Ch # 03:-

Plasma as Fluids

If you have many particles in a system then we have a complexities in a system

$n = 10^{12}$ ion-electron pairs per cm^3

- Plasma is a many body problem.

In order to solve many body problem, we use technique to solve the given system.

Kinetic theory for plasma

Fluid theory for plasma

Simulations

Computer programming

Research

Theory

Experimental.

Kinetic Theory:- exact description of Plasma (Microscopic lev

Fluid theory:- Macroscopic level.

Plasma parameters are describing the quantities are expressed in distribution function.

- Mathematically complex system.

We can discuss and learn fluid theory in this chapter so,

Fluid Theory:-

In this Theory, the model used to describe plasma is used in fluid mechanics in which the identity of individual particle is neglected and only the motion of fluid element is taken into account.

→ In plasma, we have ions, electrons and neutral particles.

Fully ionized plasma $\xrightarrow[\text{equally}]{}$ ions, electrons.

(Every particle is important in kinetic theory) but in fluid theory we have take plasma as fluid.

Ions → Ions Fluid.

Electrons → Electron Fluid.

Plasma is charged medium it is not a neutral medium.

Fluid Model:-

Maxwell equations-

(In vaccum) :- $\nabla \cdot E = \rho/\varepsilon_0$ → Guass's Law.

$\nabla \cdot B = 0$

$\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$ → Faraday's law.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\rho = qn \rightarrow$ charge density
$J = qnv \rightarrow$ Current density.

$\rho = J = 0$ In vaccum.
When we apply in a plasma, so we cannot take this b/c plasma is charged medium
So we take dielectric

$\varepsilon_0 \rightarrow \varepsilon$

$$K_e = \frac{\varepsilon_{med}}{\varepsilon_{vac}}$$

(Dielectric is different in difference medium)

$$K_m = \frac{\mu_{med}}{\mu_{vac}}$$

$K_m = K_e = 1$ in plasma

$\varepsilon_{med} = \varepsilon_{vac}$

$\varepsilon = \varepsilon_0$

$\mu = \mu_0$

$$\vec{\nabla} \times \vec{B} = \mu \vec{J} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$D = \varepsilon E$

$\varepsilon = \varepsilon_0$

$D = \varepsilon_0 E$

$\vec{\nabla} \cdot \varepsilon \vec{E} = \rho$

$\vec{\nabla} \cdot \vec{D} = \rho$

$B = \mu M$

$$\vec{\nabla} \times \frac{\vec{B}}{\mu} = J + \varepsilon \partial \vec{E}/\partial t$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$ (Transform)

$\vec{\nabla} \cdot \mu \vec{H} = 0$

$\vec{\nabla} \cdot \vec{H} = 0$

$D = \varepsilon E$

$E = D/\varepsilon$

$$\vec{\nabla} \times \frac{\vec{D}}{\varepsilon} = -\mu \frac{\partial \vec{H}}{\partial t}$$

In plasma:-

$$\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$$

$$\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$$

$$\vec{\nabla} \times \vec{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

These equations now use in Fluid model.

→ Lecture 13/11/23

In Plasma (not fully ionized)

Electron Ion Neutral

Free charges.

Electric and Magnetic Field.

Collective Behaviour

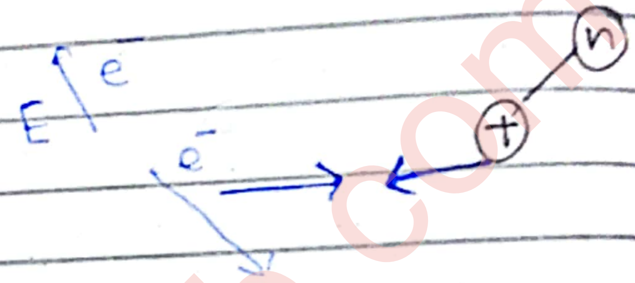
Collisions

$e^- e^-$

ii, ei, in

ne, nn

Collisions.



Three types of particles so their will be three types of equation of motion (as for electron, ions, neutra

Equation of motion (in Plasma)

$$m\frac{d\vec{V}}{dt} = q(\vec{E} + \vec{V}\times\vec{B}) \rightarrow (1)$$

this is for single particle. This eq used in ch#2 so after mod used in ch #3 not used direct

The Fluid Equation of motion:-

In the fluid approximation we consider the plasma to be composed of two or more interpenitrating fluid one for each speice. So we need three eq of motion for the +ve ions, electron and neutral atom.

* The neutral fluid will interact with the ions and electrons only

through collisions.

* The electron and ion fluids will intact with each other even in the absence of collision b/c of E and B feilds they generate.

● There are certain modification which we have to make in one (1) eq of motion (eq ① used in Ch #2) when we take plasma as a fluid.

* As eq ① is for single particle so when we have a large no of particle we replace the velocity V of single particle with the average velocity of the particle U

$$V \to U$$

$$m\frac{du}{dt} = q(\vec{u} \times \vec{B} + \vec{E}) \to ②$$

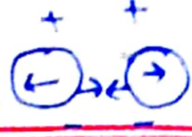
* We multiply eq ② with $n = N/V$ the number density of particle

$$mn\frac{d\bar{u}}{dt} = qn(U \times B + \bar{E}) \to ③$$

*(Third modification is will collisions.)

Force due to collision drag

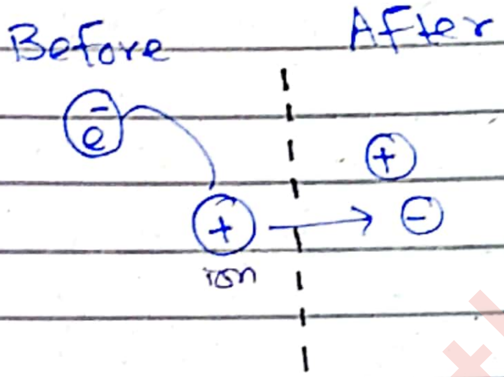
Electron and ion can collide with each other and also one electron with another similarly one ion with the



So the four possible collision frequencies.
$\nu_{ee}, \nu_{ei}, \nu_{ie}, \nu_{ii}$

path will be same not change

- Neutral particle do head on collision whereas in case of charged particle the situation is different



$\boxed{\nu_{ee}, \nu_{nn}, \nu_{ii} \quad \text{self collison}}$

- In plasma mixxme so electrons are moving rapidly or composed to the ion.

$\boxed{\text{Space plasma is mostly collision less.}}$

Change in momentum for collision = ?

Initial momentum collision $= mu_o = P_i$
final " " $= mu_{"} \quad P_f$

Change in momentum for collision $= \Delta P = P_f - P_i$

$$\Delta P = mu - mu_o$$
$$\Delta P = m(u - u_o)$$

if τ is the mean free time b/w

the collisions.

$$F = -\frac{\Delta P}{\Delta t} = -\frac{m(u-u_0)}{\tau} \rightarrow \text{Drag Force.}$$

Total collisional drag force $= F_D = nF$

$$\boxed{F_D = -\frac{nm(u-u_0)}{\tau}} \rightarrow \text{collisional Drag force}$$

As $1/\tau = \nu \rightarrow$ Collisional freq.

$$\vec{F}_D = -mn\nu(\vec{u}-\vec{u}_0)$$

Modifications.
1. Velocity
2. No density
3. Collisional drag
4. Gradient Force.

$$mn\frac{d\vec{u}}{dt} = qn(\vec{E}+\vec{u}\times\vec{B}) + \vec{F}_D$$

$$mn\frac{d\vec{u}}{dt} = nq(\vec{E}+\vec{u}\times\vec{B}) - mn\nu(\vec{u}-\vec{u}_0)$$

*The fourth modification is the addition of another force which is named as pressure gradient force.

$\Delta P = nk_BT$
Δn

$P = nk_BT$
(pressure defined in terms of no. density n)

The pressure change in plasma b/c so the

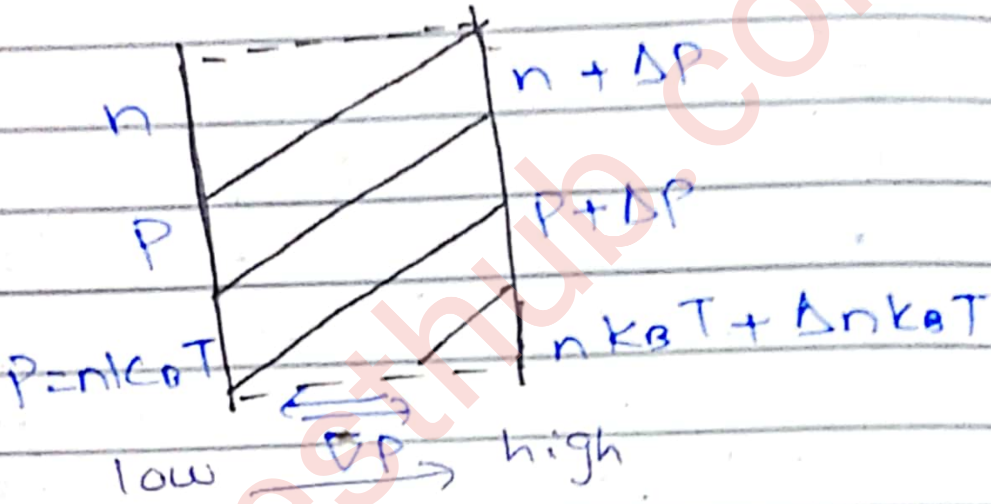
Density gradient always present b/c it will make equal like high to low ~~[illegible]~~
(Perfume) ~~[illegible]~~

Gradient means low to high.

Pressure scalar quantity no Gradient

Change in n so we have a force.

$P = F/A$



$F = A\Delta P$

$\vec{F} = -A\vec{\nabla} P$

$\vec{F} = -A\,\Delta P\,\hat{x}$

$A = 1$

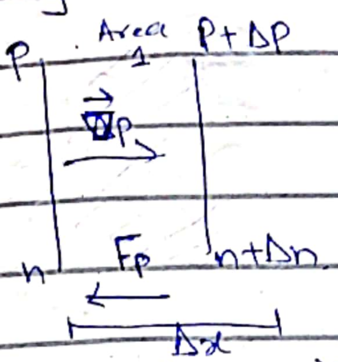
$\vec{F} = -\Delta P\,\hat{x}$

$\vec{F} = -\vec{\nabla} P$

(1)

* Pressure gradient force.

$P = n k_B T$



if we take unit area = 1 $\quad F = P$

$\vec{F} = \Delta P \quad\quad \Delta P = 0$

$\Delta P \sim \Delta n$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \partial/\partial z\,\hat{z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} + \partial^2/\partial z^2$$

$\vec{F} = \partial P/\partial x$ direction

$\vec{F} = \vec{\nabla} P$

$$\boxed{\vec{F}_p = -\vec{\nabla} P}$$

Pressure gradient force.

if system doesn't have no density change then this term will not occur.

- Divergence: both must be ~~scalar~~ vector $\vec{\nabla} \cdot \vec{A}$
- Curl: both must be vector $\vec{\nabla} \times \vec{A}$
- Gradient: scalar + vector gives vector result. $\vec{\nabla} a$ always from low to high

Net force per particle:-

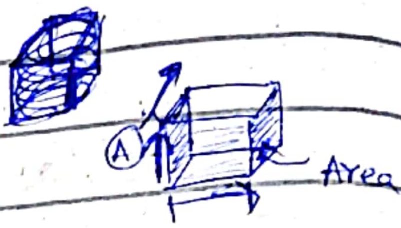
NP (where $N = nV$)

Net force per particle $= \vec{F}_p/N$

" " $= \vec{F}_p/nV$.

$$= \frac{f_p}{n V} = \frac{\vec{F_p}}{n A \Delta x}$$

$V = \text{Area} \times \text{Length}$



Net force per particle $= \frac{f_p}{n \Delta x} = \frac{\Delta P_x}{n \Delta x}$

$$= -\frac{\partial P_x}{n \partial x}$$

$$= -\frac{\vec{\nabla} P}{n}.$$

$V = A \Delta x$

$A = 1$

$V = \Delta x.$

if n is the no. density so multiplying Net force per particle with n gives the total force

$$= -\frac{n \vec{\nabla} P}{n}$$

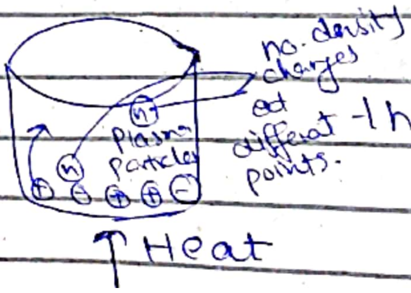
Pressure Gradient force $= -\vec{\nabla} P$

$$mn \frac{d\vec{u}}{dt} = qn(\vec{E} + \vec{v} \times \vec{B}) - mn\nu(v - u_0) - \vec{\nabla} P.$$

Four modifications completed.

(5) 5^{th} modification
imp one in plasma.

The Convective Derivative:-



Heat transfer through fluid take place by convectional method.

Layers:

High Energy	Low no density	High velocity
Low Energy	High density	low velocity

As it is heating so its no. density decreases while the rest of fluid no density increases so average velocity will be different in different regions (diff for volume element 1 and 2.

Average velocity of particles depend on time and space $u(x,t)$. if one dimension if $u(x,y,t)$ it becomes 2 D and if $u(x,y,z,t)$ it is in 3 Dimension.

So average velocity is different for different region for the same volume element.

If we have function of several variables it's derivative is expressed in terms of partial derivative.

$$\vec{u}(x,t) = \vec{u}(x,y,z,t).$$

$$\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + \frac{\partial \vec{u}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \vec{u}}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \vec{u}}{\partial z}\frac{\partial z}{\partial t}$$

We know that: $\frac{\partial x}{\partial t} = U_x \quad \frac{\partial y}{\partial t} = U_y \quad \frac{\partial z}{\partial t} = U_z.$

$$\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + U_x\frac{\partial \vec{u}}{\partial x} + U_y\frac{\partial \vec{u}}{\partial y} + U_z\frac{\partial \vec{u}}{\partial z}$$

$$\boxed{\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}}$$

$U = U_x \hat{x} + U_y \hat{y} + U_z \hat{z}$

$\vec{\nabla} = \frac{\partial}{\partial x} \hat{x} + \partial/_{\partial y} \hat{y} + \partial/_{\partial z} \hat{z}$

$U.\vec{\nabla} = U_x \frac{\partial}{\partial x} + U_y \frac{\partial}{\partial y} + U_z \partial/_{\partial z}$

$(\vec{U}.\vec{\nabla})\vec{U} = U_x \frac{\partial \vec{U}}{\partial x} + U_y \frac{\partial \vec{U}}{\partial y} + U_z \frac{\partial \vec{U}}{\partial z}$

$$\boxed{mn\left[\frac{\partial}{\partial t} + \vec{U}.\vec{\nabla}\right]\vec{U} = qn(\vec{E} + \vec{V} \times \vec{B}) - mn\nu(\vec{U} - \vec{U}_0) - \nabla}$$

Modified form of eq of motion.

$\nu = 1/_T$

- Inclusion of pressure gradient force
- Replacing time derivative with convective derivative

Exam Question:- Write in headings. all five.

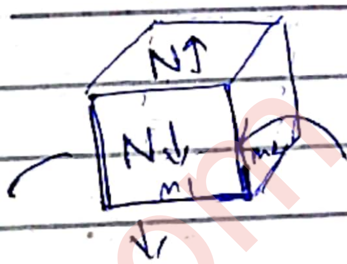
(For collision less plasma ignore less term as this is b/c of drag force).

$$mn\left[\frac{\partial}{\partial t} + \vec{U}.\vec{\nabla}\right]\vec{U} = qn(\vec{E} + \vec{V} \times \vec{B}) - \Delta P$$

imp

Equation of Continuity:-

The conservation of matter requires that the total number of particles N in a volume V can change only if there is a net ~~flux~~ of particles across the surface S bounding the volume



$$\boxed{\frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0}$$

↳ eq of continuity

- +ve Diverges
- -ve Diverges

↳ also means wheather particles are entering or leaving.

Product of nu is the particle flux density.

Equation of state:-

In order to describe the state of the system we need eq of state for this we use eq:

$P \propto \rho^{\gamma}$

$P = C\rho^{\gamma}$

$\rho = mn$ mass density

where $\gamma = C_p / C_v$

$P = C(mn)^{\gamma}$

↳ eq of state is a relation b/w P and n.

C = constant

PV satisfies system is ideal gas.

↳ we have equations for adiabatic, isothermal, iso

$$P = C(mn)^{\gamma} = C\rho^{\gamma}, \quad \therefore C = \text{constant}$$

$$\vec{\nabla}P = C\gamma\rho^{\gamma-1}\vec{\nabla}\rho$$

$$\frac{\vec{\nabla}P}{P} = \frac{\cancel{C}\gamma\rho^{\gamma-1}\vec{\nabla}\rho}{\cancel{C}\rho^{\gamma}}$$

$$\frac{\vec{\nabla}P}{P} = \gamma\frac{\vec{\nabla}\rho}{\rho}$$

$$P = nK_BT$$

$$\vec{\nabla}P = nK_B\vec{\nabla}T + K_BT\vec{\nabla}n$$

isothermal plasma

$$\vec{\nabla}T = 0$$

$$\vec{\nabla}P = K_BT\vec{\nabla}n$$

$$\frac{\vec{\nabla}P}{P} = \frac{K_BT\vec{\nabla}n}{nK_BT}$$

$$\frac{\vec{\nabla}P}{P} = \frac{\vec{\nabla}n}{n}$$

$$\rho = mn$$

$$\vec{\nabla}\rho = m\vec{\nabla}n$$

γ = gamma = adiabatic factor.

$$\frac{\vec{\nabla}P}{P} = \frac{\gamma\cancel{m}\vec{\nabla}n}{\cancel{m}n}$$

$$\boxed{\frac{\vec{\nabla}P}{P} = \gamma\frac{\vec{\nabla}n}{n}}$$

$\gamma \Rightarrow$ adiabatic factor.

$\gamma = 1$

For isothermal plasma.

$\vec{\nabla}P/P = \frac{\vec{\nabla}n}{n}$

$\frac{\vec{\nabla}P}{P} = P\vec{\nabla}n/n$

$\frac{\vec{\nabla}P}{P} = \gamma\frac{\vec{\nabla}n}{n}$

$\gamma = \frac{N+2}{N}$

N → no of degree of freedom.

$N = 1$

$\gamma = 3/1 = 3$

$\frac{\vec{\nabla}P}{P} = 3\frac{\vec{\nabla}n}{n}$ → Degree of freedom is 1

$\frac{\vec{\nabla}P}{P} = \frac{2+2}{2}\frac{\vec{\nabla}n}{n}$ → $N = 2$

$\frac{\vec{\nabla}P}{P} = 2\frac{\vec{\nabla}n}{n}$

$\frac{\vec{\nabla}P}{P} = \frac{5}{3}\frac{\vec{\nabla}n}{n}$ → $N = 3$ Degree of freedom is $N = 3$

*Complete Set of Fluid Equations:-

Charge density:-

* $\sigma = n_i q_i + n_e q_e$

charge density. (mass per unit volume

* $j = n_i q_i V_i + n_e q_e v_e$

Current density

$\rho = mn$

mass density

(charge per unit volume).

All are Fluid equation

* Maxwell eq

$$\nabla \cdot E = \frac{\sigma}{\epsilon_0} \rightarrow (1)$$

$$\nabla \cdot B = 0 \rightarrow (2)$$

$$\vec{\nabla} \times \vec{E} = -\partial B/\partial t \rightarrow (3)$$

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow (4)$$

Now fluid eq:-

$j = i, e$

j = is some dummy index here not current density

$$m_j q_j \left(\frac{\partial}{\partial t} + \vec{U}_j \cdot \vec{\nabla}\right)\vec{U}_j = q_j n_j (E + \vec{U}_j \times \vec{B}) - \vec{\nabla} P_j$$

⇒ Equation of motion.

Collision less plasma. (5)

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j U_j) = 0 \rightarrow (6)$$

$$P_j = C \rho_j^{\gamma}$$

$$\rho_j = m_j n \rightarrow (7)$$

Electrostatic

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{E} = -\vec{\nabla}\phi \rightarrow \text{scalar potential.}$$

Guass's Law states that

$$\vec{\nabla} \cdot E = \sigma / \varepsilon_0$$

$$\vec{\nabla} \cdot \vec{E} = q_e n_e + q_i n_i / \varepsilon_0$$

$$q_i = e$$

$$q_e = -e$$

$$\vec{\nabla} \cdot \vec{E} = -en_e + en_i / \varepsilon_0$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$-\vec{\nabla} \cdot \vec{\nabla}\phi = -en_e + en_i / \varepsilon_0$$

$$-\nabla^2\phi = \frac{-en_e + en_i}{\varepsilon_0}$$

$$\boxed{\nabla^2\phi = \frac{en_e - en_i}{\varepsilon_0}}$$ → Possion eq. → same for gravity

*

Plasma as a Fluid

↓ Fluid drift ⊥ to B → (Case I)

↓ Fluid drift ∥ to B

(I) →

(II) ↑ fluid Plasma

When plasma is taken as fluid we have many particles moving in the system so we except the fluid to have drifts ⊥ to B if the individual guiding centre have such drifts.

$$mn\left[\frac{\partial v}{\partial t} + V\cdot\nabla V\right] = qn\left(\vec{E} + \vec{V}\times\vec{B}\right) - \vec{\nabla}P$$

① ② ③ ④

$V \rightarrow$ average velocity } on ch # 4

Since in eq ① we have $\vec{\nabla} p$ term so drifts related with pressure gradient term results when plasma is taken as a fluid.

Here we take

$$\frac{\partial}{\partial t} \rightarrow -i\omega$$

and we take $V_\perp$ (as discussing $\perp$ component).

Considering the ratio of eq ① and ③.

$$\frac{①}{③} \approx \left| \frac{mn \frac{\partial v}{\partial t}}{qn(V \times B)} \right|$$

→ For ratio we take this magnitude term.

$$\approx \left| \frac{-mn i\omega V_\perp}{q n V_\perp B} \right| \approx \frac{m \cancel{n} i\omega \cancel{V_\perp}}{q \cancel{n} \cancel{V_\perp} B}$$

$$\approx \frac{i\omega}{\omega_c}$$

ω_c = cyclotron freq.

ω = wave freq.

Assumptions:-

1. $\frac{①}{③} \approx \rightarrow 0$ (Ratio approaches to zero)

For highly magnetized plasma ratio $①/③ = 0$ $\boxed{\omega = 0}$

$\omega_c >> \omega$

$\boxed{(V_\perp \cdot \nabla) V_\perp}$ → Non linear term

(2) term $mn(V_1 . \nabla)V_1$ → non linear term.

we neglect term when doing linear analysis.

$0 \simeq qn(E + \vec{V} \times \vec{B}) - \vec{\nabla} P$ → (2)

(2) Assumption:-

eq (1) if we take electrons as inertialess $m \to 0$

we can take any assumption to put 0 in eq (1)

Take cross product of eq (2)

~~Also~~

$$0 \times \vec{B} = qn(\vec{E}\times\vec{B}) + qn(\vec{V}_\perp \times \vec{B})B - \nabla\vec{P}\times\vec{B} \quad (3)$$

$$(\vec{A}\times\vec{B})\times\vec{C}$$

$$(\vec{V}_\perp \times \vec{B})\times\vec{B} = (\vec{V}_\perp \cdot \vec{B})B - (\vec{B}\cdot\vec{B})V_\perp$$

$$(\vec{V}_\perp \times \vec{B})\times\vec{B} = V_\perp B\cos 90 \cdot B - B^2 V_\perp$$

$$(\vec{V}_\perp \times \vec{B})\times\vec{B} = -B^2\vec{V}_\perp \rightarrow (4)$$

By using (4) in (3)

$$0 = qn(\vec{E}\times\vec{B}) - qnB^2\vec{V}_\perp - \vec{\nabla}P\times\vec{B}$$

$$\vec{V}_\perp B^2 = qn(\vec{E}\times\vec{B}) - \vec{\nabla}P\times\vec{B}$$

$$qn\vec{V}_\perp = \frac{qn(\vec{E}\times\vec{B})}{B^2} - \frac{\vec{\nabla}P\times\vec{B}}{B^2}$$

$$\vec{V}_\perp = \frac{\vec{E}\times\vec{B}}{B^2} - \frac{\vec{\nabla}P\times\vec{B}}{qnB^2}$$

$$\vec{V}_\perp = \vec{V}_E + \vec{V}_D$$

$$V_E = \frac{\vec{E}\times\vec{B}}{B^2}$$

where V_E is the electric drift

$$\vec{V}_D = -\frac{\vec{\nabla}P\times\vec{B}}{qnB^2}$$

$\vec{V}_D$ is the diamagnetic drift and the source for this drift is the pressure gradient force.

Analysis of diamagnetic drift so we have different then.

For ions:- $\vec{V}_{Di} = \dfrac{-\vec{\nabla}P\times\vec{B}}{enB^2}$

For electrons:-

$$\vec{V}_{De} = \frac{+\vec{\nabla}P \times \vec{B}}{+en_e B^2}$$

$$\vec{V}_{DE} = \frac{\vec{\nabla}P \times \vec{B}}{en_e B^2}$$

$P = \gamma_n k_B T$

$\vec{\nabla}P = \gamma k_B T \vec{\nabla} n$

$\vec{\nabla}P_i = \gamma k_B T_i \vec{\nabla} n_i$

$\vec{\nabla}P_e = \gamma k_B T_e \vec{\nabla} n_e$

↳ we just taking gradient in no. density

$$\vec{V}_{Di} = \frac{-\gamma k_B T_i \vec{\nabla} n_i \times \vec{B}}{en_i B^2} \rightarrow (5)$$

$$\vec{V}_{De} = \frac{\gamma k_B T_e \vec{\nabla} n_e \times \vec{B}}{en_e B^2} \rightarrow (6)$$

Current density:-

$\vec{j} = n_i q_i V_i + n_e q_e V_e$

$\vec{j} = n_i e V_i - n_e e V_e \qquad q_e = -e.$

$\vec{j} = e n_i \vec{V}_{Di} - n_e e \vec{V}_{De} \rightarrow (7)$

using (5) and (6) in (7)

$$\vec{j} = \frac{-\cancel{e n_i} \gamma k_B T_i \vec{\nabla} n_i \times \vec{B}}{\cancel{e n_i} B^2} - \frac{\cancel{n_e e} \gamma k_B T_e \vec{\nabla} n_e}{\cancel{e n_e} B^2}$$

$$\vec{j} = \frac{-\gamma k_B T_i \vec{\nabla} n_i \times \vec{B}}{B^2} - \frac{\gamma k_B T_e \vec{\nabla} n_e \times \vec{B}}{B^2}$$

$\vec{\nabla} n_i \approx \vec{\nabla} n_e \approx \vec{\nabla} n$

same

$$j = -\left(\frac{\gamma k_B T_i}{B^2}\vec{\nabla} n \times \vec{B} + \frac{\gamma k_B T_e \nabla n \times \vec{B}}{B^2}\right)$$

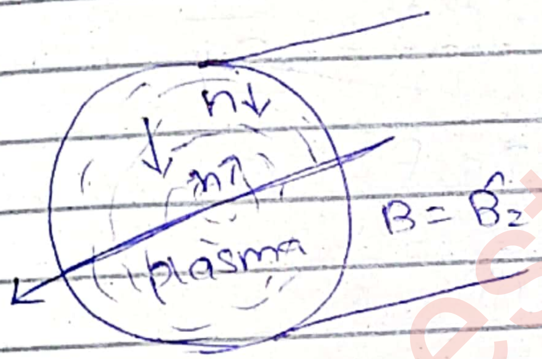
$$\vec{j} = (\gamma k_B T_i + \gamma k_B T_e)\frac{\vec{\nabla} n \times \vec{B}}{B^2}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\boxed{\vec{j} = (\gamma k_B T_i + \gamma k_B T_e)\frac{\vec{B} \times \vec{\nabla} n}{B^2}}$$

Current density in case of diamagnetic drift

Consider a cylinder and take plasma in it



Lower density ↙ outer side.

High density ↙ centre

inward $\sim -\hat{r}$

$\vec{\nabla} n \sim -\hat{r}$

$\vec{B} = B\hat{z}$

$(\hat{r}, \hat{\theta}, \hat{z})$

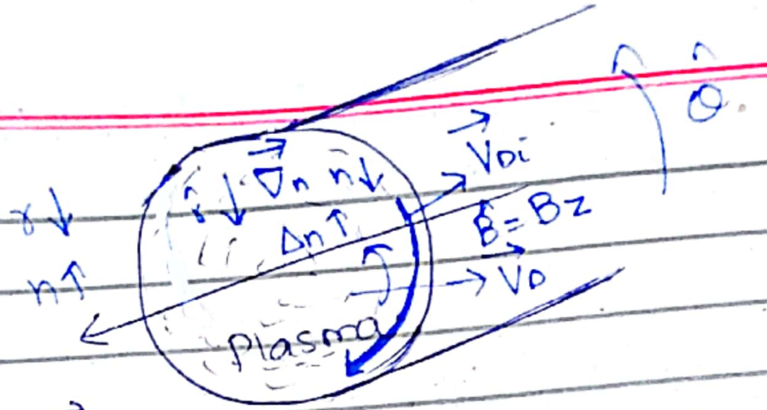
Clockwise direction $\sim$ -ve

Anticlockwise direction $\sim$ +ve

$\vec{V}_{Di} \simeq -(-\hat{r} \times \hat{z}) = -\hat{\theta}$ (clockwise)

$\hat{z}\ \hat{r}\ \hat{\theta}$

$\vec{V}_{DE} = +(-\hat{r} \times \hat{z}) = \hat{\theta}$ (anticlockwise)



$\vec{\nabla} n \sim \partial n / \partial r$

n↑ r↑   n↓ r↓

$\frac{\partial n}{\partial r} \sim +ve$ $\frac{\partial n}{\partial r} \sim +ve$

n↑ r↓

$\frac{\partial n}{\partial r} \sim -ve$ mean $\boxed{\frac{\partial n}{\partial r} < 0}$

(5) $$\vec{V}_{Di} = \frac{-r k_B T_i \frac{\partial n}{\partial r} B_-(-\hat{\theta})}{enB^2 - \hat{r} \times \hat{z}}$$

$$= \frac{r k_B T_i \, \partial n / \partial r \, B(\hat{\theta})}{en_i Bz}$$

$$\vec{V}_{De} = \frac{r k_B T_e \frac{\partial n}{\partial r} B_-(\hat{\theta})}{en_e B^2}$$

$$'' = \frac{r k_B T_e \frac{\partial n}{\partial r} B(\hat{\theta})}{en_e B^2}$$

$\left|\frac{\partial n}{\partial r}\right|$ → direction leta huma hum ha $\partial n / \partial r$ ka mode lena yani positive rakna ha

$$\vec{V}_{Di} = \frac{-\gamma k_B T_i \left|\frac{\partial n}{\partial r}\right| B\hat{\theta}}{e n_i B^2}$$

$$\vec{V}_{De} = \frac{\gamma k_B T_e \left|\frac{\partial n}{\partial r}\right| B\hat{\theta}}{e n_e B^2}$$

The Drifts parallel to B:

As B.feild is along z-axis $\vec{B} = B\hat{z}$
so the component of the fluid eq of motion is

$$mn\left[\frac{\partial}{\partial t} + (\vec{V}\cdot\vec{\nabla})\right]\vec{V} = qn(\vec{E} + \vec{V}\times\vec{B}) - \vec{\nabla}P \rightarrow (1)$$

V, V_x, V_y, V_z z-component of eq (1)

$V \rightarrow V_\perp$, $V_\parallel$

$$mn\left[\frac{\partial}{\partial t} + (\vec{V}\cdot\vec{\nabla})\right]V_z = qn(E_z + V_z B\cancel{\hat{z}\times\hat{z}}) - \partial P/\partial z \rightarrow (2)$$

@Plasma as a fluid

$$mn\left[\frac{\partial}{\partial t} + V_z\frac{\partial}{\partial z}\right]V_z = qnE_z - \frac{\partial P}{\partial z} \rightarrow (3)$$

$$mn\left[\frac{\partial}{\partial t}V_z + \underbrace{V_z\frac{\partial}{\partial z}V_z}_{\text{non Linear term}}\right] = qnE_z - \partial P/\partial z$$

non Linear term ($\rightarrow$ we can ignore it so)

$$mn\left[\frac{\partial V_z}{\partial t}\right] = qnE_z - \frac{\partial P}{\partial z}$$

if we take $m \rightarrow 0$ we are taking electrons and we are applying the approx that electrons are inertialess

$$0 = -enE_z - \partial P/\partial z \rightarrow (4)$$

→ When you take along the plasma (ch #4) as a fluid then their will be no magnetic feild component while in (Ch #3) their is Magnetic component present.

$$enE_z = -\partial P/\partial z$$

When Plasma is propagating $\parallel$ to B.feild that system is taken as electrostatic potential

$$\vec{\nabla}\times\vec{E} = 0$$

$$\vec{E} = -\vec{\nabla}\phi$$

ϕ is the electrostatic potential

$$E_z = -\frac{\partial\phi}{\partial z} \to (6)$$

using (6) in eq (5)

$$\underbrace{+en\frac{\partial\phi}{\partial z}}_{\text{electrostatic force}} = \underbrace{+\partial P/\partial z}_{\text{Pressure gradient force}} \to (7)$$

→ electrostatic force balance with Pressure gradient force. (equillibrium)

$$P = \text{cont}\,\rho^{\gamma}$$ eq of state.

$$\vec{\nabla}P = \text{cont}\,\gamma\rho^{\gamma-1}\vec{\nabla}\rho$$

$$\frac{\vec{\nabla}P}{P} = \frac{\text{cont}\,\gamma\rho^{\gamma-1}\vec{\nabla}\rho}{\text{cont}\,\rho^{\gamma}}$$

ρ mean charge per unit volume

$\because \rho = mn$

$$\frac{\vec{\nabla}P}{P} = \gamma\frac{\vec{\nabla}\rho}{\rho}$$

$$\frac{\vec{\nabla}p'}{p} = \frac{\gamma \cancel{m}\vec{\nabla}n}{\cancel{m}n}$$

$$\frac{\vec{\nabla}P}{P} = \frac{\gamma\vec{\nabla}n}{n}$$

$$P = nK_BT$$

$$p = \gamma nk_BT$$

$$\vec{\nabla}P = \gamma K_BT\vec{\nabla}n + \gamma K_Bn\vec{\nabla}T$$

For considering electron so $T = T_e$ and also the temp of electron is not equal to temp of ion in Fluid.

$$\gamma = 1 \quad \vec{\nabla}T = 0$$

→ Isothermal Plasma

$$\vec{\nabla}P = K_BT\vec{\nabla}n$$

$$\frac{\partial P}{\partial z} = K_BT_e\frac{\partial n}{\partial z}$$

$$\vec{\nabla}P = \frac{\gamma P\vec{\nabla}n}{n}$$

$$\gamma = 1$$

$$\vec{\nabla}P = \frac{\cancel{n}K_BT_e\vec{\nabla}n}{\cancel{n}}$$

$$\boxed{\vec{\nabla}P = K_BT_e\vec{\nabla}n}$$

$$\frac{\partial P}{\partial z} = K_BT_e\frac{\partial n}{\partial z} \rightarrow (8)$$

Using (8) in (7)

$$en\frac{\partial\phi}{\partial z} = K_BT_e\frac{\partial n}{\partial z}$$

$$\frac{\partial\phi}{\partial z} = \frac{K_BT_e}{en}\frac{\partial n}{\partial z}$$

Integrating both sides with dz

$$\int \left(\frac{k_B T_e}{e}\right)^{-1} \frac{\partial \phi}{\partial z} dz = \int \frac{1}{n} \frac{\partial n}{\partial z} dz$$

$$\frac{e}{k_B T_e} \int \frac{\partial \phi}{\partial z} dz = \int \frac{1}{n} \frac{\partial n}{\partial z} dz$$

$$\frac{e\phi}{k_B T_e} = \ln n + c \rightarrow (9)$$

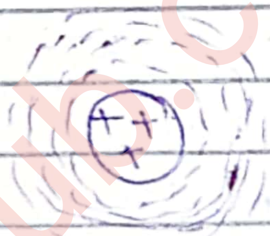
$\phi = 0 \qquad n = n_0$

equillibrium no density.

Initial condition:

$$0 = \ln n_0 + c$$

$$c = -\ln n_0 \rightarrow (10)$$



using (10) in (9)

$$\frac{e\phi}{k_B T_e} = \ln n - \ln n_0$$

$$\text{"} = \ln n/n_0 .$$

Initially $\phi = 0$ then afterer, once we have peturbation so ϕ will vary either increase or decrease.

Taking exponential on both sides

$$\exp\left(\frac{e\phi}{k_B T_e}\right) = n/n_0$$

$$n_e = n_0 \exp\left(\frac{e\phi}{k_B T_e}\right)$$

$$\boxed{n_e = n_{oe} \exp(e\phi/k_B T_e)}$$

$\hookrightarrow$ Maxwell Boltzman relation.

Thermal dist.

$$\vec{\nabla} P = \frac{\gamma P \vec{\nabla} n}{n}$$

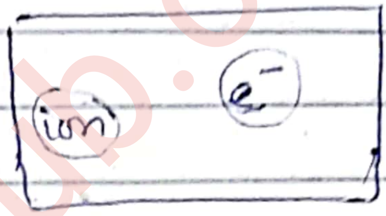
For ions:

$$\boxed{n_i = n_{i0} \exp\left(\frac{-e\phi}{k_B T_i}\right)}$$

→ $Mn \frac{\partial V_z}{\partial t}$

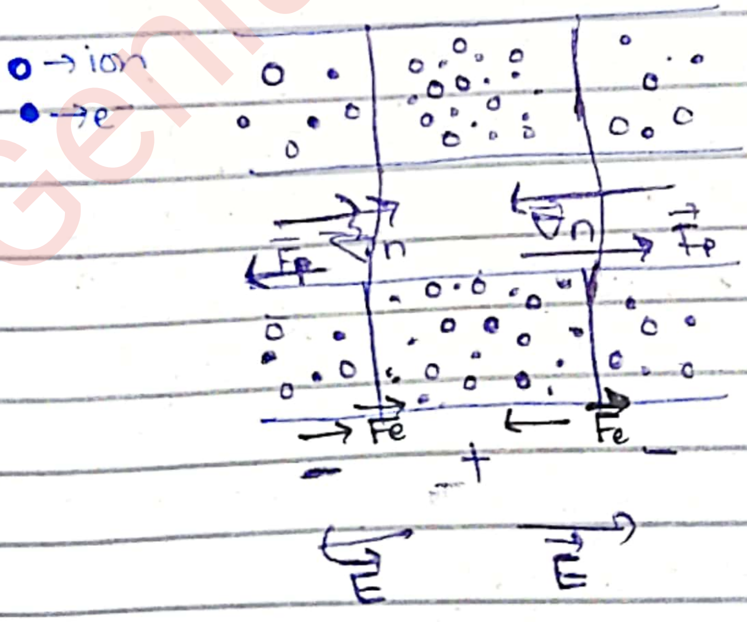
$\partial/\partial t \rightarrow \omega \rightarrow 0$

in electron we can't take $\omega = 0$ so we can take m zero



→ Speed of electron is high than ions. When we study electrons we can say ions act in background and are static.

Physical interpretation



Electric feild

+ → −

$\vec{F} = q\vec{E}$

$\vec{F}_e = -e\vec{E}$

↳ electrostatic force

Paper Question
How ions are mobile than elec

$\vec{F}_D$ $\rightarrow$ $\rightarrow$ $\vec{F}_P$

$\rightarrow$ $\leftarrow$ F

$\vec{F}_D = -\vec{\nabla} P$

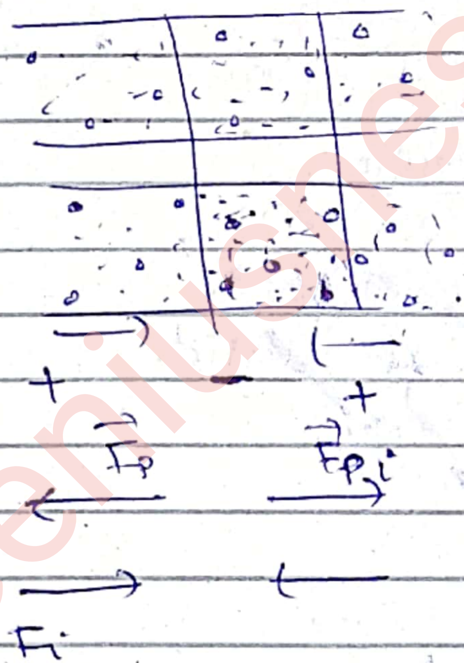
$\vec{F}_P = -k_B T_e \vec{\nabla}_n$

$\vec{F}_p = \vec{F}_e$ $\quad n_e \exp(e\phi / k_B T_e)$

Maxwell
Boltzman
dist.

→ How can This be in equillibrium.

When we study ions:-



$\vec{F}_p = \vec{F}_D^{\,F}$

→ The Plasma Approximation:

charge density $n_i = n_e$

$$\vec{\nabla}\cdot\vec{E} = \omega/\varepsilon_0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{en_i - en_e}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = 0 \rightarrow (1)$$

$$\vec{E} = -\vec{\nabla}\phi$$

$$\boxed{\nabla^2\phi = 0}$$

→ Posion eq.

eq (1) and (2) holds when $n_e = n_i$.

When we describe wave motion in plasma, we take $n_i = n_e$ but $\vec{\nabla}\cdot E \neq 0$

eq of motion.

$$mn\left[\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right]\vec{v} = q_n(\vec{E} + \vec{v}\times\vec{B}) - \vec{\nabla}P.$$

In wave description Plasma as fluid eq of motion to find the electric fluid and we don't use Poisson eq/Guass's law

Plasma approximation.
- $n_i = n_e$ $\quad \vec{\nabla}\cdot\vec{E} \neq 0$
- eq of motion to find E-feild.

* Plasma approximation is a mathentical shortcut that we can use to describe for wave motions in Plasma.