^2B}(1)\exp\left(\frac{-r^2}{a^2}\right) + \frac{2kTer}{ea^2B} \times$

$\exp(-r^2/a^2) - 1/a^2 \, (-2r).$

$\frac{\partial V_E}{\partial r} = \frac{2kTe}{ea^2B}\exp(-r^2/a^2) - \frac{4r^2kTe}{ea^4B}\exp\left(\frac{-r^2}{a^2}\right)$

$'' = \frac{2kTe}{ea^2B}\exp(-r^2/a^2)\left[1 - \frac{2r^2}{a^2}\right] \rightarrow (2)$

As V_E is max so

$\partial V_E/\partial r = 0.$

$\frac{2kTe}{ea^2B}\exp\left(\frac{-r^2}{a^2}\right)\left[1 - \frac{2r^2}{a^2}\right] = 0.$

$1 - 2r^2/a^2 = 0.$

$1 = 2r^2/a^2$

$r^2/a^2 = 1/2$

$r^2 = a^2/2$

$r = a/\sqrt{2}$

Consider eq (2) we have.

$V_E = \frac{2kTe}{ea^2B}\exp\left(\frac{-r^2}{a^2}\right)\left[1 - \frac{10^{-2}}{\sqrt{2}}\right].$

$'' = \frac{2 \times 0.2 \times 1.6 \times 10^{-19} \; 1/\sqrt{2} \times 10^{-2} \exp(-1/2)}{1.6 \times 10^{-19} \times (1 \times 10^{-2})^2 \times 0.2}$

$V_E = 85 \text{ km/s}.$

Compose this with v_g due to earth's gravitational feild?

Sol: Compare gravitational force mg with earth's electric force eF for an ion (mg) for electron would be smaller.

$g = 9.8 ms^{-2}$

for K ions:-

$Mg = 39 \times 1.67 \times 10^{-27} \times 9.8$

$Fg = 6.38 \times 10^{-25} N$

Now E.F is given as

$eE = e(V_E . B)$.

$V_E = E/B$.

$eE = (1.6 \times 10^{-19})(85 \times 0.2)$

$Fe = 2.73 \times 10^{-18}$.

thus gravitational drift is

$\frac{Fg}{Fe} = \frac{6.38 \times 10^{-25}}{2.73 \times 10^{-18}} = 2.33 \times 10^{-7}$

c) To what value B be lowered?

Sol: Larmor radius is

$r_L = \frac{Mv_\perp}{qB} = \frac{M}{qB}\sqrt{\frac{2KTe}{M}}$.

and $r_L = a$ as given so,

$a = \frac{M}{qB}\sqrt{\frac{2KTe}{M}}$

$B = \frac{M}{qa}\sqrt{\frac{2KTe}{M}}$.

mass $= 39 \times 1.6 \times 10^{-27}$

$$B = \frac{6.5 \times 10^{-26}}{1.6 \times 10^{-19} \times 1 \times 10^{-12}} \sqrt{\frac{2 \times 0.2 \times 1.6 \times 10^{-19}}{6.5 \times 10^{-26}}}$$

$$\boxed{B = 4 \times 10^{-2}\,T.}$$

2.18) A 20keV electron in a large mirror fusion device has a pitch angle θ of 45° at the midplane where $B = 0.7\,T$ compute its Larmor radius = ?

$E = 20keV.$ Deuterium

$B = 0.7\,T.$ $m = 2$

$r_L = V_\perp / \omega_c = ?$ $q = \pm 1$

$$r_L = \frac{V_\perp m}{qB}.$$

$V_\perp = V \sin\theta$

$E = \frac{1}{2} m v^2$

$v^2 = 2E/m.$

$v = \sqrt{2E/m}.$

$$r_L = \frac{m \sqrt{2E/m} \sin\theta}{qB}$$

$$r_L = \frac{2 \times 1.6 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.7} \sqrt{\frac{2 \times 20 \times 10^3 \times 1.602 \times 10^{-19}}{2 \times 1.6 \times 10^{-27}}} \sin$$

$$\boxed{r_L = 0.3\,m.}$$

(14) In plasma heating by adiabatic compression, the invariance of μ requires that $KT_\perp$ increase as B increase. The magnetic feild, however, cannot accelerate particles b/c Lorentz force $qv \wedge B$ is always perpendicular to velocity. How do particles gain energy?

Sol: Maxwell tells us that Electric feild will be induced by changing Magnetic feild. The induced E.F is what ~~...~~ accelerates the particles.

(8) Suppose the Earth M.F is 3×10^{-5} T at 1/03 ---- plane

a) compute the ion and electron ∇B drift velocities?

Sol: The grad B drift given by

$$v_{\nabla B} = \frac{1}{2} v_\perp r_L \left| \frac{B \times \nabla B}{B^2} \right|$$

$$\text{"} = \frac{1}{2} v_\perp r_L \left| \frac{\nabla B}{B} \right| \rightarrow (1)$$

we know $v_\perp = \sqrt{2E/m}$ while

$$r_L = \frac{mv_\perp}{eB} = \frac{m}{eB}\sqrt{2E/m}.$$

put in eq (1)

$$v_{\nabla B} = \frac{1}{2}\sqrt{\frac{2E}{m}} \cdot \frac{m}{eB}\sqrt{2E/m} \left| \frac{\nabla B}{B} \right|.$$

$$\text{"} = \frac{E}{eB} \left| \frac{\nabla B}{B} \right| \rightarrow (2)$$

Since $B \sim r^{-3}$ we obtain.

$$\nabla B = \frac{\partial B}{\partial r}\hat{r}$$

$$" = \frac{-3}{r^4}$$

$$" = -3B/r$$

$$\left|\frac{\nabla B}{B}\right| = 3/r$$

So

$$V_{\nabla B} = \frac{3E}{eBr} = \frac{3Er^3}{eB_0R_e^3 r} = \frac{3Er^2}{eB_0R_e^3}$$

$$= \frac{3E(eV)r^2}{B_0R_e^3} \longrightarrow (3).$$

as $B = B_0R_0^3/r^3$.

putting values in (3) eq.

$$V_{\nabla B(e)} = \frac{3\times 30\times 10^3 \times (5\times 6\times 10^6)^2}{3\times 10^{-5}\times(6\times 10^6)^3}$$

$$= 1.3\times 10^4 \, m/s.$$

$$V_{\nabla B}(i) = \frac{3\times 1\times (5\times 6\times 10^6)^2}{3\times 10^{-5}(6\times 10^6)^3}$$

$$= 0.42 \, m/s.$$

b) Does electron drift eastward or westward?

Sol: The M.F is azimuthal, from north to south, i.e the $-\hat{\theta}$ direction. The gradient of M.F is clearly in the radial direction. so we have $B \times \nabla B \sim \hat{r} \times \hat{\theta} = \phi$ which is eastward. which is for electron while ions which cones without minus sign cancel minus sign an go in $-\phi$ which is ~~east to~~ westward.

c) How long it take to encircle?

Sol: well it has to travel distance $L = 2\pi (5 R_e)$ wit velocity $V_{\nabla B_e}$ So.

$$T = \frac{L}{V_{\nabla B e}} = \frac{2\pi\, 5(R_e)}{V_{\nabla B e B}} = \frac{2\pi 5 (6.4 \times 10^{6})}{1.3 \times 10^{4}}$$

$T = 4.5$ hours.

d) Compute ring current density in A/m^2?

Sol: current density given by J
$J = nev$ so using the grad B-velocity in this expression we get

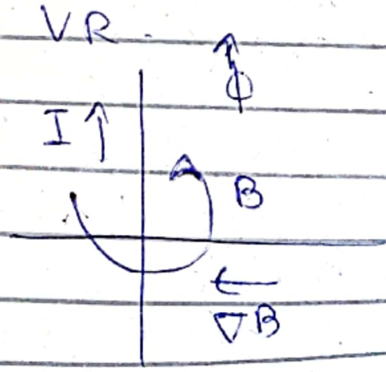
$$J = e n_e V_{\nabla B}$$
$$= 10^{7} \times 1.6 \times 10^{-19} \times 1.3 \times 10^{4}$$
$$J = 2 \times 10^{8}\ A/m^2$$

9. An electron lies at rest in M.F — — (a) Draw a diagram showing I, B, V_E, $V_{\nabla B}$ and V_R.

Sol: $\int B \cdot dl = \mu_o n I$

$B \int dl = \mu_o n I$

$B \propto \frac{\mu_o n I}{L}$



$$V_E = \frac{E \times B}{B^2} \quad \hat{r} \times \hat{\theta}$$

$V_E \sim z$

$V_{\nabla B} = -B \times \nabla B$

$" = -\theta \times -\hat{r}$

$" = -\hat{z}$

e' gyrate parallel to B.

$V_R = R \times B$

$= \hat{\theta} \times \hat{\theta}$

$= 0 = 0.$

$V_E = 0.$

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(a)

$R_m = 5$

$V_{\perp i} = V_{\parallel i}$ since μ is conserved

$V_{\perp f} = V_{\perp i}$

only $V_{\parallel}$ will increase.

and will increase until the pitch angle θ reaches the loss one

$$\sin^2\theta_m = \frac{V_{\perp f}^2}{V_{\perp f}^2 + V_{\parallel f}^2}$$

$$" = \frac{\cancel{V_{\perp f}^2}}{\cancel{V_{\perp}^2} f\left(1 + V_{\parallel f}^2 / V_{\perp f}^2\right)}$$

$$" = \frac{1}{1 + V_{\parallel f}^2 / V_{\perp f}^2}$$

smiller = $\frac{\quad}{Rm}$

" $= 1/5$

which means $\frac{V_{\parallel}^2 f}{V_{\perp}^2 i} = 4$

$V_{\parallel}^2 f = V_{\perp}^2 i \, 4$

$V_{\parallel} f = 2 V_{\perp} i$

Energy is $E_f = \frac{1}{2} M (V_{\parallel}^2 f + V_{\perp}^2 f)$

$E_f = \frac{1}{2} M (4 V_{\perp} i^2 + V_{\perp}^2 f)$

" $= \frac{1}{2} M (4+1) V_{\perp} i^2$

" $= \frac{5}{2} M V_{\perp} i^2$

$E_i = \frac{1}{2} M (V_{\parallel} i^2 + V_{\perp} i^2).$

$= \frac{1}{2} M (1+1) V_{\perp} i^2$

$E_i = M V_{\perp}^2$

$E_f = \frac{5}{2} E_i$

$E_f = \frac{5}{2} (1 \text{ keV}).$

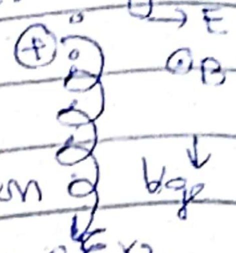
" $= 2.5$ keV.

(1.4) Show that V_E is same for two ions of equal mass and charge but different energies ------ so that fractional change in $V_{\perp}$ is small?

Sol: Let initial energy be $\mathcal{E}_0$ and Larmor radii r_1 and r_2

$r_1 \to$ for big circle

$r_2 \to$ small circle



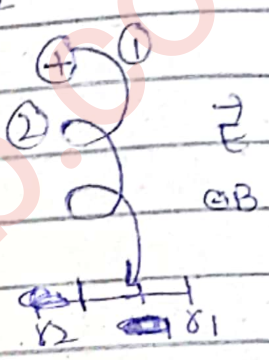
energy at (1)

Ions energy after acceleration by electric feild. $\mathcal{E}_1 = \mathcal{E}_0 - eEr_2$

energy at (2)

Ions after deacceleration.

$$\mathcal{E}_2 = \mathcal{E}_0 + eEr_2$$



Also

$$v_\perp = \sqrt{2E/m}$$

$$r_L = \frac{mv_\perp}{qB}$$

$$" = \frac{m}{|q|B}\sqrt{\frac{2E}{m}}$$

$$r_{1,2} = \frac{M}{|q|B}\sqrt{\frac{2\mathcal{E}_{1,2}}{M}}$$

$$" = \frac{M}{|q|B}\sqrt{\frac{2}{M}}\sqrt{\mathcal{E}_0 \pm eEr_{1,2}}$$

$$" = \frac{M}{qB}\sqrt{\frac{2}{M}\mathcal{E}_0}\left(1 \pm \frac{eEr_{1,2}}{\mathcal{E}_0}\right)^{1/2}$$

Bionomial expansion.

$$(1 \pm x)^n = 1 \pm nx + \frac{(n)(n-1)}{2!}x^2 + \text{---}$$

$$\left(1 \pm \frac{eEr_{1,2}}{\mathcal{E}_0}\right)^{1/2} = 1 \pm \frac{1}{2}\frac{e\,\mathcal{E}_{1,2}}{\mathcal{E}_0} + \text{---}$$

using 1st order term of expansion series.

$$r_{1,2} = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}\left(1 \pm \frac{1}{2}\frac{eE_{r_{1,2}}}{\varepsilon_0}\right)$$

$$'' = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}} \pm \frac{1}{2}\frac{eE_{r_{1,2}}}{\varepsilon_0} \cdot \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}$$

$$r_{1,2} \pm \frac{eE_{r_{1,2}}}{2\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}} = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}$$

$$r_{1,2}\left[1 \mp \frac{eE}{2\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right] = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}.$$

$$r_{1,2} = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}\left[1 \mp \frac{eE}{2\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right]^{-1}$$

Again bionomial expansion.

$$(1 \mp x)^{-n} = 1 \pm nx + \frac{n(n-1)}{2!}x^2 \pm - - -$$

ignoring higher terms we get

$$r_{1,2} = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}\left[1 \pm \frac{eE}{2\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right].$$

$$r_1 = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}\left[1 + \frac{1}{2}\frac{eE}{\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right].$$

$$r_2 = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}\left[1 - \frac{1}{2}\frac{eE}{\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right].$$

$$r_1 - r_2 = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}\left[\cancel{1} + \left(\frac{1}{2}\frac{eE}{\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right) \cancel{-1}\right.$$
$$\left. \frac{1}{2}\frac{eE}{\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right]$$

$$'' = \frac{1}{w_c}\sqrt{\frac{2\varepsilon_0}{M}}\left[\frac{\cancel{2}\,eE}{\cancel{2}\,\varepsilon_0 w_c}\sqrt{\frac{2\varepsilon_0}{M}}\right]$$

$$a = \frac{2eE_r}{\cancel{\varepsilon_0}\, w_c^2}\,\frac{2\cancel{\varepsilon_0}}{M}$$

which is independent of ε_0, The quity centre moves a distance $2(r_1-r_2)$ in a time $2\pi/\omega_c$ so:

$V = S \times T$.

$$V_{gc} = 2(r_1-r_2)\left(\omega_c/2\pi\right).$$

$$'' = \cancel{2}\left(\frac{2eE}{M\omega_c^2}\right)\left(\frac{\cancel{\omega_c}}{\cancel{2}\pi}\right)$$

$$V_{gc} = \frac{2eE}{M\omega_c \pi}.$$

using $\omega_c = \frac{|q|B}{m} \rightarrow B = \frac{m\omega_c}{q}$.

$$V_{gc} = \frac{2\cancel{e}E\cancel{M}}{\cancel{M}\pi\cancel{e}B}$$

$$'' = \frac{2E}{\pi B} \simeq E/B.$$

The factor $2/\pi$ would be 1 if did not make the crude approximation

$V_{gc} \rightarrow$ independent of ε_0