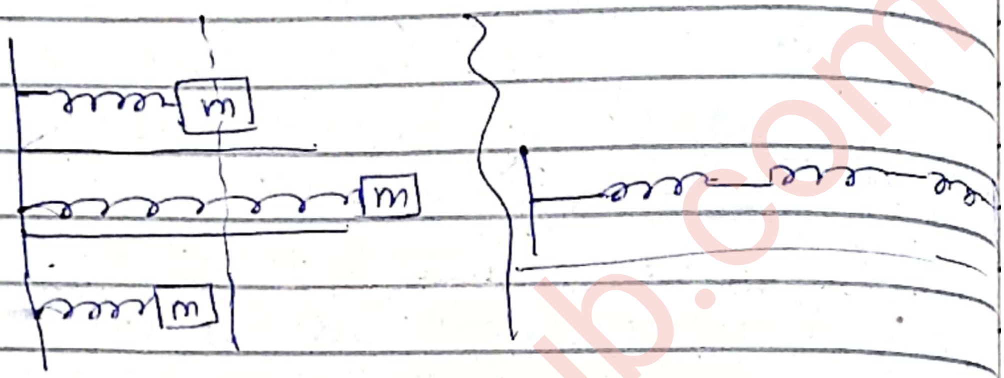
Ch #4
Waves in Plasma.

Source → Disturbance in a medium
Particles of medium (to and fro motion) → Produce oscillations.



if oscillations propagates we have wave in a system.

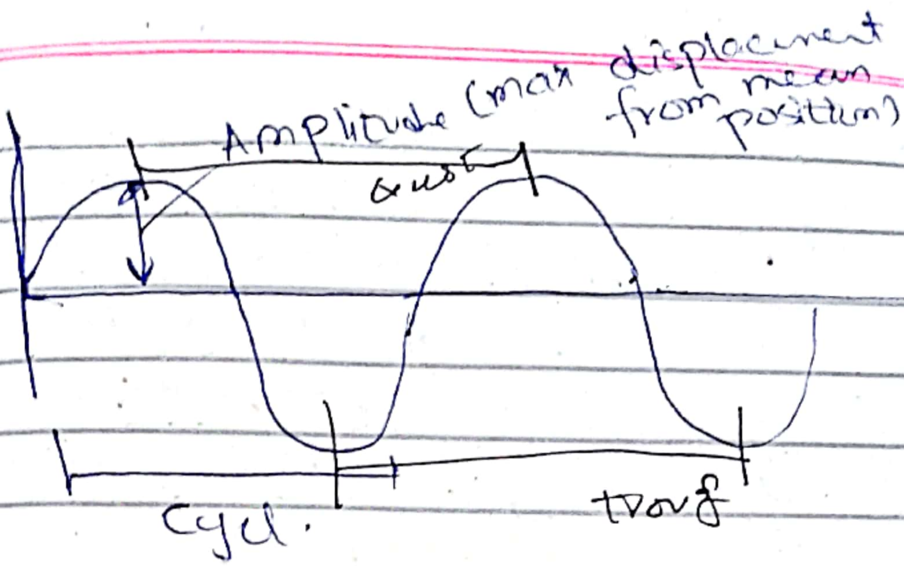
Types of wave:-

Waves in medium

- Mechanical wave → Medium Required
- Non Mechanical wave → They don't need medium for propogation
  EM wave

$\psi = \psi_0 e^{i(kx - \omega t)}$ ψ = amplitude

$\psi = \psi_0 e^{i\theta}$ $\theta = kx - \omega t$ = phase



t = Time period.

$f = 1/t$

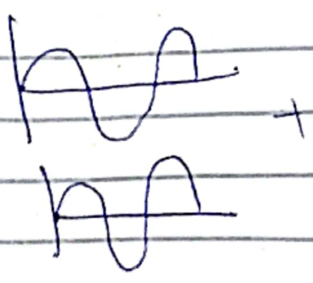
$\omega = 2\pi f$
↓
angular freq.

Distance b/w two consecutive crest and trough is wavelength.
- It tells how much it expands. crest, trough in terms wavelength
- Phase specifies or determine the position of particle.

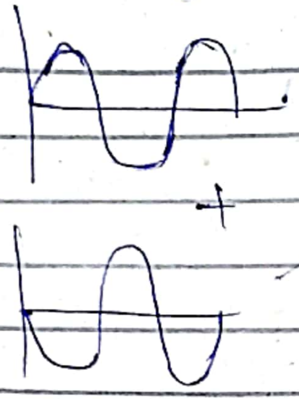
$\vec{K} = K\hat{x}$
↳ Direction of wave propagation
↓
wave no.
$K = 2\pi/\lambda$

Propagation vector. is the direction of propagation of wave

Supperposition of wave:-



Constructive interference
Crest → Crest
trough → trough
Resultant wave is with higher amplitude and it is in phase



When trough falls on crest and crest on trough is destructive interference and the combining waves are out of phase

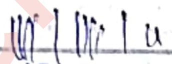
~~waves are produced to transfer~~

waves are produced ~~to~~ ~~transmission~~ for energy and particle transportation

Types of waves.

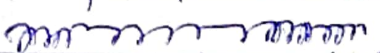
i) Longitutional.

compressions and rarefactions.



Compression: high pressure and high concentration of particle.

Rarefractions:- low pressure and low concentration of particle.



Rarefraction . compression

Parallel to propagation of wave

ii) transverse wave

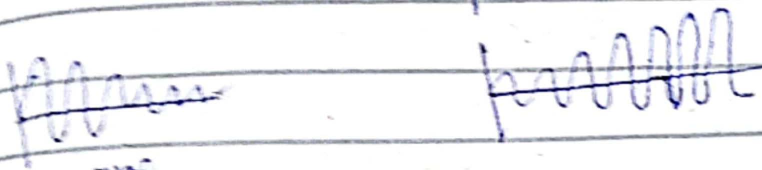
oscillations of the particles are ⊥ to the direction of propagation of wave vector

wave instabilities:-

Amplitude grow with wave propagation

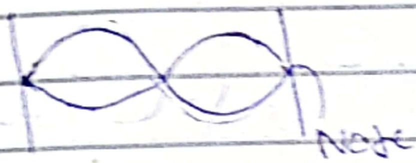
Wave Grow.

Amplitude decrease with wave propagation
wave damps



wave damp

Standing wave:-



wave localized Than you have Standing/ stationary wave.

Phase Velocity:-

speed of light.

$c = \omega/k$

$\omega = ck$

$v = dx/dt$

$v = \omega/k$ — $k \rightarrow$ wave no.

$v = \frac{2\pi f}{2\pi/\lambda}$ $\quad k = 2\pi/\lambda$

$\boxed{v = f\lambda}$ $\quad \because f = 1/t$

$v = \frac{\lambda}{t}$

$\psi = \psi_0 e^{i(kx-\omega t)}$

$\theta = kx - \omega t$ phase.

$$\frac{d\theta}{dt} = k\frac{dx}{dt} - \omega.$$

$$'' = kv - \omega.$$

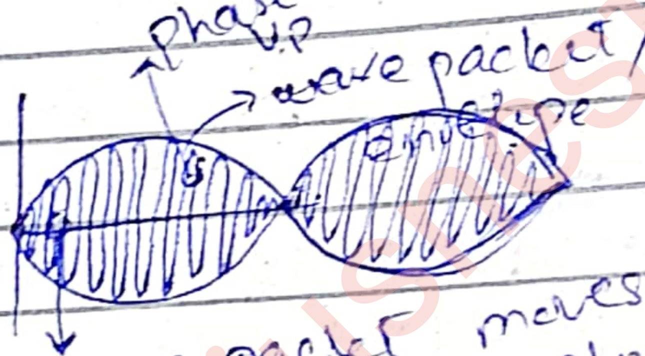
$$\boxed{0 = kv - \omega}$$ → when talking about

$$\omega = kv$$

$$\boxed{\frac{\omega}{k} = v.}$$

Phase velocity is the velocity of the particle

phase velocity.



wave packet moves with group velocity.

PLASMA OSCILLATIONS

To generate wave we need oscillators oscillation propagation generate wave in a system

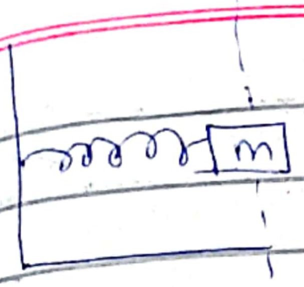
Dispersion relation:

In electromagnetic wave

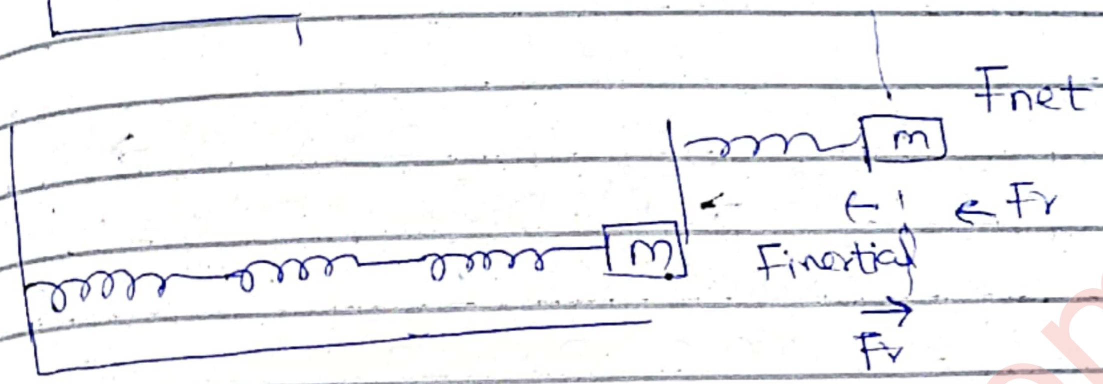
$\omega = ck$ EM wave/Light

It is a relation b/w ω and k

angular freq — wave no.



oscillators
(No wave)



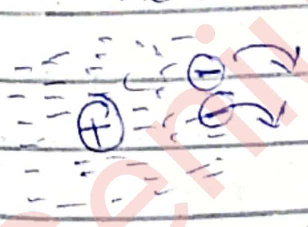
$\omega = \sqrt{K/m}$ — spring constant

oscillation freq

This is not a dispersion relation b/c K is a spring constant and these are only oscillators

We only consider $k \rightarrow$ wave no and ω

If we consider Plasma and we have Debye sheilding.



Debye Sheilding

Electron leave → plasma sheilding disturb

⊙ In plasma $n_i = n_e$

but when electron leave $n_i \neq n_e$

$\phi \neq 0$

So electric feild increase.

⊛ Source
- External E.F (disturb the system)
- Internal E.F

$-\oplus \ominus \quad E$

After restoring e^- will not static. It will oscillate for some time. Those oscillations are known as plasma oscillations.

Change inbalance create potential → this potential diff → Internal Electric feild → Restoring force. → Electron start oscillating about their mean position. So we have plasma oscillations then if electron oscillations propogate then we have wave or ion oscillations propagate.

If we perturb the debye sphere by applying some external electric feild both ion and electron respond to the electric feild

As ions are massive so when we take dynamics of electron. We take ions immobile.

$\vec{F}_{ext} = q \vec{E}_{ext}$

$m\vec{a}_e = q_e \vec{E}_{ext}$ $\qquad \because M >> m_i$

$\vec{a}_e = \frac{q_e \vec{E}_{xt}}{m}$ $\qquad a_i << a_e$

$M\vec{a}_i = q_i \vec{E}_{ext}$

$\vec{a}_i = q_i \vec{E}_{xt} / m$

As ions are massive as compared to the electron. So we take ions are immobile. When we study the dynamics of electron. As oscillations of e^- is very large as theirs mass is small whereas in case of ions their acceleration is small as their mass is large.

- Charge imbalance is created when e^- leaves the debye sphere and in this way an internal electric feild is sets up.

$$\vec{\nabla}\cdot\vec{E} = \sigma/\varepsilon_0$$

$$\sigma = en_i - en_e$$

$$\vec{\nabla}\cdot\vec{E} = \frac{en_i - en_e}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = e(n_i - n_e)/\varepsilon_0 \rightarrow (1)$$

internal E·F / restoring feild.

- This internal EF push the electron back to words their equillibrium position due to interia they keep on oscillating or moving. So they oscillate with frequency called the Plasma frequency.
- We now derive the expression for the plasma angular frequency ω_p in order to derive expression we assume.

1. There is no Magnetic feild (unmagnetized Plasma)
2. There is no thermal motion ($k_BT = 0$) → cold plasma.
3. Ions are fixed in space or taken immobile.
4. The plasma is infinite in extent (large no densities)

(5) The electron motion occurs in x-direction

$\vec{\nabla} = \partial/\partial x \, \hat{x}$

$\vec{E} = E\hat{x}$

$\vec{\nabla} \times \vec{E} = 0$

$E = -\vec{\nabla}\phi \rightarrow$ external potential

↓ internal EF in one direction.

Electrostatic oscillations b/c magnetic feild is not applied.

- **eq of motion** for electrons in convective derivative form:-

$$mn_e\left[\frac{\partial}{\partial t}\vec{V} + (\vec{V_e}\cdot\vec{\nabla})\vec{V_e}\right] = q_{e}n_e\vec{E} \rightarrow (1)$$

$$mn_e\left[\partial/\partial t\,\vec{V} + (\vec{V_e}\cdot\vec{\nabla})\right]\vec{V_e} = -en_e\vec{E} \rightarrow (1)$$

- **eq of continuity**:-

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\vec{V_e}) = 0$$

- **Possion eq**:

$$\vec{\nabla}\cdot\vec{E} = en_i - en_e/\varepsilon_o.$$

$$-\vec{\nabla}\cdot\vec{\nabla}\phi = en_i - en_e/\varepsilon_o$$

$$-\nabla^2\phi = en_i - en_e/\varepsilon_o$$

$$\nabla^2\phi = e(n_e - n_i)/\varepsilon_o \rightarrow (3)$$

eq (1) to (3)

These equations can be solved easily by the method of linearalization. By this we mean that amplitude of oscillation is small and ion containing higher powers of amplitude factors are neglected.

We separate the dependent variables into two parts an equillibrium part and perturb part. Equillibrium part indicate by Subscript 0 and perturb are indicated by subscript 1.

$n_e = n_e + n_i$

$\vec{V_e} = \vec{V_0} + \vec{V_1} \rightarrow (i)$

$\vec{E} = \vec{E_0} + \vec{E_1} \rightarrow (ii)$

* ions are massive and not perturb. ions are working at background

initially before perturbation

$V_0 = 0$

as initially no E.F

$E_0 = 0$

We take uniform equillibrium, no density

$\nabla n_0 = 0$

$\frac{\partial n_0}{\partial t} = 0$

$\frac{\partial V_0}{\partial t} = 0$

$\frac{\partial E_0}{\partial t} = 0$

$m(n_0 + n_i)\left[\frac{\partial}{\partial t}(V_0 + V_1) + (V_0 + V_1)\cdot\nabla\right] V_0 + V_1 = -e(n_0 + n_1) E_0 + E_1$

Putting $V_0 = 0$, $E_0 = 0$

$m(n_0 + n_i)\left[\frac{\partial}{\partial t}\vec{V_1} + (\vec{V_1}\cdot\vec{\nabla})\right] = -e(n_0 + n_i) E_1$

Since n_0 is greater than n_i so,

$n_0 >>> n_1$

$$mn_0\left[\frac{\partial \vec{V_1}}{\partial t} + (\vec{V_1}\cdot\nabla)\vec{V_1}\right] = -en_0\vec{E_1}$$

$$m\left[\frac{\partial}{\partial t}\vec{V_1} - (\vec{V_1}\cdot\nabla)\vec{V_1}\right] = -e\vec{E_1}$$

Appling method of linearization

→ as second term is non-linear term so neglect this

$$\boxed{m\frac{d\vec{V_1}}{dt} = -e\vec{E_1}}$$ → Linearlized form of eq ①

$$\frac{\partial n_0}{\partial t} = \nabla_1(n_0\vec{V_e}) = 0$$

$$\frac{\partial}{\partial t}(n_0+n_1) + \nabla\cdot\left[(n_0+n_1)(\vec{V_e}^{\,0}+\vec{V_1})\right] = 0$$

$$\frac{\partial n_1}{\partial t} + \nabla\cdot\left[(n_0+n_1)\vec{V_1}\right] = 0$$

$$\frac{\partial n_1}{\partial t} + \nabla\cdot(n_0\vec{V_1}) + \underbrace{\vec{\nabla}\cdot(n_1\vec{V_1})}_{} = 0$$

non linear term so ignore this b/c it has a very low amplitude as two perturbed quantities are multipling here.

$$\boxed{\frac{\partial n_1}{\partial t} + \nabla\cdot(n_0\vec{V_1}) = 0}$$

↓

linearlized form of eq ②

Taking possion eq :-

$$\nabla^2\phi = e/\varepsilon_0 (n_e - n_i)$$

$$\nabla\cdot E = \frac{e}{\varepsilon_0}(n_i - n_0) \rightarrow (3'')$$

$$\nabla\cdot(\cancel{E_0}^{\,0} + E_1) = \frac{e}{\varepsilon_0}[n_0 - (n_0 + n_1)]$$

$$\boxed{\nabla\cdot E_1 = -\frac{e}{\varepsilon_0}(n_1)} \rightarrow \text{linearized form of eq (3)}$$

$$\boxed{\partial E_1/\partial x = e/\varepsilon_0 (n_1)} \rightarrow (4) \rightarrow \text{In 1 D}$$

$$m\frac{d\vec{v}_1}{dt} = -eE_1 \rightarrow (1)$$

$$\frac{\partial n_1}{\partial t} + \nabla\cdot(n_0 v_1) \rightarrow (2)$$

$$\nabla\cdot E_1 = -e/\varepsilon_0 (n_1) \rightarrow (3)$$

These are 3 linearized form of eq

The oscillating quantities are assumed to behave sinusoidally. → in x not in y b/c we take this in 1D.

sinosidual approximation

$$\vec{V}_1 = V_1 e^{i(kx - \omega t)}\hat{x} \rightarrow (1')$$

$$n_1 = n_1 e^{i(kx-\omega t)} \rightarrow (2')$$

$$\vec{E}_1 = E_1 e^{i(kx-\omega t)}\hat{x} \rightarrow (3')$$

$$\frac{\partial\vec{v}_1}{\partial t} = -iv_1\omega e^{i(kx-\omega t)}\hat{x} \rightarrow (5)$$

Using (5) and (3') in eq (1) we get

$$\left[+im v_1\omega \cancel{e^{i(kx-\omega t)}}\cancel{\hat{x}}\right] = +eE_1\cancel{e^{i(kx-\omega t)}}\cancel{\hat{x}}$$

$$\boxed{im v_1\omega = eE_1} \rightarrow (6)$$

Rewrite :-

$$\frac{\partial n_1}{\partial t} + n_0 \vec{\nabla}\cdot\vec{V_1} + \vec{V_1}\cdot\cancel{\vec{\nabla} n_0}^{\,0} = 0$$

$$\vec{\nabla}\cdot(a\vec{A}) = a\,\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla} a$$

$$\frac{\partial n_1}{\partial t} + n_0\left(\frac{\partial}{\partial x}\hat{x}\cdot V_1\hat{x}\right) = 0$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial V_1}{\partial x} = 0 \rightarrow (4')$$

$$\frac{\partial n_1}{\partial t} = -i\omega n_1 e^{i(kx-\omega t)} \rightarrow (7)$$

$$\frac{\partial V_1}{\partial x} = ikV_1 e^{i(kx-\omega t)} \rightarrow (8)$$

Using (7) and (8) in (4')

$$-i\omega n_1 e^{i(kx-\omega t)} + n_0 ikV_1 e^{i(kx-\omega t)} = 0$$

$$n_0 \cancel{i} kV_1 \cancel{e^{i(kx-\omega t)}} = \cancel{i}\omega n_1 \cancel{e^{i(kx-\omega t)}}$$

$$\boxed{n_0 kV_1 = \omega n_1} \rightarrow (9)$$

$$\boxed{\frac{n_0 kV_1}{\omega} = n_1} \rightarrow (5')$$

$$E_1 = E_1 e^{i(kx-\omega t)}$$

$$\frac{\partial E_1}{\partial t} = ikE_1 e^{i(kx-\omega t)} \rightarrow (10)$$

Using (10) and (2') in eq (4)

$$ikE_1\cancel{e^{i(kx-\omega t)}} = \frac{e}{\varepsilon_0} n_1 \cancel{e^{i(kx-\omega t)}}$$

$$\boxed{ikE_1 = \frac{e}{\varepsilon_0} n_1} \rightarrow (11)$$

Eliminate V_1, n_1 and E_1 from eq

(6), (9) and (11)

Using (6) in (11)

$$ik(-imV_1\omega) = e/\varepsilon_0 \, n_1 \rightarrow (12)$$

From eq (9) $\frac{n_0 k V_1}{\omega} = n_1 \rightarrow (5')$

Using (5') in (12) eq

$$-ik\,imV_1\omega = \frac{e}{\varepsilon_0}\frac{n_0 k V_1}{\omega}$$

$$m\omega = \frac{e}{\varepsilon_0}\frac{n_0}{\omega}$$

$$\omega^2 = \frac{en_0}{\varepsilon_0 m}$$

$$\omega = \sqrt{\frac{en_0}{\varepsilon_0 m}}$$

11/12/23 Lecture {Clear Rewrite of previous eq's}

→ Eq of motion:-

$$m n_e \left[\frac{\partial}{\partial t}\vec{V} + (\vec{V_o}\cdot\vec{\nabla})\vec{V_o}\right] = q n_e E \rightarrow (i)$$

$$m n_e \left[\frac{\partial}{\partial t}\vec{V} + (\vec{V}\cdot\vec{\nabla})\right] V_e = -e n_e E \rightarrow (1)$$

→ Eq of continuity:-

$$\frac{\partial n_e}{\partial t} + \nabla(n_e V_o) = 0 \rightarrow (2)$$

→ Possion eq:-

$$\nabla\cdot E = e n_i - e n_e/\varepsilon_o$$

$$-\nabla\cdot(\nabla\phi) = \text{''}$$

$$\nabla^2\phi = \frac{e(n_e - n_i)}{\varepsilon_o} \rightarrow (3)$$

As: ions are massive working in background, so perturbed take

$$n_e = n_o + n_i \rightarrow (i)$$

$$V_e = V_o + V_i \rightarrow (ii)$$

$$E_e = E_o + E_i \rightarrow (iii)$$

Initially $\boxed{V_o = 0}$ as no E.F so, $\boxed{E_o = 0}$

also we take no uniform density

$$\nabla n_e = 0$$

$$\frac{\partial n_e}{\partial t} = 0 \;;\; \frac{\partial V_o}{\partial t} = 0 \;;\; \frac{\partial E_o}{\partial t} = 0$$

Putting (i), (ii), (iii) in eq (1)

$$m(n_o + n_i)\left[\frac{\partial}{\partial t}(V_o + V_i) + (V_o + V_i)\cdot\nabla\right](V_o + V_i) = -e(n_o + n_i)(E_o + E_i).$$

Putting $V_o = 0$, $E_o = 0$

$n_o >> n_i$

$$mn_0\left[\frac{\partial v_1}{\partial t}+(v_1\cdot\nabla)v_1\right]=-en_0\vec{E_1}$$

$$m\left[\frac{\partial}{\partial t}\vec{v_1}-(\vec{v_1}\cdot\nabla)\vec{v_1}\right]=-eE_1$$

as applying linerization so second term is non-linear term so neglect this:-

$$\boxed{m\frac{d\vec{v_1}}{dt}=-e\vec{E_1}} \rightarrow \text{linearized eq of } 1$$

$$\frac{\partial n_0}{\partial t}=\nabla_1(n_0\vec{v_e})=0$$

$$\frac{\partial}{\partial t}(n_0+n_1)+\nabla\left[(n_0+n_1)(\cancel{v_e}^{0}+v_1)\right]=0$$

$$\frac{\partial n_1}{\partial t}+\nabla\cdot\left[(n_0+n_1)(v_1)\right]=0$$

$$\frac{\partial n_1}{\partial t}+\nabla\cdot(n_0\vec{v_1})+\vec{\nabla}\underbrace{(n_1\vec{v_1})}_{\text{Non-Linear term.}}=0$$

$$\boxed{\frac{\partial n_1}{\partial t}+\nabla\cdot(n_0\vec{v_1})=0} \rightarrow 2 \text{ Linearized eq}$$

$$\nabla\cdot E=\frac{e(n_i-n_e)}{\epsilon_0}$$

$$\nabla(\cancel{E_0}^{0}+E_1)=\frac{e}{\epsilon_0}\left[n_0-(n_0+n_1)\right]$$

$$\nabla E_1=\frac{e}{\epsilon_0}\left[\cancel{n_0}-\cancel{n_0}-n_1\right]$$

$$\boxed{\nabla E_1=-\frac{e\,n_1}{\epsilon_0}} \rightarrow \text{linearized eq } 3$$

$$\frac{\partial E_1}{\partial x}=-\frac{e}{\epsilon_0}(n_1)\rightarrow 4$$

Sinosidual approximations:-

$$\vec{v_1}=v_1e^{i(kx-\omega t)}\hat{x}\rightarrow 1'$$

$$n_1=n_1e^{i(kx-\omega t)}\rightarrow 2'$$

$$E_1=E_1e^{i(kx-\omega t)}\hat{x}\rightarrow 3'$$

$$\frac{\partial V_1}{\partial t} = -iV_1 \omega e^{i(kx-\omega t)} \hat{x} \rightarrow (5)$$

Using (5) and (3') in eq (1) we get:-

$$+m(iV_1\omega \cancel{e^{i(kx-\omega t)}} \hat{x}) = +eE_1 \cancel{e^{i(kx-\omega t)}} \hat{x}$$

$$\boxed{+im\omega V_1 = eE_1 \rightarrow (6)}$$

eq of motion

$im\omega V_1 = eE_1$

$$\frac{\partial n_1}{\partial t} + n_0 \vec{\nabla} \vec{V_1} + \vec{V_1} \cancel{\vec{\nabla} n_0}^{0} = 0$$

$$\vec{\nabla}(a\vec{A}) = a\vec{\nabla}\vec{A} + \vec{A} \cdot \vec{\nabla} a.$$

$$\frac{\partial n_1}{\partial t} + n_0\left(\frac{\partial}{\partial x}\hat{x} \cdot V_1 \hat{x}\right) = 0$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial V_1}{\partial x} = 0 \rightarrow (4')$$

$$\frac{\partial n_1}{\partial t} = -i\omega n_1 e^{i(kx-\omega t)} \rightarrow (7)$$

eq of continuity

$-i\omega n_1 + n_0 ik V_1 =$

$$\frac{\partial V_1}{\partial x} = ikV_1 e^{i(kx-\omega t)} \rightarrow (8)$$

Putting (7), (8) in (4')

$$\boxed{-i\omega n_1 e^{i(kx-\omega t)} + n_0 ikV_1 e^{i(kx-\omega t)} = 0}$$

$$n_0 \cancel{i} kV_1 \cancel{e^{i(kx-\omega t)}} = +\cancel{i}\omega n_1 \cancel{e^{i(kx-\omega t)}}$$

$$n_0 kV_1 = \omega n_1 \rightarrow (9)$$

$$\frac{n_0 kV_1}{\omega} = n_1 \rightarrow (5')$$

From eq (3')

$$\frac{\partial E_1}{\partial t} = ikE_1 e^{i(kx-\omega t)} \rightarrow (10)$$

Using (10) and (2') in eq (4)

$$ikE_1 \cancel{e^{i(kx-\omega t)}} = -\frac{e}{\varepsilon_0} n_1 \cancel{e^{i(kx-\omega t)}}$$

$$\boxed{ikE_1 = -\frac{e}{\varepsilon_0} n_1 \rightarrow (11)}$$

Gauss's Law

Eliminate V_1, n_1 and E_1 — (6), (9) & (11)

Using (6) in (11)

$$ik\left(\frac{imwV_1}{e}\right) = \frac{-e}{\varepsilon_0}n_1 \quad \text{?}$$

$$m\omega V_1 k = \frac{e^2}{\varepsilon_0}n_1$$

$$\omega = \frac{e^2 \quad n_1}{\varepsilon_0 V_1 k m} \rightarrow (12)$$

using (5) in eq (12)

$$\omega = \frac{e^2}{\varepsilon_0 \cancel{V_1} \cancel{k} m} \frac{n_0 \cancel{k} \cancel{V_1}}{\omega}$$

$$\omega^2 = \frac{e^2 n_0}{\varepsilon_0 m}$$

$$\boxed{\omega = \sqrt{\frac{e^2 n_0}{\varepsilon_0 m}}}$$ Angular freq

Ions:-

$$\omega_i = \sqrt{\frac{e^2 n_0}{\varepsilon_0 M}}$$

$\omega = 2\pi f$

$f = \omega/2\pi$

$$f_{pi} = \frac{1}{2\pi}\sqrt{\frac{n_0 e^2}{\varepsilon_0 M}}$$

Electron:-

$$\omega_e = \sqrt{\frac{e^2 n_0}{\varepsilon_0 m}}$$

$$f_{ei} = \frac{1}{2\pi}\sqrt{\frac{n_0 e^2}{\varepsilon_0 m}}$$

$$f_{pi} = \frac{1}{2\pi}\sqrt{\frac{n_o e^2}{\varepsilon_o M}}$$

$$f_{ei} = \frac{1}{2\pi}\sqrt{\frac{n_o e^2}{\varepsilon_o m}}$$

→ Take a system when no. density is $n_o = 10^{18} m^{-3}$ Find angular freq (?) and linear frequency(?) for electron and ion?

$\omega_{pe} = 9\ GHz \cong 9 \times 10^9\ Hz \sim rad/sec$ (angular displacement/unit)

$f_e = 28\ GHz$.

$\omega \sim (n_o)^{1/2}$ → plasma angular freq increases with n_o and plasma angular freq decreases with m.

- Electron plasma oscillations $\xrightarrow{\text{propagating}}$ Electron plasma wave

- Ion plasma oscillations. $\xrightarrow{\text{propagate}}$ Ion plasma wave.

Assignment:-

eq of motion, possion eq, continuity eq by changing variations in temp?

① Ions are stationary = eq of motion trasformed in the derivation

$$mn_e\left[\frac{\partial V_e}{\partial t} + (\vec{V_e}.\vec{\nabla})\vec{V_e}\right] = -eE_1 - \nabla P_e$$

unmagnetized plasma (isothermal).

$P_e = \gamma n_e k_B T_e$

$\gamma = \frac{N+2}{N}$ → Book Article 4.29.

in 1D

$\gamma = \frac{1+2}{1}$

$\boxed{\gamma = 3}$

Make
- Linearized form of eq.
- Also apply sinosidual approximation

$P_e = 3 k_B T_e n_e$

$\nabla P_e = 3 k_B T_e \vec{\nabla} n_e$

electrons are following Maxwell Boltzman eq (MB). dist so

$\vec{\nabla} T_e = 0$

$3 k_B T_e \vec{\nabla}(n_0 + n_1)$

$3 k_B T_e \vec{\nabla} n_1 + 3 k_B T_e \cancel{\vec{\nabla} n_0}^{\,0}$

Eq of motion:-

$$mn_e\left[\frac{\partial \vec{V_e}}{\partial t}+(\vec{V_e}\cdot\vec{\nabla})\vec{V_e}\right]=-eE_1-\vec{\nabla}P_e \rightarrow (1)$$

$$P_e=\gamma n_e k_B T_e$$

$$\gamma=N+2/N$$

$$N=1$$

$$\gamma=1+2/1=3$$

$$P_e=3n_e k_B T_e$$

$$\nabla P_e=3n_e k_B\vec{\nabla}T_e+3T_e k_B\vec{\nabla}n_e$$

$$'' = 3T_e k_B\vec{\nabla}n_e+3n_e k_B\vec{\nabla}T_e$$

So we see that electron following Boltzman dist so $\nabla T_e=0$.

$$\nabla P_e=3T_e k_B\vec{\nabla}n_e \rightarrow (2)$$

Now put (2) in (1)

$$mn_e\left[\frac{\partial V_e}{\partial t}+(\vec{V_e}\cdot\vec{\nabla})\vec{V_e}\right]=-eE_1-3T_e k_B\vec{\nabla}n_e$$

$n_e=n_0+n_1$, $V_e=V_0+V_1$, $E_e=E_0+E_1$

Ions working on background and massive so $\vec{\nabla}n_0=0$

$$=3k_B T_0\vec{\nabla}n_e$$

$$=3k_B T_e\vec{\nabla}(n_0+n_1)$$

$$=3k_B T_e\cancel{\vec{\nabla}n_0}^{0}+3k_B T_e\nabla n_1$$

$$=3k_B T_e\vec{\nabla}n_1$$

$$mn_e\left[\frac{\partial V_e}{\partial t}+(\vec{V_e}\cdot\vec{\nabla})\vec{V_e}\right]=-en_eE_1-3k_B T_e\nabla n_1 \quad (1)$$

$$m(n_0+n_1)\left[\frac{\partial v_{e1}}{\partial t}\right] = -e(n_{e0}+n_{e1})E_1 - 3k_B T_e \vec{\nabla}(n_{e0}-n_{e1})$$

$$\Rightarrow \nabla n_{e0} = 0$$

$$m(n_0+n_1)\left[\frac{\partial(v_0+v_1)}{\partial t} + \left((v_0+v_1)\cdot\nabla\right)(v_0+v_1)\right] =$$

$$-e(n_0+n_1)(E_0+E_1) - 3k_B T_e \vec{\nabla} n_1$$

$n_0 >> n_1$

$v_0 = 0 \quad E_0 = 0 \quad n_1 = 0$

$$mn_0\frac{\partial v_1}{\partial t} + \underbrace{(\vec{v_1}\cdot\vec{\nabla})\vec{v_1}}_{\text{non linear}} = -en_0E_1 - 3k_BT_e\vec{\nabla}n_1$$

$$\boxed{mn_0\frac{\partial v_1}{\partial t} = -en_0E_1 - 3k_BT_e\nabla n_1} \rightarrow (1)$$

Linearized form of eq of motion

$$mn_0\frac{\partial v_1}{\partial t} = -en_0E_1 - 3k_BT_e\frac{\partial n_1}{\partial x}\hat{x}$$

$$\frac{\partial n_e}{\partial t} + \nabla(n_eV_e) = 0 \rightarrow (2)$$

$$\frac{\partial}{\partial t}(n_0+n_1) + \nabla(n_0+n_1)(\overset{0}{V_0}+V_1) = 0$$

$$\frac{\partial n_1}{\partial t} + \nabla(n_0V_1) + \underbrace{\vec{\nabla}(n_1V_1)}_{\text{non linear}} = 0$$

$$\boxed{\frac{\partial n_1}{\partial t} + \nabla(n_0V_1) = 0} \rightarrow (2) \text{ linearized}$$

$$\nabla^2\phi = \frac{e}{\epsilon_0}(n_e - n_i)$$

$$\nabla\cdot E = e/\epsilon_0\,(n_i - n_e) \rightarrow (3)$$

$$\nabla(E_0+E_1) = \frac{e}{\epsilon_0}\left[n_0 - (n_0+n_1)\right]$$

$$\nabla\cdot E_1 = \frac{e}{\epsilon_0}\left[\cancel{n_0} - \cancel{n_0} - n_1\right]$$

$$\boxed{\nabla\cdot E_1 = -\frac{e\,n_1}{\epsilon_0}} \rightarrow (3) \text{ linearized}$$

$$\partial E_1/\partial x = -\frac{e}{\varepsilon_0}(n_1) \rightarrow 3''$$

The oscillating quantities V_1, n_1, E_1 are assumed to behave sinusidually

$$V_1 = \vec{V_1}\, e^{i(kx-\omega t)}\hat{x} \rightarrow 1'$$

$$n_{e1} = n_{e}e^{i(kx-\omega t)} \rightarrow 2'$$

$$E_1 = E_1 e^{i(kx-\omega t)}\hat{x} \rightarrow 3'$$

Derivative of 1'

$$\frac{\partial V_1}{\partial t} = -iV_1\omega\, e^{i(kx-\omega t)}\hat{x} \rightarrow 4$$

$$\frac{\partial n_1}{\partial x} = ikn_1 e^{i(kx-\omega t)} \rightarrow 2''$$

using 2'' 3' and 4 in 1

$$V_1 m n_0(-i\omega e^{i(kx-\omega t)})\hat{x} = -e\, n_0 E_1 e^{i(kx-\omega t)} -$$

$$-m n_0 i\omega V_{e1} = -en_{e0}E_1 - 3ik_BT_e k n_{e1}$$

$$\boxed{m n_e i\omega V_{e1} = en_{e0}E_1 + 3ik_BT_e k n_{e1}} \rightarrow 2'$$

$$\boxed{-i\omega n_{e1} + n_{e0} ik V_{e1} = 0} \rightarrow 3'$$

$$ikE_1 = \frac{e}{\varepsilon_0}(n_{i0} - (n_{e0} + n_{e1}))$$

$$n_{i0} = n_{e0} = n_0$$

$$\boxed{ikE_1 = \frac{e}{\varepsilon_0} n_{e1}} \rightarrow 4'$$

From 4' $\quad n_{e1} = \frac{-\varepsilon_0 ikE_1}{e}$

Governing eq's
↓
Linearized
↓
apply sinsodinal app
↓
Eliminate unknown variables
↓
oscillation freq / Dispersion relation

From (3') $V_e = i\omega n_{e1}/n_{e0} ik$

$$\frac{m n_0 i\omega \cdot n_{e1}}{n_0 ik} = e n_{e0} E_1 - \frac{3ik_B T_e k \varepsilon_0 ik E_1}{e}$$

$$-\frac{m i\omega^2 n_{e1}}{k} = e n_{e0} E_1 + \frac{3k_B T_e k^2 \varepsilon_0}{e} E_1$$

$$-\frac{m i\omega^2 \varepsilon_0 ik E_1}{ke} = e n_{e0} E_1 + \frac{3k_B T_e k^2 \varepsilon_0 E_1}{e}$$

$$m\omega^2 \varepsilon_0 = e^2 n_{e0} + 3k_B T_e k^2 \varepsilon_0$$

$$\omega^2 = \frac{e^2 n_{e0}}{m\varepsilon_0} + \frac{3k_B T_e k^2}{m} \rightarrow (1)$$

as $\omega_p^2 = \frac{e^2 \varepsilon_0}{m\varepsilon_0} \rightarrow (2)$

Using (2) in (1)

$$\boxed{\omega^2 = \omega_{pe}^2 + \frac{3k_B T_e k^2}{m}} \rightarrow (3)$$

ions fixed in background

As

$\frac{1}{2} mv^2 = K_B T$

$V_{th}^2 = 2K_B T/m$

$\frac{V_{th}^2}{2} = \frac{K_B T}{m}$ {Arranged according to need}.

(9) using (9) in (3)

$$\boxed{\omega^2 = \omega_{pe}^2 + \frac{3}{2} K^2 V_{th}^2} \rightarrow (4)$$

$\omega(K)$

eq (3) is dispersion relation for electron plasma wave.

→ Dispersion Relation is Relation b/w Angular Frequency and wave number K
eq (3) is the eq of wave as it satisfy the dispersion relation.

Dispersion Relation

we can find

Wave phase velocity.

$V_p = \omega/K$

Group velocity.

$V_g = \frac{\partial \omega}{\partial K}$.

÷ eq (4) by K^2.

$$\frac{\omega^2}{K^2} = \frac{\omega_{pe}^2}{K^2} + \frac{3}{2} V_{th}^2$$

$$V_p^2 = \frac{\omega_{pe}^2}{K^2} + 3/2\, V_{th}^2$$

$$\boxed{V_p = \sqrt{\frac{\omega_{pe}^2}{K^2} + 3/2\, V_{th}^2}}$$ → Phase velocity for electron plasma wave.

For group velocity :-

$$2\omega \frac{\partial \omega}{\partial k} = \frac{3}{\cancel{2}} \cancel{2} k V_{th}^2$$

$$\frac{\partial \omega}{\partial k} = \frac{3}{2} \frac{k}{\omega} V_{th}^2$$

$$V_g = \frac{3}{2} \frac{1}{\omega / k} V_{th}^2$$

$$\boxed{V_g = \frac{3}{2} \frac{V_{th}^2}{V_p}}$$ → Group velocity for electron plasma wave.

$$V_g V_p = \frac{3}{2} V_{th}^2 \qquad \because V_{th}^2 \sqrt{\frac{2 k_B T_e}{m}}$$

⇒ Increase in group velocity. Decrease in Phase velocity. (as Inversely Related).

$$V_g \simeq 1/V_p$$

Equation :- → Curve plotting

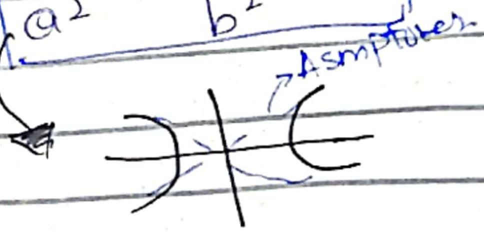
$$\omega^2 = \omega_{pe}^2 + \frac{3}{2} k^2 V_{th}^2$$

(5)

$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$$ → Hyperbola form in Transverse axis

(6)

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$ → Hyperbola formed in Horizontal transverse axis



Lecture 12/12/23

heading Assignment

→ In exam write in detail all assumptions each and every ~~thing~~ matched with previous one

Plasma Oscillations

Ion plasma oscillations (low freq) → Propagating → Ion plasma wave

Electron Plasma oscillations (high freq) → Propagating → Electron Plasma wave

Electron Plasma wave:-

We take thermal effects of electron

$$m n_e\left[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e\cdot\vec{\nabla})\vec{v}_e\right] = -e n_e \vec{E}_1 - \vec{\nabla} P_e \rightarrow (1)$$

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}\cdot(n_e \vec{v}_e) = 0 \rightarrow (2)$$

$$\nabla\cdot E = \frac{e}{\varepsilon_0}(n_i - n_e) \rightarrow (3)$$

$$P_e = \gamma k_B T_e n_e$$

$$\gamma = \frac{N+2}{N}$$

$$N = 1$$

$$P_e = 3 k_B T_e n_e$$

$$\vec{\nabla} P_e = 3 k_B T_e \vec{\nabla} n_e + 3 k_B n_e \nabla T_e$$

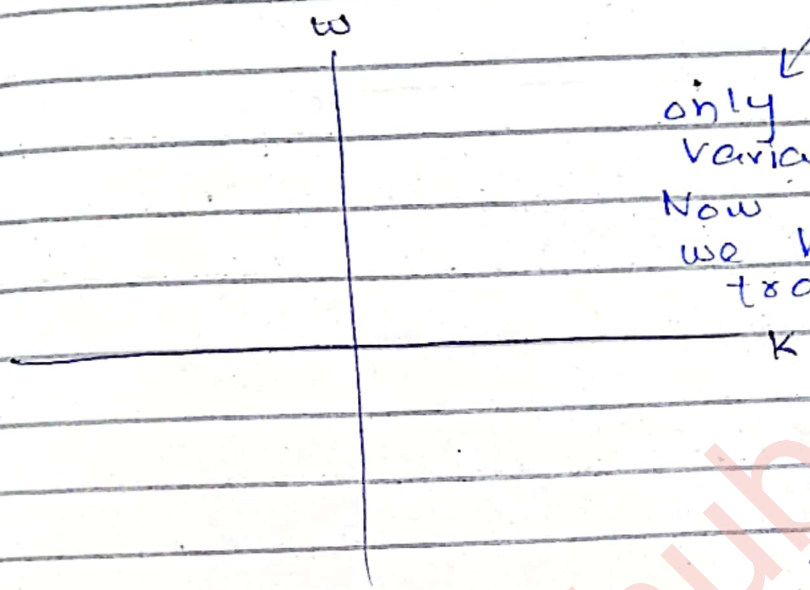
If we assume that electrons we following Boltzmann dist then $\nabla T_e = 0$

$$\vec{\nabla} P_e = 3 k_B T_e \vec{\nabla} n_e$$

$$\omega^2 = \omega_{pe}^2 + 3/2\, k^2 v_{th}^2 \rightarrow ③$$

$$\omega^2 - \omega_{pe}^2 = \frac{3 k^2 v_{th}^2}{2} \rightarrow ④$$

$\omega(k)$



$\omega \rightarrow$ Dependent
$k \rightarrow$ Independent variable.

only variables decide
Now decide whether we have hyperbola transverse or in k horizontal axis.

$\div$ eq ④ by ω_{pe}^2 we get

$$\frac{\omega^2}{\omega_{pe}^2} - 1 = \frac{3}{2}\frac{k^2 v_{th}^2}{\omega_{pe}^2}$$

$$\frac{\omega^2}{\omega_{pe}^2} - \frac{3}{2}\frac{k^2 v_{th}^2}{\omega_{pe}^2} = 1 \rightarrow ⑦$$

Similar things with eq

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{\omega^2}{\omega_{pe}} - \frac{3}{2}\frac{k^2 v_{th}^2}{\omega_{pe}^2} = 1$$

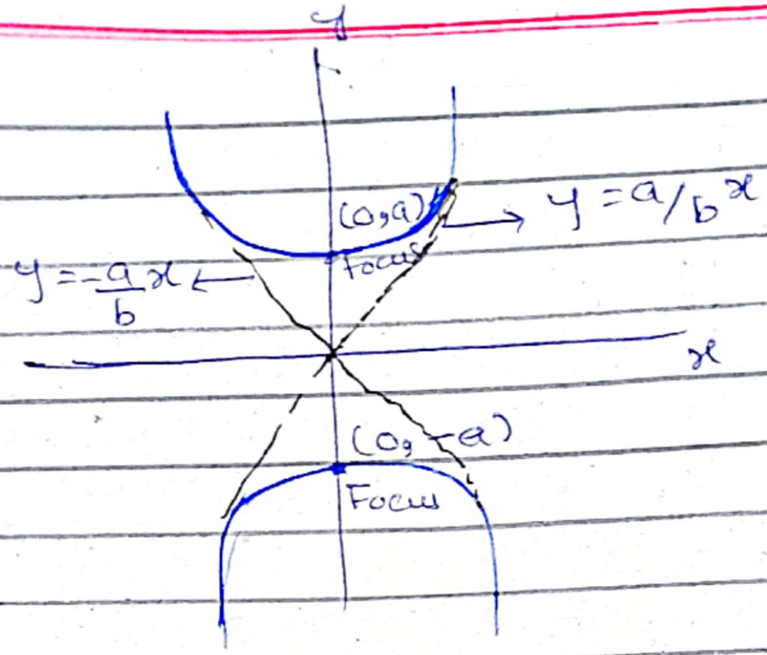
$a^2 = \omega_{pe}^2$

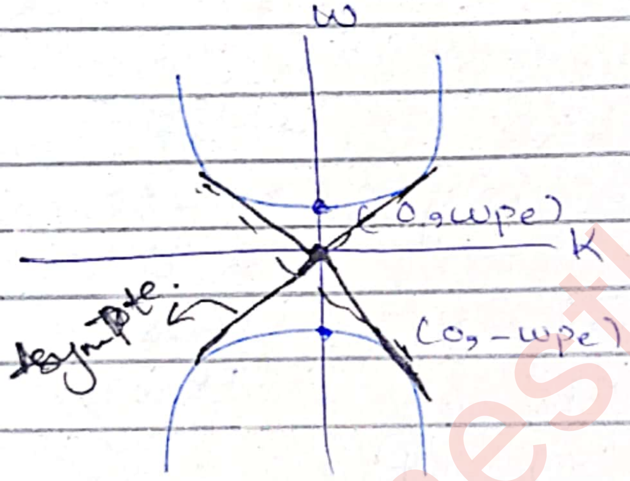
$b^2 = \frac{2\omega_{pe}^2}{3 v_{th}^2}$

Comparing ④ and ⑤ will give transverse hyperbola.

→ If eq ⑦ matches with ⑤ then transverse axis.

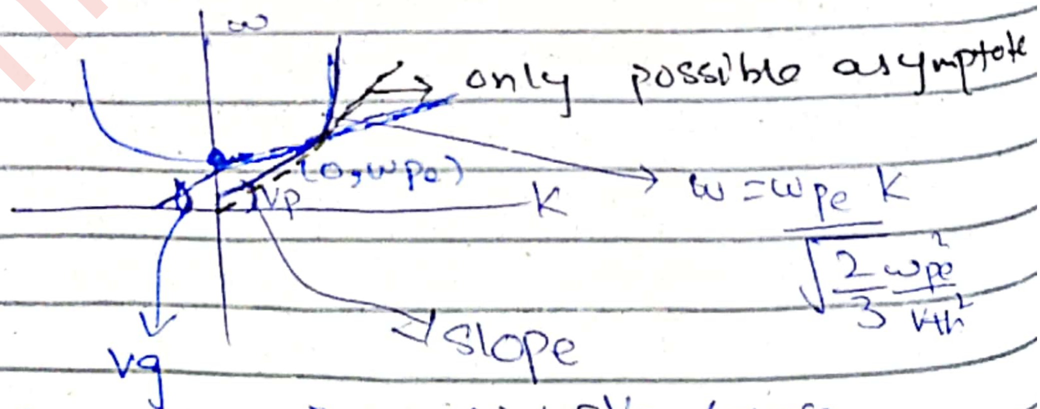
→ If eq ⑦ matches with ⑥ eq then horizontal axis.





No of cycles not -ve in plasma

as ω_{pe} is not physical result so only possible root is $+\omega_{pe}$



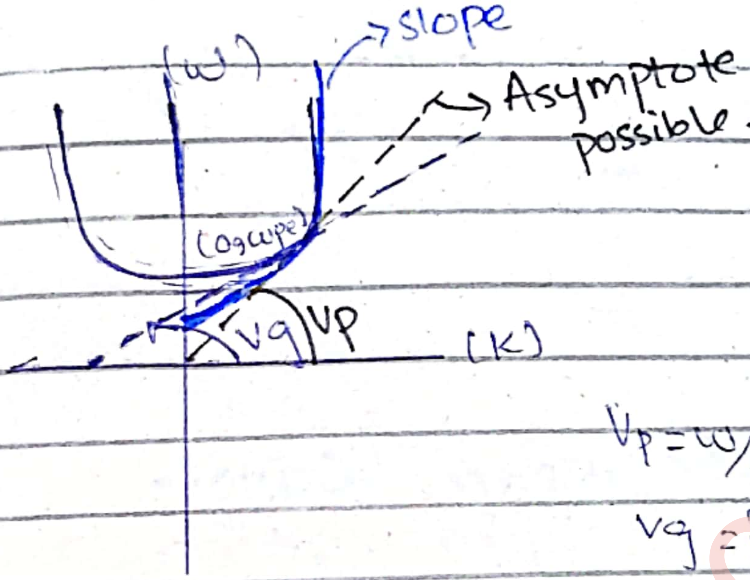
(group velocity

$\omega/k = v_p$ (phase velocity).

$\frac{\partial \omega}{\partial k} = v_g$.

$\sqrt{\frac{3}{2} v_{th}^2}$

Transverse hyperbola axis



$V_p = \omega/k = \sqrt{\frac{3}{2} v_{th}^2}$

$v_g = \frac{\partial \omega}{\partial k} = v_g$

$$\frac{\omega^2}{\omega_{pe}^2} - 1 = \frac{3}{2} k^2 \frac{v_{th}^2}{\omega_{pe}^2}$$

→ If k is small

$$\frac{\omega^2}{\omega_{pe}^2} - 1 = 0.$$

$$\omega^2 = \omega_{pe}^2$$

$$\omega = \omega_{pe}.$$

→ For small k. only plasma oscillation

→ If k is large

means that factor $\frac{3}{2} k^2 \frac{v_{th}^2}{\omega_{pe}}$ is very dominating.

then

$$\frac{\omega^2}{\omega_{pe}^2} = \frac{3}{2} k^2 \frac{v_{th}^2}{\omega_{pe}^2}$$

→ large k Thermal effect in wave.

information is travelling at thermal velocities.

$$V_{sound} = \sqrt{\frac{\gamma P}{\rho}} = 343\,m/s.$$

Ion Plasma Wave:-

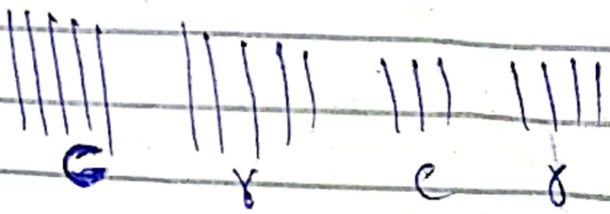
Propagation of oscillation

* Ions are massive particle →
when massive particle propagates
compression and rarefractions
produce.

Regions of Compressions. ↙ High density

Regions of Rare fraction. ↙ low Density

When sound waves propogate
series of compressions and
rarefractions produce
Similarly ion wave does.



{Sound Propagate

→ In Plasma if we compress ionize gas due to compression are rarefraction this will be density gradient.

* Compressions of ion and electron are different or it will create

→ In sounds compression transfer collisions in rarefractions through collisions.
Similarly,
Plasma is ionized → high density → high ions → high electric feild so ion wave propagate

* Electric feild E which is the source of communication in plasma.
* Since the motion of ions are massive so ion wave will be produced b/c of low frequency oscillation.

* In ion wave interia of fluid is caused by ion rather than e^-

* As mass of $m_i >> m_e$ so the thermal motion of e^- is greater as compared to ion.

$$\frac{1}{2} m v_{th}^2 = k_B T$$

For Ion

$\frac{1}{2} m_i V_{thi}^2 = K_B T_i$

$V_{thi} = \sqrt{\frac{2 K_B T_i}{m}}$

For electron

$V_{the} = \sqrt{\frac{2 K_B T_e}{m}}$

$V_{thi} << V_{the}$

$T_i < T_e$

Governing eq:-

eq. of motion:-

$$Mn_i\left[\frac{\partial}{\partial t} + (\vec{V}_i \cdot \vec{\nabla})\right]\vec{V}_i = en\vec{E} - \vec{\nabla} P_i \rightarrow (1)$$

→ when $\vec{E}$ is common electric feild fed by both e^- and ion.

* Now As ion wave in plasma are identical with the sound wave in neutral medium so ion waves are lognitudinal waves and Ions oscillations are longitudinal oscillations.

As we are not taking any B-feild so our oscillations are electrostatic

$\vec{\nabla} \times \vec{E} = 0$

$\vec{E} = -\vec{\nabla}\phi \rightarrow (2)$

propagation vector always the direction of feild.

$$Mn_i\left[\frac{\partial}{\partial t} + (\vec{V}_i \cdot \vec{\nabla})\right]\vec{V}_i = -en\vec{\nabla}\phi - \vec{\nabla} P_i \rightarrow (3)$$

$P_i = \gamma_i k_B T_i n_i \rightarrow (3')$

using (3') in (3) by taking its gradient

$$\nabla P_i = \gamma_i k_B T_i \vec{\nabla} n_i$$

$$M n_i \left[\frac{\partial}{\partial t} + (\vec{V_i} \cdot \vec{\nabla}) \right] \vec{V_i} = -e n_i \vec{\nabla}\phi - \gamma_i k_B T_i \nabla n_i \rightarrow (4)$$

eq. of continuity:-

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot (\vec{n_i} \vec{V_i}) = 0 \rightarrow (5)$$

Poission eq:-

$$\vec{\nabla} \cdot \vec{E} = \frac{e}{\varepsilon_0}(n_i - n_e) \rightarrow (6)$$

→ when $n_e = n_{e0} \exp\left(\frac{e\phi}{k_B T_e}\right)$ (Boltzman relation.)

We applied Boltzman relation b/c when ions are moving on large time scale the electron will thermalize.

Now linearize equations.

$\phi = \phi_0 + \phi_1$

$\vec{V_i} = \vec{V}_{i0} + \vec{V}_{i1}$

$n_i = n_{i0} + n_{i1}$

---- Dont put vector sign on ϕ as it is scaler quantity and make 60% in mind

$n_{i0} >> n_{i1}$

Linearized eq ①

$$\boxed{M n_i \frac{\partial V_{i1}}{\partial t} = -e n_{i0} \frac{\partial \phi_1}{\partial x} - \gamma_i k_B T_i \frac{\partial n_{i1}}{\partial x}} \rightarrow ⑦$$

$$\vec{\nabla}\phi = \vec{\nabla}(\phi_0 + \phi_1)$$

$$E_0 = 0 \rightarrow \phi_0 = 0$$

$$\vec{\nabla}\phi = \vec{\nabla}\phi_1$$

In 1D. so,

$$\vec{\nabla}\phi = \frac{\partial \phi_1}{\partial x}\hat{x}$$

linearized form of eq ②

$$\frac{\partial n_{i1}}{\partial t} + \vec{\nabla}\cdot(n_{i0} + n_{i1})(\cancel{V_{i0}}^{0} + V_{i1}) = 0$$

non linear form

$$\frac{\partial n_{i1}}{\partial t} + \vec{\nabla}\cdot(n_0 \vec{V}_{i1}) + \vec{\nabla}\underbrace{(\cancel{n_{i1}} V_{i1})}_{} = 0$$

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_{i0} + \vec{V}_{i1})$$

$$\frac{\partial n_i}{\partial t} + \vec{V}_{i1}\cdot\cancel{\vec{\nabla} n_{i0}}^{0} + n_{i0}\vec{\nabla}\cdot\vec{V}_{i1} = 0$$

$$\boxed{\frac{\partial n_{i1}}{\partial t} + n_{i0}\frac{\partial V_{i1}}{\partial x} = 0} \rightarrow ⑧$$

linearizing form of eq ③ in ion wave

$$-\vec{\nabla}\cdot\vec{\nabla}\phi = \frac{e}{\varepsilon_0}(n_i - n_e)$$

$$-\vec{\nabla}^2\phi = \frac{e}{\varepsilon_0}(n_i - n_0)$$

$$-\nabla^2(\phi_0+\phi_1) = \frac{e}{\varepsilon_0}(n_{i0}-n_{i1})-(n_{e0}-n_{e1})$$

$$-\nabla^2\phi_1 = \frac{e}{\varepsilon_0}(n_{i1}-n_{e1}) \rightarrow (9)$$

$$n_e = n_{e0}\exp\left(\frac{e\phi}{k_B T_e}\right)$$

$$\cancel{n_{e0}+n_{e1} = n_{e0}\exp\left(\frac{e\phi}{k_B T_e}\right)}$$

$$n_{e0}+n_{e1} = n_{e0}\left[1+\frac{e\phi}{k_B T_e}+\underbrace{\frac{1}{2}\left(\frac{e\phi}{k_B T_e}\right)^2}\right]$$

neglect

we can neglect this factor as $\frac{e\phi}{k_B T} \lll 1$ (less than 1)

as $T_e \gg 1$

$$\frac{e\phi}{k_B T} \lll 1$$

$$e^x = 1+x+\frac{x^2}{2!}+\dots$$

$$\cancel{n_{e0}}+n_{e1} = \cancel{n_{e0}}+\frac{n_{e0}\exp}{k_B T_e}$$

$$n_{e1} = \frac{n_{e0}\exp}{k_B T_e} \rightarrow (10)$$

using (10) in (9)

$$-\nabla^2\phi_1 = \frac{e}{\varepsilon_0}\left(n_{i1}-n_{e0}\frac{e\phi}{k_B T_e}\right)$$

$$\boxed{-\frac{\partial^2\phi_1}{\partial x^2} = \frac{e}{\varepsilon_0}\left(n_{i1}-n_{e0}\frac{e\phi}{k_B T_e}\right)} \rightarrow (11)$$

All the perturbed quantities are sinosidinely so apply sinosidual approximations:-

$$\phi_1 \sim \phi_1 \exp^{(ikx-\omega t)}$$
$$n_1 \sim n_i \exp^{(ikx-\omega t)}$$

$$\frac{\partial}{\partial t} \to -i\omega$$

$$\frac{\partial}{\partial x} \to ik$$

$$\boxed{-i\omega n_{i1} + n_{i0} i k V_{i1} = 0} \to (12)$$

$$\boxed{-M n_{i0} i\omega V_{i1} = -e n_{i0} ik\phi_1 - \gamma_i k_B T_i ik n_{i1}} \to (13)$$

$$\frac{\partial}{\partial x} \Rightarrow ik$$

$$\frac{\partial^2}{\partial x^2} \to (ik)(ik) = -k^2$$

$$+k^2\phi_1 = \frac{e}{\varepsilon_0}\left(n_{i1} - \overbrace{n_{e0}\frac{e\phi}{k_B T_e}}^{n_{e1}}\right)$$

$$\boxed{k^2\phi_1 = \frac{e}{\varepsilon_0}(n_{i1} - n_{e1})}$$

if we take quasineutrality condition at perturbed stage.

$$n_{i1} = n_{e1}$$

$$\phi_1 = 0$$

Finding ion wave by applying plasma approximation

$\frac{e}{\varepsilon_0}(n_{i1} - n_{e1}) = 0$

$n_{i1} - n_{e1} = 0$

$n_{i1} = n_{e1}$

$$\boxed{n_{i1} = n_{e0} \frac{e\phi_1}{K_B T}} \rightarrow (14)$$

Using plasma approximation

$n_{e1} = n_{i1}$

$\phi_1 \neq 0$ eq of motion

$\phi_1 = 0$ Possion eq.

Eliminating V_{i1}, n_{i1} and ϕ_1 from eq (12), (13) and (14)

From eq (14)

$n_{i1} = n_{e_0} \frac{e\phi_1}{k_B T}$

$$\boxed{\phi_1 = \frac{n_{i1}}{n_{e_0}} \frac{K_B T}{e}} \rightarrow (14')$$

$$\boxed{n_{i1} = \frac{n_{10} K V_{i1}}{\omega e}} \rightarrow \text{using eq (12) we get } n_{i1}$$

(12')

Putting (12') in (14')

$\phi_1 = \frac{\cancel{n_{i0}} K V_{i1} k_B T_e}{\omega e \cancel{n_{e0}}}$

$\phi_1 = \frac{K V_{i1} k_B T_e}{\omega e} \rightarrow (15)$

Putting (15) in (13) we get

$$-Mn_{io}\, i\omega k V_{i1} = -en_{io} ik\left(\frac{kV_{i}k_BT_e}{\omega e}\right) - \gamma_i k_B T_i k\left(\frac{n_{io}kV_{i1}}{\omega e}\right)$$

$\rightarrow$ $\cancel{\omega^2 = \alpha k^2 \dots}$

$$i\omega Mn_o V_{i1} = \left(en_o ik \frac{KT_e}{en_o} + \gamma_i kT_i ik\right)\frac{n_o ikv_{i1}}{i\omega}$$

$$\omega^2 = k^2\left(\frac{KT_e}{M} + \gamma_i \frac{kT_i}{M}\right)$$

$$\frac{\omega}{k} = \left(\frac{KT_e + \gamma_i kT_i}{M}\right)^{1/2} \equiv V_s .$$

$$\omega^2 = k^2\left(\frac{K_BT_e + \gamma_i k_BT_i}{M}\right)$$

$$\omega = \sqrt{\frac{k_BT_e + \gamma_i k_BT_i}{M}}\, k$$

Dispersion relation for ion-accoustic wave.

large # ↓

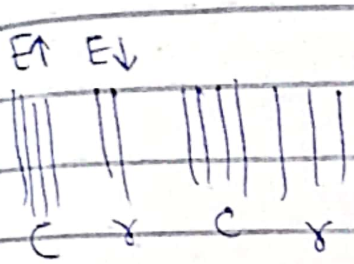
Ions → High E.F

↓

less #

↓

low E.F



Role:-

Ions thermelize than they are involved in Debye Sheilding

Sheilding not perfect but generated EF

$n_{e1} = n_{i1}$

$\nabla \cdot E \neq 0$

Plasma approximation.

E-feild is zero in possion eq.

Eelectric feild is not zero in eq of motion.

Here $n_{e1} \neq n_{i1}$

here not apply plasma approximation.

Possion eq:-

$$k^2 \phi_1 = \frac{e(n_{i1} - n_{e1})}{\varepsilon_0} \rightarrow (1)$$

eq of motion:-

$$-M n_{i0} i\omega v_{i1} = -e n_{i0} ik\phi_1 - \gamma_i k_B T_i ik n_{i1} \rightarrow (4)$$

eq of continuity.

$$-i\omega n_{i1} + n_{i0} i k V_{i1} = 0 \rightarrow (2)$$

$$n_{e1} = n_{e0} \frac{e\phi_1}{K_B T_e} \rightarrow (3)$$ Boltzman Relation.

Using eq (2) in (1)

$$K^2 \phi_1 = \frac{1}{\varepsilon_0}\left[e n_{i1} - \frac{e^2 \phi_1 n_{e0}}{k_B T_e}\right]$$

$$K^2 \phi_1 + \frac{e^2 \phi_1 n_{e0}}{\varepsilon_0 k_B T_e} = \frac{e n_{i1}}{\varepsilon_0}$$

$$\phi_1 \left(K^2 + \frac{e^2 n_{i0}}{\varepsilon_0 k_B T_e}\right) = \frac{e n_{i1}}{\varepsilon_0}$$

as $\lambda_{De} = \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n_{e0}}}$

$$\phi_1 \left(k^2 + \frac{1}{\lambda_{De}^2}\right) = \frac{e n_{i1}}{\varepsilon_0}$$

$$\phi_1 \left(k^2 \lambda_{De}^2 + 1\right) = \frac{e n_{i1} \lambda_{De}^2}{\varepsilon_0}$$

From eq (2) value of n_{i1} is :-

$$n_{i1} = \frac{n_{i0} \cancel{i} k V_{i1}}{\cancel{i} \omega}$$

$$n_{i1} = \frac{n_{i0} k V_{i1}}{\omega} \rightarrow (6)$$

Using (6) in (5)

$$\phi_1 (k^2 \lambda_{De}^2 + 1) = \frac{e}{\varepsilon_0} \lambda_{De}^2 \frac{n_{i0} k V_{i1}}{\omega}$$

$$\phi_1 = \frac{e}{\varepsilon_0}\frac{\lambda_{De}^2 n_{io} k V_{i1}}{\omega(1+k^2\lambda_{De}^2)} \rightarrow (7)$$

Using (6), (7) in eq (4) we get

then eq (4) Taking i and $-ve$ sign common

$$-M n_{io} \cancel{i} \omega V_{i1} = -\cancel{i}(e n_{io} k \phi_1 + \gamma_i k_B T_i k n_{i1})$$

$$M n_{io} \omega V_{i1} = e n_{io} k \left(\frac{e}{\varepsilon_0}\frac{\lambda_{De}^2 n_{io} k V_{i1}}{\omega(1+k^2\lambda_{De}^2)}\right) + \gamma_i k_B T_i k \frac{n_{io} k V_{i1}}{\omega}$$

$$M n_{io} \omega \cancel{V_{i1}} = \cancel{V_{i1}}\left[\frac{e n_{io} k e}{\varepsilon_0}\frac{\lambda_{De}^2 n_{io} k}{\omega(1+k^2\lambda_{De}^2)} + \frac{\gamma_i k_B T_i k n_{io} k}{\omega}\right]$$

$$\omega = \frac{e n_{io} k e}{M \cancel{n_{io}} \varepsilon_0}\frac{\lambda_{De}^2 \cancel{n_{io}} k}{\omega(1+k^2\lambda_{De}^2)} + \frac{\gamma_i k_B T_i k \cancel{n_{io}} k}{M \cancel{n_{io}}\, \omega}$$

$$\omega^2 = \frac{e^2 k^2 n_{io} \lambda_{De}^2}{M\varepsilon_0(1+k^2\lambda_{De}^2)} + \frac{\gamma_i k_B T_i k^2}{M}$$

$$\frac{\omega^2}{k^2} = \frac{\cancel{e^2} \cancel{n_{io}} \cancel{\varepsilon_0} k_B T_e}{\cancel{e^2} \cancel{n_{eo}} M \cancel{\varepsilon_0}(1+k^2\lambda_{De}^2)} + \frac{\gamma_i k_B T_i}{M}$$

as $n_{io} = n_{eo} = n_0$

$$\frac{\omega^2}{k^2} = \frac{k_B T_e}{M(1+k^2\lambda_{De}^2)} + \frac{\gamma_i k_B T_i}{M}$$

$$\omega = \sqrt{\frac{k_B T_e}{M(1+k^2\lambda_{De}^2)} + \frac{\gamma_i k_B T_i}{M}}\; k \rightarrow (*)$$

without plasma approximation. $\boxed{n_{ei} \neq n_{ii}}$

$$\omega = \sqrt{\frac{k_B T_e}{M} + \frac{\gamma_i k_B T_i}{M}}\; k \rightarrow (*')$$

$\boxed{n_{ei} = n_{ii}}$

with plasma approximation.

When (*) is equal to (*')

$k^2\lambda_{De}^2 \ll 1$

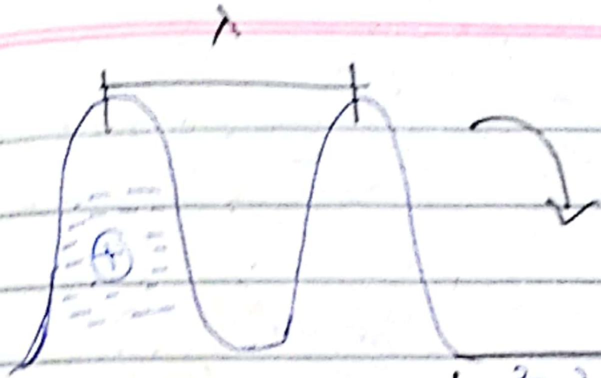
This is good approximation for fluid theory.

$(k = 2\pi/\lambda)$

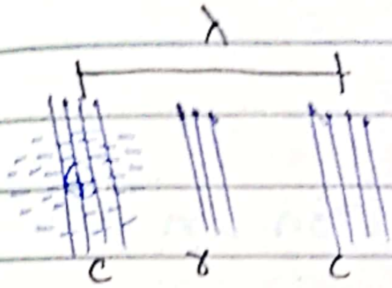
$$\frac{2\pi}{\lambda}\lambda_{De} \ll 1$$

$\boxed{\lambda_{De} \ll \lambda} \rightarrow$ ion wave wavelength.

$k\lambda_{De} \lll 1$

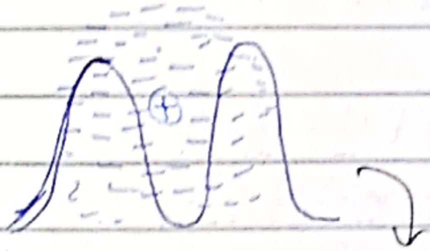


$\boxed{k^2\lambda_{De}^2 << 1}$



In Fluid theory it is good approximation

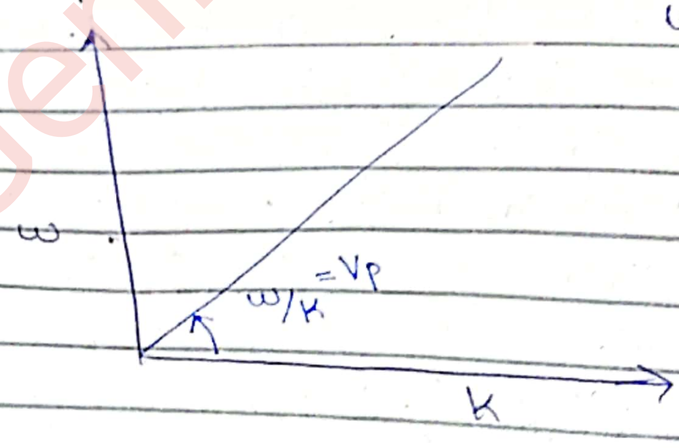
$$\frac{2\pi\lambda_{De}}{\lambda} << 1$$



$\boxed{k^2\lambda_{De}^2 >> 1}$

Kinetic Theory.

Graph:-



$\omega = \text{const } k$

$$\frac{\omega}{k} = \text{const} = V_p$$

$$V_g = \frac{\partial \omega}{\partial k} = \sqrt{\frac{k_B T_e}{M(1+k^2\lambda_{De}^2)} + \frac{\gamma_i k_B T_i}{M}}$$

$$V_p = \frac{\omega}{k} = \sqrt{\frac{k_B T_e}{M} + \frac{\gamma_i k_B T_i}{M}}$$

→ in Ion wave group and phase velocity ~~of~~ are same but different in e^-

Comparsion of electron wave and ion wave

* Ion wave are low freq waves and are produced by propagation of low freq. oscillation.	* Electron wave are high freq waves and are produced by propagation of high freq oscillati
* In ion wave ions are moving whereas electrons are thermalize by following some thermal distribution (Maxwell – Boltzman dis.) and playing their role in sheilding process	* In electron wave electrons are moving whereas ions are fixed.
* Ion wave is studied on large time scale	Electron wave are studied on short time.

Dispersion relation for ion

→ with applying plasma approximation

$\omega = \sqrt{---}\ K$

→ without applying plasma approximation.

$\omega = \sqrt{\ }\ K$

⊛ Dispersion relation for electron.

⊛ The expression for phase velocity and group velocity.

Phase:-

→(1)

Group:- →(2)

⊛ The expression for phase velocity and group velocit

Phase:-

→(1')

Group:-

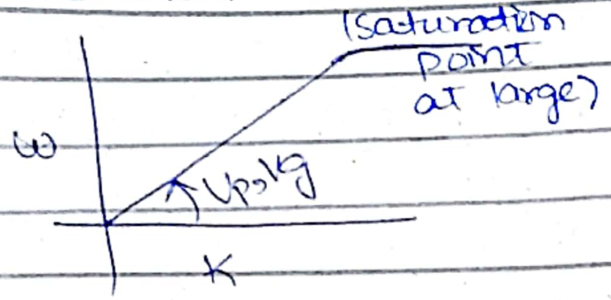
→(2')

⊛ We conclude in ion wave phase and group velocity expression are same.

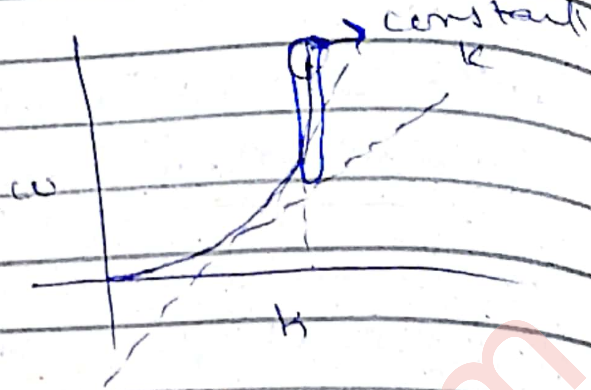
⊛ We conclude that phase velocity and group velocity are not same in electron wave

$V_p = V_g$ | $V_p \neq V_g$

Sketch



Sketch.



Ion wave is a constant velocity curve. (at small K value)

↳ For small value ~~and large value of K~~

For large K value the ion dispersion relation curve become constant freq. curve (saturation point)..

Electron wave is a constant frequency curve.

↳ For small value of K

Electron dispersion wave / curve becomes constant velocity curve at large K values.

Sound Waves:-

→ introduction of ion waves let's briefly review sound waves in ordinary air. Neglecting viscosity we can write Navier - Stokes equillibrium which describes these waves.

$$\rho\left[\frac{\partial v}{\partial t}+(v\cdot\nabla)\right]v=-\nabla p$$

$$=-\frac{\gamma p}{\rho}\nabla\rho$$

eq of continuity

$$\frac{\partial\rho}{\partial t}+\nabla(\rho v)=0$$

Linearizing about a stationary equilli -brium with uniform ρ_0 and p_0 we have

$$-i\omega\rho_0 v_1=-\frac{\gamma p_0}{\rho_0}ik\rho_1$$

$$-i\omega\rho_1+\rho_0 ik\cdot v_1=0$$

Where we have again taken a wave dependence of the term

$$\exp\left[i(k\cdot r-\omega t)\right].$$

For a plane wave with $k=k\hat{x}$ and $v=v\hat{x}$ find, upon eliminating ρ_1

$$-i\omega \rho_0 v_1 = \frac{-\gamma P_0}{\rho_0} ik \frac{\rho_0 ikv_1}{i\omega}$$

$$\omega^2 v_1 = k^2 \frac{\gamma P_0}{\rho_0} v_1$$

$$\frac{\omega}{k} = \left(\frac{\gamma P_0}{\rho_0}\right)^{1/2} = \left(\gamma \frac{KT}{M}\right)^{1/2} \equiv C_s$$

This is expression for velocity C_s of sound waves in a neutral gas. The waves are pressure waves propagating from one layer to next by collisions among the air molecules. In plasma with no neutrals and few collisions, an analogous phenomena occurs. This is called ion acoustic wave. Or simply an ion wave.