Ch # 2

Single Particle Motions

→ Assuming that electrons, ions and neutrals particles are one single
⊖ , ⊕ , ⓝ

→ Plasma is confined in space
Drift → Velocity
↳ Distraction (guiding centre shift)
not confined if you were not take a class and This is The guiding centre.

→ In this chapter,
we can study plasma in which the things have drift
Shift in guiding centre → drift.

If we have to confine (plasma)
charged particles.

We need Electric and Magnetic feilds.

$E(r,t)$, $B(r,t)$

Possibilities.

* Uniform E and B feilds

$$\frac{\partial E}{\partial t} = 0 \quad , \quad \frac{\partial B}{\partial t} = 0$$

feilds are uniform in time.

* $$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

Space variation:

$$\bar{E} = E\hat{x}$$

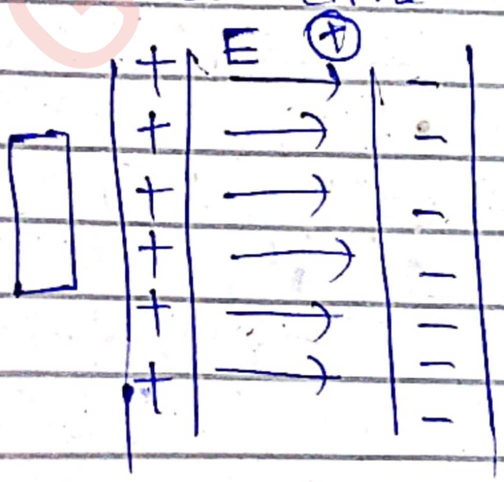
$\bar{\nabla}\cdot\bar{E} = 0$, $\bar{\nabla}\cdot\bar{B} = 0$

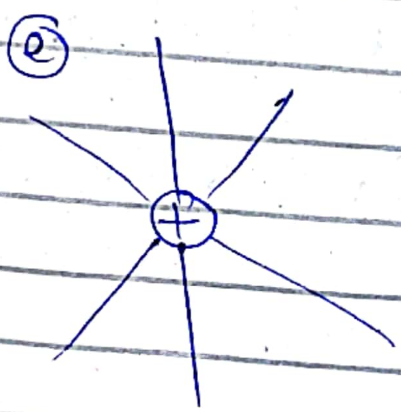
$\bar{\nabla}\times\bar{E} = 0$, $\bar{\nabla}\times\bar{B} = 0$

$\bar{\nabla}\cdot\bar{E} = 0$, $\bar{\nabla}\cdot\bar{B} = 0$

→ Uniform Electric and Magnetic Feild :-

This is uniform on both space and time.





So fig ① is uniform. So there is a some electric flux and This is a uniform electric field and fig ② has not same electric feild flux So there is not a uniform E.F

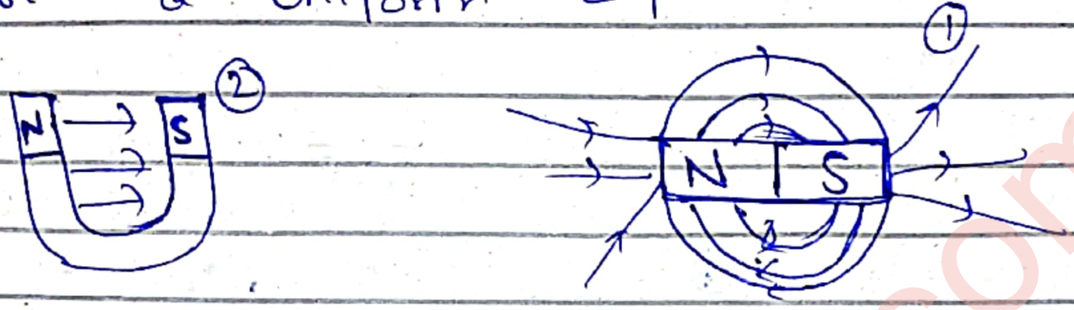
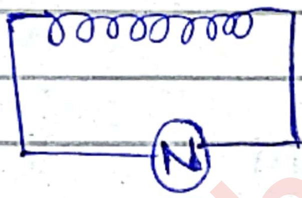


Figure ②



Uniform Magnetic and Electric feild
both has some electric flux

Case 1: Uniform E and B field:

(i) $\bar{E} = 0$ no electric feild / uniform B feild

$$F = \frac{m\,d\vec{v}}{dt} = q\,(\bar{E} + \bar{v} \times \bar{B}) \rightarrow ①$$

$\bar{E} = 0$ in eq ①

$$\boxed{\bar{F} = \frac{m\,d\vec{v}}{dt} = q\,(\bar{v} \times B)} \rightarrow ②$$

For simplicity we assume that B is directed along z-axis

$$\bar{B} = B\hat{z} \rightarrow ③$$

Resolving:-

There is no effect on neutral particles but there is effect on (+) and (−)

eq (2) is component taking $\boxed{x\text{-component}}$

$$m\frac{d\vec{V}_x}{dt}\hat{x} = qV_yB\hat{x}$$

$$\frac{dV_x}{dt} = \dot{V}_x$$

$$m\dot{V}_x = qV_yB \rightarrow (4)$$

$$\bar{V}\times\bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ V_x & V_y & V_z \\ 0 & 0 & 0 \end{vmatrix}$$

$\boxed{y\text{-Component}}$

$$\bar{V}\times\bar{B} = V_yB\hat{x} - V_xB\hat{y} + z(0)$$

$$m\dot{V}_y = -qV_xB \rightarrow (5)$$

$$\bar{V}\times\bar{B} = V_yB\hat{x} - V_xB\hat{y} + 0$$

$\boxed{z\text{-component}}$

$$\bar{V}\times\bar{B} = V_yB\hat{x} - V_xB\hat{y}$$

$$m\dot{V}_z = 0 \rightarrow (6)$$

$$\dot{V}_z = 0 \qquad V_z = \text{const}$$

$\Rightarrow \dot{V}_z = 0$

V_z velocity of particle along z-axis is constant

eq (4) and (5) are the coupled eq's

In order to solve eq (4) and (5) we have to decoupled these equations

Take derivative of eq (4)

$$m\cancel{\dot{V}_x}^{0} + m\ddot{V}_x = \cancel{\dot{q}V_yB}^{0} + q\dot{V}_yB + q\cancel{V_y\dot{B}}^{0} \rightarrow (7)$$

As we have to take B uniform so eq (7) takes the following form.

$$m\ddot{V}_x = qV_yB \rightarrow (8)$$

using (5) in eq (8)

From (5) $\dot{V}_y = -\frac{qV_xB}{m} \rightarrow (5)'$

Inserting (5)' in eq (8)

$$m\ddot{V}_x = q\left(-\frac{qV_xB}{m}\right)B.$$

$$\boxed{\ddot{V}_x = -\frac{q^2B^2V_x}{m^2}} \rightarrow (9) \text{ Decoupled eq.}$$

Taking derivative of eq (5)

$$\dot{m}\dot{V}_y + m\ddot{V}_y = -\dot{q}V_xB + q\dot{V}_xB - qV_x\dot{B} \rightarrow (7)'$$

As we have taken B uniform so eq (7)' takes the following form.

$$m\ddot{V}_y = -q\dot{V}_xB \rightarrow (8)'$$

using (4) in eq (8)'

From (4) $\dot{V}_x = \frac{qV_yB}{m} \rightarrow (5)'$

Inserting (5)' in eq (8)'

$$m\ddot{V}_y = q\left(-\frac{qV_yB}{m}\right)B$$

$$\boxed{\ddot{V}_y = -\frac{q^2B^2V_y}{m^2}} \rightarrow (10)$$

$$\bar{F} = q\bar{V}\times\bar{B}$$

$$F_c = F_B$$

$$\frac{mv^2}{r} = qvB\sin\theta.$$

$$\cancel{r} \cdot \frac{mv^2}{r} = qvB\sin 90^\circ$$

$$\frac{mv}{r} = qB$$

$$V = r\omega$$

$$\frac{m(\cancel{r}\omega)}{\cancel{r}} = qB$$

$$m\omega = qB$$

$$\boxed{\omega_c = \frac{qB}{m}} \rightarrow (11)$$ cyclotron freq when you have magnetic

If $B = 0$ Then $\omega = 0$

Magnetic feild causes oscillations. which is ~~[illegible]~~ ω_c.

using eq (11) in eq (9) and (10) we get.

$$\ddot{V}_x = -\omega_c^2 V_x \rightarrow (12)$$

$$\ddot{V}_y = -\omega_c^2 V_y \rightarrow (13)$$

For ions
$\omega_c = {}^{+}eB/M$ Mass of ion.

$$\omega_c = \frac{-eB}{m}$$
mass of e^-

Solution of eq (10)

$$\boxed{V_x = V_\perp e^{i\omega_c t}}$$

z axis – no force on particles → ∥ velocity

x, y axis has force on particles

Velocity ⊥ to B feild
⊥ velocity.

$z = x + iy$

$z = x - iy$

$$\boxed{V_y = \pm i V_\perp e^{i\omega_c t}}$$

$$\vec{F} = q(\vec{V} \times \vec{B}) \rightarrow (1)$$

$$V_x = V_\perp e^{i\omega_c t} \rightarrow (2)$$

$$V_y = i V_\perp e^{i\omega_c t} \rightarrow (3)$$

Real part

$$\omega_c = \left|\frac{qB}{m}\right|$$

$$\omega_c = \pm \frac{qB}{m}$$

ion $= \omega_c = eB/M$

$e^- = \omega_c = eB/m$

$$\omega_c^2 = \frac{e^2B^2}{m^2}$$

Velocities field
⇓
trajectories feild

eq jab aa gy honay pata chal gya ka kis ws korsi aai ha

$$V_x = V_\perp \cos\omega_c t + V_\perp e \sin\omega_c t \rightarrow (4)$$

$$V_y = \pm i V_\perp (\cos\omega_c t + i\sin\omega_c t) \rightarrow (5)$$

Taking real part of eq (4) and (5)

$$V_x = V_\perp \cos\omega_c t \rightarrow (6)$$

$$V_y = \pm i^2 V_\perp \sin\omega_c t$$

$$V_y = \mp V_\perp \sin\omega_c t \rightarrow (7)$$

It is following some trajectory. Trajectory describes some real component and real can't be imaginary.

$$V_x = dx/dt = V_\perp \cos\omega_c t \rightarrow (8)$$

$$V_y = dy/dt = \mp V_\perp \sin\omega_c t \rightarrow (9)$$

To find the trajectory we integrate eq (8) and (9) w.r.t t.

$$\int \frac{dx}{dt}\,dt = \int_0^t V_\perp \cos\omega_c t\,dt$$

$$x\Big|_{x_0}^{x} = \frac{V_\perp \sin\omega_c t}{\omega_c}\Big|_0^t$$

$$\boxed{x - x_0 = \frac{V_\perp \sin\omega_c t}{\omega_c}} \quad (10)$$

x_0 → must be known. particle position when $t=0$

Taking y component

$$\int_{y_0}^{y} \frac{dy}{dt}\,dt = \mp V_\perp \int_0^t \sin\omega_c t$$

$$y - y_0 = \pm \frac{V_\perp \cos\omega_c t}{\omega_c}\Big|_0^t$$

$$y - y_0 = \pm \frac{V_\perp \cos\omega_c t}{\omega_c}$$

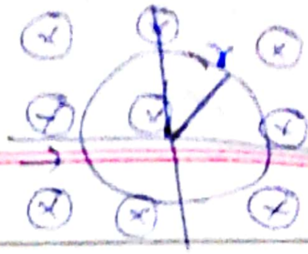
$$\boxed{y - y_0 = \pm \frac{V_\perp \cos\omega_c t}{\omega_c} \mp \frac{V_\perp}{\omega_c}} \quad (11)$$

$V = V_\perp + V_{\parallel}$

$\boxed{V_\perp = r\omega_c}$ → ω_c due to cyclotron freq in plasma.

$\therefore \omega_c = \pm \frac{qB}{m}$ {All three factors varying so not change remain same}

$\boxed{r = V_\perp/\omega_c}$ Larmor radius. (charge particle in a path)



Larmor Radius.

[plasma particle are tracing the path in trajectory

→ x_0 is not varying so not changing.
x is varying {according to eq (10)}
→ y_0 is not varying {according to eq (11)}
y is varying

→

$r_L = \frac{V_\perp}{\omega_c}$ is not varying with time

So we need to include trajectory in it

$$x - x_0 = \frac{V_\perp}{\omega_c} \sin \omega_c t \rightarrow (12)$$

$$y - y_0 = \pm \frac{V_\perp}{\omega_c} \cos \omega_c t \rightarrow (13)$$

For juding particle final position we must include x_0, y_0.

Squaring and adding eq (12) and (13)

$$(x - x_0)^2 + (y - y_0)^2 = \frac{V_\perp^2}{W_c^2}\left(\underbrace{\sin^2 W_c t + \cos^2 W_c t}_{1}\right)$$

$$(x - x_0)^2 + (y - y_0)^2 = \frac{V_\perp^2}{W_c^2}$$

→ When centre of circle is non-zero.

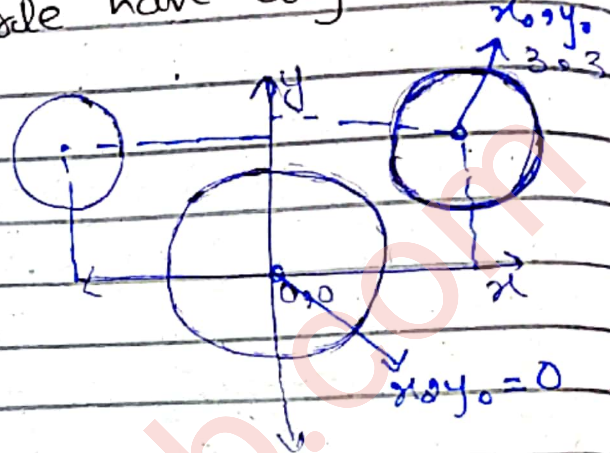
$$\boxed{(x-x_0)^2+(y-y_0)^2 = r_L^2} \rightarrow (14)$$

eq of circle.

→ When centre of circle have origin zero.

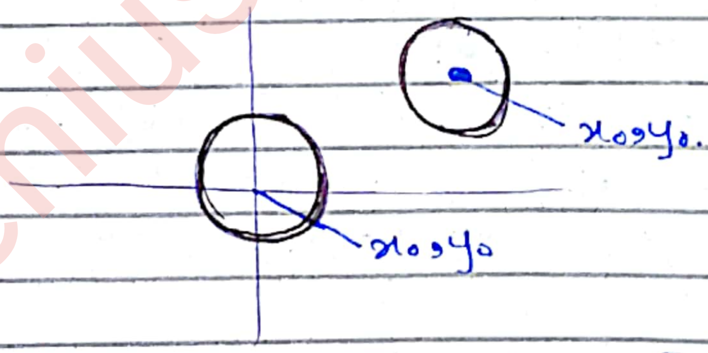
$$\boxed{x^2+y^2=r^2}$$

eq of circle.



→ eq (14) describes the circular orbit about a guiding centre (x_0, y_0) which is fixed and radius of circle r_L

origin changes if circle not at centre. the x_0, y_0 is non-zero.



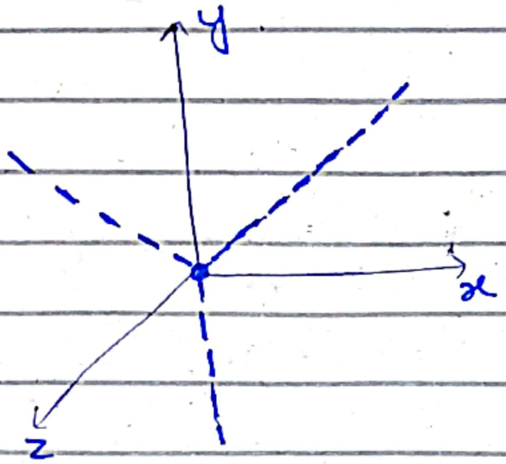
→ Guiding centre is not shifted as it moving about its same position.

→ Uniform Electric feild and Magnetic feild (E & B) Then No drift

$E = 0$

hence guiding centre is not shifted.

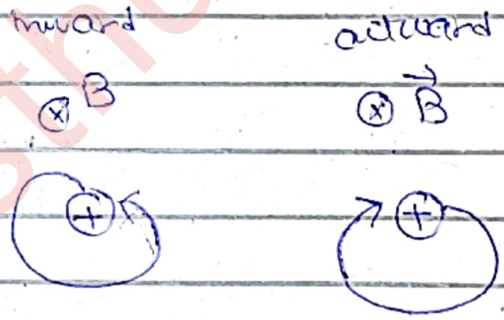
Trajectory:-



⊙ $\hat{z}$

$\vec{B} = B\hat{z}$

In $\boxed{\text{FMT}}$



In FMT
+ve in clockwise
-ve in anticlockwise.

→ Plasma particles are Diamagnetic.
→ Plasma contain charge particles.

→ when there is magnetic feild of charged particle also there magnetic feild outside. If both add then charge instead of getting stable becomes more energetic.

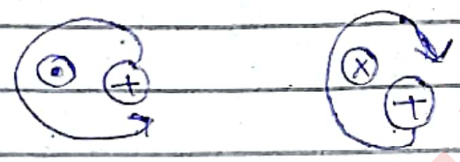
→ If both cancel each other then plasma particle become stable.

→ Positive charge & electron move in opposite direction if +ve particle will move clockwise.

$\odot \ \vec{B} = B\hat{z}$

+ve Right hand Rule.
−ve left hand Rule.

Possibilities:-

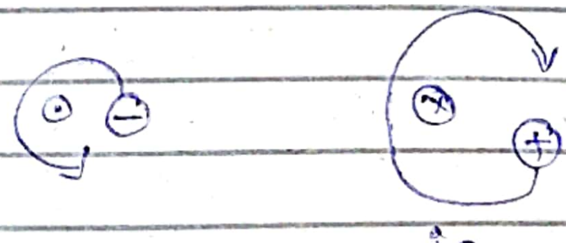


Here become more energetic because it satisfies $\vec{B} = B\hat{z}$ in outward direction.

Plasma will stable here in inward direction (because B outside and inside is opposite in direction)

For electrons:-

$E = 0 \qquad B = \text{uniform}.$


ion

$r_L = \frac{V_\perp}{\omega_c}$

This is trajectory when E=0 and B is uniform.

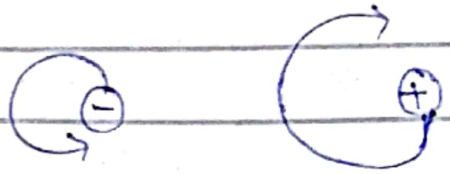
$r_L = \frac{v_\perp}{|\omega_c|}$

$r_L = \frac{v_\perp m}{qB}$

$M >> m$

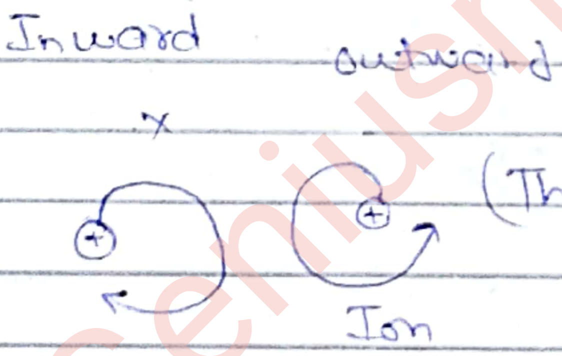
$m = M$

Why circle of ⊖ is less than ⊕



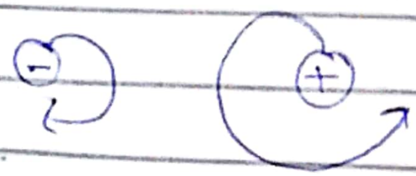
Because $r_L = \frac{v_\perp m}{qB}$

$\otimes \vec{B} = -B\hat{z}$



(This one will satisfy)

For electron :-



GS is not shifted → No drift in plasma

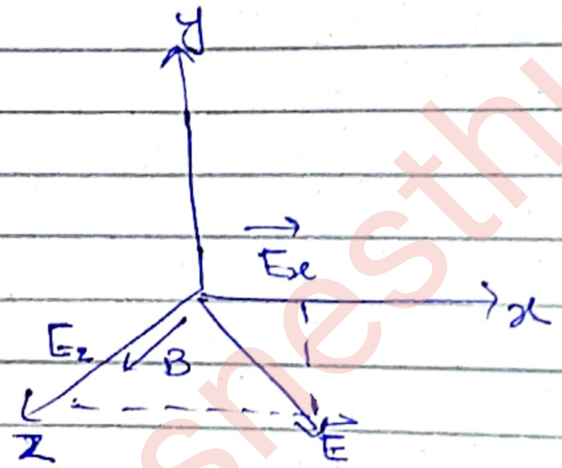
ion + electron

CASES:

① $E = 0$, B was uniform

② E is finite, $x-z$ plane. B is uniform $\vec{B} = B\hat{z}$

Figure:



$$\vec{F} = q[\vec{E} + (\vec{V} \times \vec{B})] \rightarrow ①$$

$$m\frac{d\vec{v}}{dt} = q[\vec{E} + \vec{V} \times \vec{B}] \rightarrow ②$$

Resolving eq ② into components

$$m\dot{V}_x = qE_x + qV_y B \rightarrow ③$$

$$V \times B = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ V_x & V_y & V_z \\ 0 & 0 & B \end{vmatrix}$$

$\vec{V} \times \vec{B} = V_y B\hat{x} - V_x B\hat{y} + 0$

$m\dot{V}_y = -qV_x B \rightarrow (4)$

$m\dot{V}_z = q(E_z + 0)$ b/c $E_y = 0$

$m\dot{V}_z = qE_z \rightarrow (5)$

In order to solve futhere we have to decouple eq (3) and (4)

Taking time derivative of eq (3)

$m\ddot{V}_x = q\cancel{\dot{E}_x}^{\,0} + q\dot{V}_y B + q\cancel{V_y \dot{B}}^{\,0}$

as E and B are uniform so E_x is zero $\dot{E} = 0, \dot{B} = 0$

$m\ddot{V}_x = qB\dot{V}_y \rightarrow (6)$

Inserting eq (4) in eq (6) we get:

$m\ddot{V}_x = qB\left(\frac{-qV_x B}{m}\right)$

$\ddot{V}_x = -\frac{q^2 B^2}{m^2} V_x$

$\boxed{\ddot{V}_x = -\omega_c^2 V_x} \rightarrow (7)$ $\qquad \because \omega_c^2 = \frac{q^2 B^2}{m^2}$

Decoupled eq.

$m\ddot{V}_y = -qB\dot{V}_x$

$\ddot{V}_y = -\frac{qB}{m}\left(\frac{q}{m}\left(E_x + V_y B\right)\right)$

$\ddot{V}_y = -\frac{q^2 B E_x}{m^2} - \frac{q^2 B^2 V_y}{m^2}$

$\ddot{V}_y = -\frac{qB}{m}\left[\frac{qE_x}{m} + \frac{qBV_y}{m}\right]$

$$W_c = \pm \frac{qB}{m}$$

$$\pm W_c = \frac{qB}{m}$$

$$\ddot{V}_y = -\pm W_c \left[\frac{qE_x}{m} \pm W_c V_y\right]$$

$$\ddot{V}_y = \mp W_c \left[\frac{qE_x}{m} \pm W_c V_y\right]$$

$$\ddot{V}_y = \mp W_c \left[\frac{qE_x B}{m\,B} \pm W_c V_y\right]$$

$$\ddot{V}_y = \mp W_c \left[\pm W_c \frac{E_x}{B} \pm W_c V_y\right].$$

$$\ddot{V}_y = -W_c^2 \left[\frac{E_x}{B} + V_y\right] \rightarrow (8)$$

$$\ddot{V}_x = -W_c^2 V_x \xrightarrow{\text{Sol.}} V_x = V_\perp e^{i w_c t}$$

$$\boxed{\ddot{V}_y = -W_c^2 V_y} \rightarrow (a)$$

it not gives similar sol. as V_x so taking substitute.

$$V_y' = \frac{E_x}{B} + V_y.$$

Putting in eq (8)

$$\ddot{V}_y' = -W_c^2 V_y' ,$$

$$\ddot{V}_y' = \ddot{V}_y$$

$$V_y' = \pm i V_\perp e^{i w_c t} \rightarrow \text{Case 1}$$

$$\ddot{V}_y = -W_c^2 V_y$$

Derivative of eq (a) is

$$\boxed{\dot{V}_y' = \cancel{\frac{\dot{E}_x}{B_x}} + \dot{V}_y}$$

$$V_y = \pm i V_\perp e^{i\omega_c T}$$

$$\frac{E_x}{B} + V_y = \pm i V_\perp e^{i\omega_c t}$$

$$V_y = \pm i V_\perp e^{i\omega_c t} - \frac{E_x}{B} \rightarrow (9)$$

$$\boxed{V_x = V_\perp e^{i\omega_c T}} \rightarrow (10)$$

$$m\dot{V}_z = q E_z$$

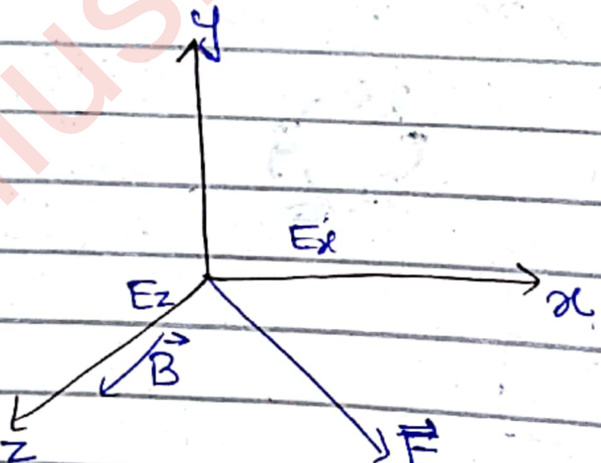
$$\dot{V}_z = q\frac{E_z}{m}$$

$$V_z = \frac{q E_z t}{m}$$

$V_z = \text{const}$ } Derivative of V_z.
$\dot{V}_z = 0$

⊙ V_z according to time changes

Figure:



Drift is a factor of velocity

→ Drift due to velocity

⊙ $V_0 = -E_x/B$

Now $\vec{E}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_x & 0 & E_z \\ 0 & 0 & B \end{vmatrix}$

$\vec{E} \times \vec{B} = \hat{x}(0) - \hat{y}\, E_x B + 0\hat{z}$.

$$\boxed{\vec{E} \times \vec{B} = -E_x B \hat{y}}$$

Drift causing term in form of vectors:-

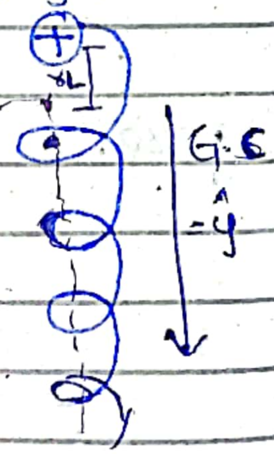
$$\boxed{\vec{V}_D = -\frac{E_x}{B}\hat{y}}$$

$$\frac{\vec{E} \times \vec{B}}{B} = -E_x \hat{y}.$$

$$\boxed{\vec{V}_D = \frac{\vec{E} \times \vec{B}}{B^2}}$$ → General expression for drift in presence of E-feild.

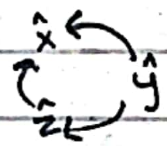
→ Ion will accelerate in direction of E-feild then it'll start moving in direction of B-feild. M.F more ion in circle.

Figure:-



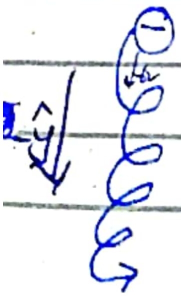
$$\frac{(E_x \hat{x} + E_z \hat{z}) \times B\hat{z}}{B^2}$$

$$\vec{V}_D = -\frac{E_x B \hat{y}}{B^2}$$



$$\boxed{\vec{V}_D = -\frac{E_x}{B}\hat{y}}$$

Ion



Drift happening both in pie's in $-y$ direction.

Previous Figure

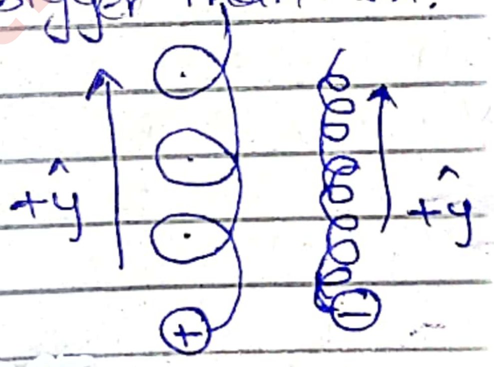
→ Why we have electron drift of big radius and ion as small?

→ Drift is not dependent on charge.
→ Drift in electron is bigger in size as of big radius while in ion smaller

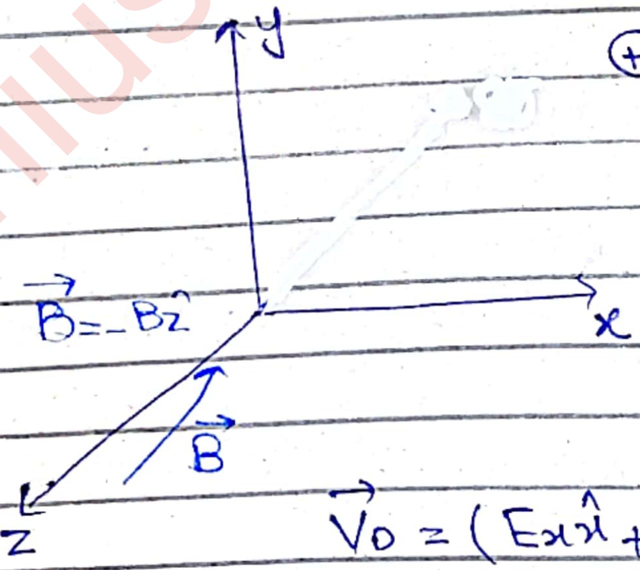
→ Radius of drift in electron less than ion b/c of mass and when any charge is in dir. of E.F then accelerate so velocity increases when $v_\perp \uparrow = r_L \uparrow$

⊙ It means drifted is related with E.F so V_0 is called Electric drift in plasma.

⊙ r_L of electron is bigger than ion.



Another Case:



$$\vec{V}_0 = \frac{(E_x\hat{x} + E_y\hat{y}) \times -B\hat{z}}{B^2}$$

$$\vec{V}_0 = \frac{E_x}{B}\hat{y} \ldots$$

limitations in drawing as we ...

Gravitational field:-

both are same almost.

→ is also uniform feild

$$V_E = \frac{\vec{E} \times \vec{B}}{B^2} \rightarrow (1)$$

electric drift $\vec{E} \times \vec{B}$

Multiplying and dividing by q

$$V_E = \frac{q\vec{E} \times \vec{B}}{qB^2}$$ as F_E = electric force $= q\vec{E}$

$$V_E = \frac{\vec{F_E} \times \vec{B}}{qB^2} \rightarrow (2)$$

$\therefore \vec{F_g} = m\vec{g}$

if we put gravitational force in it [electric has gravity]

if we replace $\vec{F_e} = \vec{F_g}$ so, $\vec{V_g} = \vec{F_g}$

$$\vec{V_g} = \frac{\vec{F_g} \times \vec{B}}{qB^2} \rightarrow \text{drift}$$

putting $\vec{F_g} = m\vec{g}$.

$$\vec{V_g} = \frac{m\vec{g} \times \vec{B}}{qB^2}$$

$$\vec{V_g} = \frac{m\vec{g} \times \vec{B}}{qB^2}$$

$J = I/R \rightarrow nqv$

current density

but:

in plasma:

$J = \Sigma nqV$

$= en_iV_i - en_eV_e$

As we have current density in plasma as

$\vec{J} = en_i\vec{v_i} - en_e\vec{v_e}$

$\vec{V_{Fi}} = \vec{V_{Fe}}$

$\vec{J} = en(\vec{V_{Fi}} - \vec{V_{Fe}})$

$n_i \simeq n_e \simeq n$

For electric drift

$$V_{Ei} = \frac{\vec{E} \times \vec{B}}{B^2} = V_{Ee}$$

$$V_{Ei} = \frac{\vec{E} \times \vec{B}}{B^2} = V_{Ee}$$

$J = 0$ for electric drift ← No current density when we study e^- drift
but for gravitational drift

$$\vec{J} = e n_i \vec{V}_{gi} - e n_i \vec{V}_{ge}$$

$n_i \simeq n_e \simeq n$ quasi neutrality.

$$\vec{J} = en(\vec{V}_{gi} - \vec{V}_{ge}) \rightarrow (1)'$$

For electrons:-

$$\vec{V}_{gi} = \frac{\overset{\text{mass of ion}}{M}\vec{g} \times \vec{B}}{eB^2} \rightarrow (2)'$$

$$\vec{V}_{ge} = \frac{\overset{\text{mass of } e^-}{m}\vec{g} \times \vec{B}}{-eB^2} \rightarrow (3)'$$

putting (2)' and (3)' in (1)'

$$\vec{J} = \frac{\cancel{e} n (M+m) \vec{g} \times \vec{B}}{\cancel{e} \cdot B^2}$$

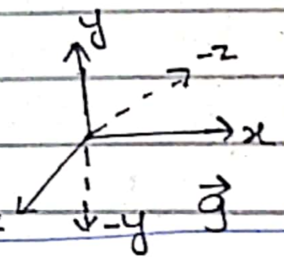
$$J = \underbrace{n(M+m)}_{\text{plasma mass density}} g \times B / B^2$$

q_n = charge density
m_n = mass density.

as $\rho = mn$ ← mass density

$$J = \rho_m \frac{\vec{g} \times \vec{B}}{B^2}$$

g ~ $-y$ directions.

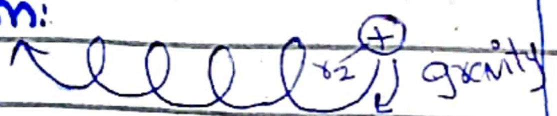


⊙ outward B

$$\vec{V}_{gi} = \frac{M \, -\hat{y} g \times \hat{z} B}{eB^2}$$

$$\vec{V}_{gi} = -\hat{y} \times \hat{z} \simeq -\hat{x}$$

Ion:

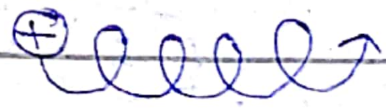


B ⊗ inward

$$\vec{V}_{gi} = \frac{M \, -y g \times -zB}{eB^2}$$

$$V_g = -y \times -z = +\hat{x}$$

Ion:



$r_L = v_\perp / \omega_c$

$r_L = \frac{v_\perp m}{qB}$

Electron:

$V_{ge} = \frac{m\vec{g} \times \vec{B}}{eB^2}$

$V\vec{g}_e = -(-\hat{y}) \times \hat{z}$

electron drift

Electron:

$V_{ge} = \frac{m\vec{g} \times \vec{B}}{eB^2}$

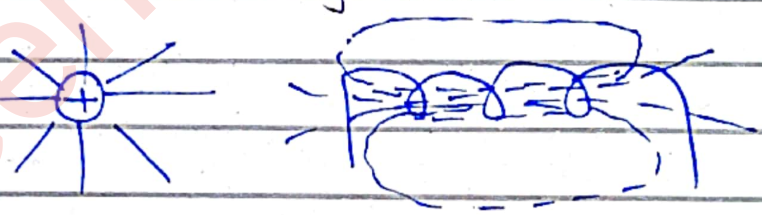
electron drift

→ Gravitational drift is charge dependent ions and electrons.

Non-Uniform E.B field:

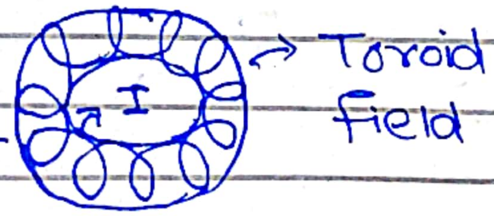
$\vec{B}(r,t)$] not varrying
$\vec{E}(r,t)$] with space and time
means they don't have gradient and curls e.t.c.

Non-uniform varry with time:
↳ Means simply it contains gradient, curl, divergence e.t.c.



← solenoid

→ Takamak is a device in which plasma is confined → Japnese device



Magnetic feild $B = \mu_0 n I \rightarrow$ in toroid.

$$B = \mu_0 N I / l$$

$$B = \frac{\mu_0 N I}{2\pi r} \rightarrow \textbf{So } B \sim I/r$$

For toroid, Solonoid B feild is $1/r$

how magnetic feild is uniform in toroid?

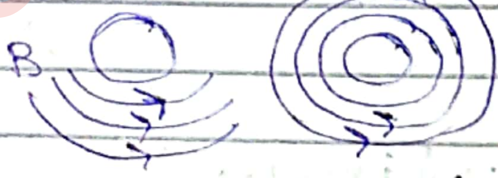
Ans: If something is moving in a circle its uniform in toroid field is uniform until its moving in same circle but if it moves from  one to other circle and it $1/r$ changes its becomes non-uniform.

Magnetic field is non-uniform in Tokamak 

Grad B-drift:-

Tokamak

same direction of B



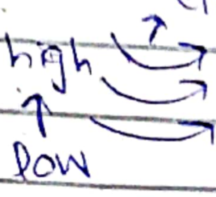
→ Magnetic field varies as we move along Tokamak. The drift in this system will be $\underbrace{\text{Grad}}\ \underbrace{B}$-drift → Variation in B

→ Gradient is directed from low to high

$$\vec{\nabla} B \perp \vec{B}$$

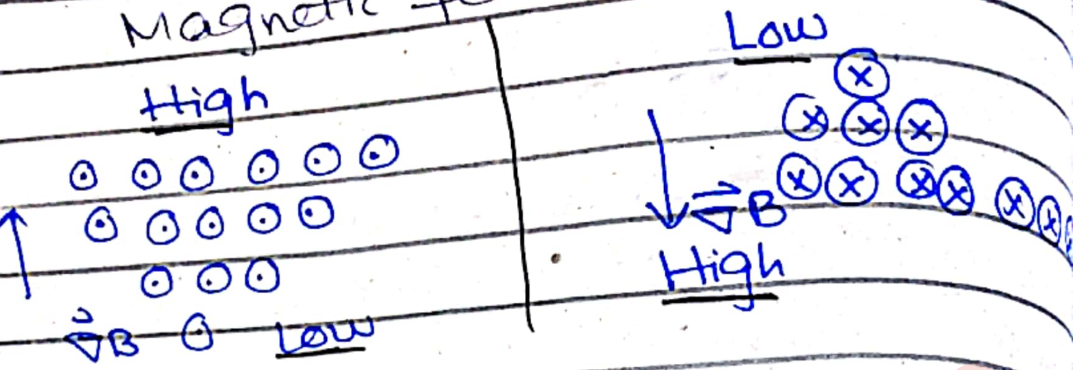
low to high

Grad B direction.



direction of B

Non Uniform Magnetic feild.



Why radius changes :-

We know that:

$$r_L = \frac{V_\perp}{\omega_c} = \frac{V_\perp m}{qB}$$

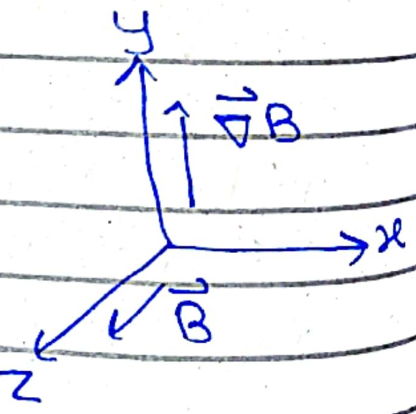
Magnetic	radius
→ B↓	r_L↑
B↑	r_L↓

→ less magnetic field great radius.
→ More magnetic field short radius.

When there is a change in radii there will be change in B, there will be drift.

Derivation:-

B is along z-axis
∇B is along y-axis.



$F = q(\vec{v} \times \vec{B}) \rightarrow (1)$

(B is varying in space)

$\vec{F} = q(\vec{v} \times \vec{B}(r)) \rightarrow (1')$

$F_x = q V_y B \rightarrow (2)$

$F_y = -qV_x B \rightarrow (3)$ → hamary kam ki force

it can be drift causing force

$F_z = 0 \rightarrow (4)$ → not important

$F_y = -qV_x B_z(y) \rightarrow (5)$ → varrying

$$\boxed{F_y = -qV_x B(r)} \rightarrow \text{general expression.}$$

Taylor's Series:-

$$f(a+h) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \dots$$

here

$$a = 0 \qquad h = r$$

$$\vec{B}(r) = B(0) + (\vec{r}\cdot\vec{\nabla})\vec{B} + \dots$$

if uniform then: $\vec{B}_z(y) = B_0\hat{z}$.

$$\vec{B}_z(y) = \Big(\underbrace{B_0}_{\text{constant term}} + y\frac{\partial}{\partial y}B + \dots\Big)\hat{z} \rightarrow (6)$$

→ Non uniformality exists because of second term:-

using eq (6) in (5)

varrying in y ↳ $F_y = -qV_x\Big[\Big(B_0 + \underbrace{y\frac{\partial B}{\partial y}}_{\text{اسکا کوئی نہ کوئی لینا دینا ہے drift k sth}} + \dots\Big)\hat{z}\Big] \rightarrow (7)$ directed in $\hat{z}$.

Lecture:

As we found earlier in uniform E and B field.

(3) — $y = y_0 \pm r_c \cos\omega_c t$

(6) — $x = x_0 \pm r_L \sin\omega_c t$

eq 2.7 of chapter

$V_x = V_\perp \cos\omega_c t \rightarrow (4)$, $V_y = \pm i V_\perp \cos\omega_c t \rightarrow (5)$

As inserting (6) in (5)

$$F_y = -qV_x\Big[B_0 + y\frac{\partial B}{\partial y}\Big]$$

using (3) (4) in (5)

$$F_y = -q V_\perp \cos\omega_c t \left[B_0 + (y_0 \pm r_L \cos\omega_c t)\frac{\partial B}{\partial y}\right] \rightarrow (8)$$

Take $y_0 = 0$ in eq (8)

$$F_y = -q V_\perp \cos\omega_c t \left[B_0 + (\pm r_L \cos\omega_c t)\frac{\partial B}{\partial y}\right]$$

varrying b/c of this term

$$F_y = -q V_\perp \cos\omega_c t \left[B_0 \pm r_L \cos\omega_c t \frac{\partial B}{\partial y}\right] \rightarrow (9)$$

B_0 is uniform magnetic feild

→ as F_y is varrying because of $\frac{\partial B}{\partial y}$ we have to take an average of expression (9)

$$F_y = -q V_\perp B_0 \underbrace{\cos\omega_c t}_{\text{varrying term}} \mp q V_\perp \underbrace{\cos\omega_c t}_{\text{varrying term}} \frac{\partial B}{\partial y}$$

$$\text{Average of } \cos\omega_c t = \frac{\overset{\text{min}}{-1} + \overset{\text{max}}{1}}{2} = 0$$

$$\text{Average of } \cos^2\omega_c t = \frac{0+1}{2} = 1/2$$

Now we can write average.

$$F_y = 0 \mp q V_\perp \frac{r_\perp}{2} \frac{\partial B}{\partial y}$$

$$\underbrace{F_y = \mp q \frac{V_\perp r_\perp}{2} \frac{\partial B}{\partial y}}_{\text{Drift causing force.}}$$

General expression:-

$$V_{\nabla B} = \frac{\vec{F_y} \times \vec{B}}{qB^2}$$

General Expression

$$\boxed{V = \frac{\vec{F} \times \vec{B}}{qB^2}}$$

putting F_y here

$$V_{\nabla B} = \frac{\mp \cancel{q} \frac{V_\perp r_\perp}{2} \frac{\partial B}{\partial y} \hat{y} \times \cancel{B}\hat{z}}{\cancel{q} B^{\cancel{2}}}$$

$$\vec{V}_{\nabla B} = \mp \frac{V_\perp r_\perp}{2B} \frac{\partial B}{\partial y} \hat{x} \rightarrow (10)$$

B vary in z
∇B varying y
V directed in x.

$$\bar{B} \times \nabla \bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & B \\ 0 & \frac{\partial B}{\partial y} & 0 \end{vmatrix}$$

$$B \times \nabla B = \hat{x}\left(-B\frac{\partial B}{\partial y}\right) - \hat{y}(0) + \hat{z}(0)$$

$$'' = \hat{x}\left(-B\frac{\partial B}{\partial y}\right) + 0 + 0$$

$$\boxed{B \times \nabla B = -B\frac{\partial B}{\partial y}\hat{x}}$$

$$\boxed{-\frac{B \times \nabla B}{B} = +\partial B/\partial y} \rightarrow (a)$$

using (a) in (10) eq, we get:

$$\boxed{\vec{V}_{\nabla B} = \pm \frac{V_\perp r_\perp}{2B^2} \vec{B} \times \nabla \vec{B}}$$ → General expression Grad B drift

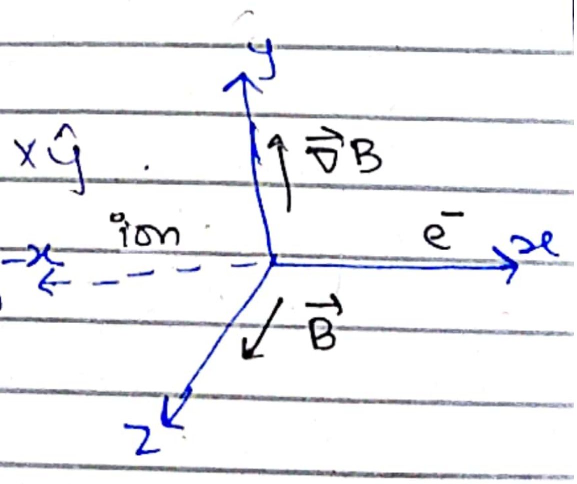
⇒ if q, or ± sign present in same eq then one is for ion and other for e^-

Our Problem was

For ion:-

$$\vec{V}_{\nabla B} = +\frac{V_\perp r_\perp}{2B} B \frac{\partial B}{\partial y} \hat{z} \times \hat{y}.$$

$$\bar{V}_{\nabla B} = -\hat{x} \rightarrow \text{for ion}$$



For electron:-

$$\vec{V}_{\nabla B} = -B \times \nabla \bar{B}$$

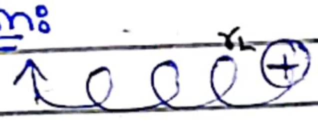
$$\vec{V}_{\nabla B} = +\hat{x} \quad \text{for electrons}$$

Where there is
$B\uparrow \ r_L \downarrow$

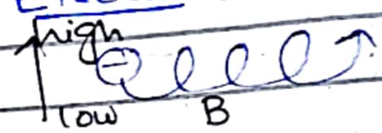
hai B-E high جہاں
چھوٹا radius waha

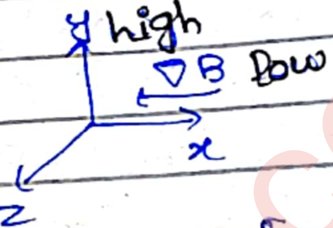
Trajectory (General case)

Ion:



Electron:





Practice:-

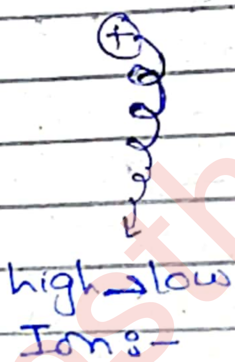
$\hat{B} \simeq \hat{z}$

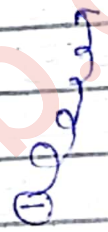
For ion:-

$V_{\nabla B} = -\hat{y}$

For e^-:-

$V_{\nabla B} = +\hat{y}$

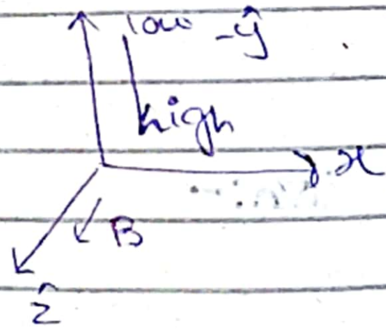


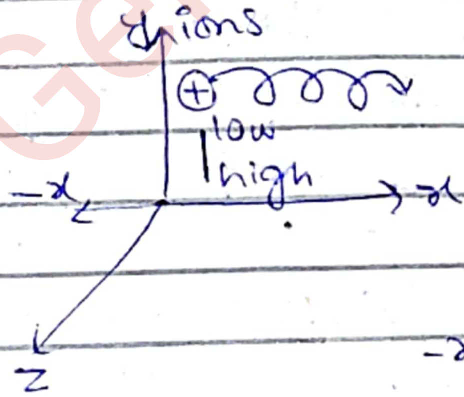


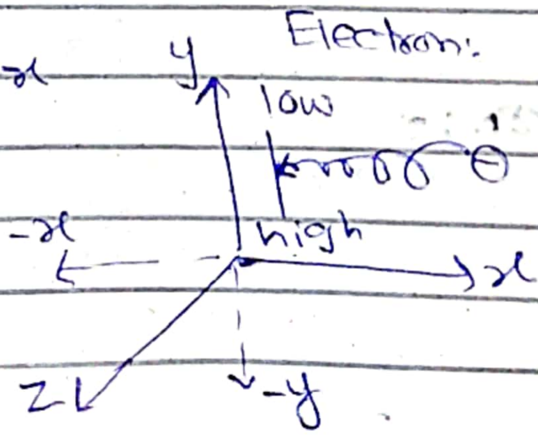
Practice Case:-

$\text{ion} = +(-y \times z) = +\hat{x}$

$e^- = -(-y \times z) = -\hat{x}$

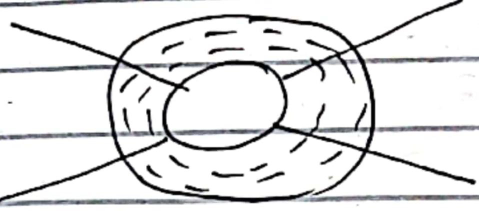






Curved B : Curvature Drift:-

Releated to Tokamak



"The drift which arises at curvature is called curvature drift"

→ Assume that the lines of forces to be curved with constant radius of curvature R_c and we take $|B|$ to be constant → Consider a case of toroid where we have curve B-feild.

$B \sim 1/R_c$

$\boxed{\text{if } R_c \text{ not varying } B \text{ also not varying}}$

Mode became magnetic field line can be clockwise or anticlockwise

The force which move the particle outward and cause drift is centrifugal force

We have gyromotion of charge particle The particle will experience a force called centrifugal force.

A shift in guiding centre arises from the centrifugal force. When plasma particles moves along the B-feild lines.

Centripetal force: $\leftarrow \vec{F}_c = \frac{mv^2}{r}\hat{r}$

centrifugal force $\leftarrow \vec{F}_{cf} = \frac{mv_\parallel^2}{r}\hat{r}$

$$\vec{F}_{cf} = \frac{mv_{\parallel}^2}{r}\hat{r}$$

$r = R_c$

$$\vec{F}_{cf} = \frac{mv_{\parallel}^2}{R_c}\hat{R}_c$$

as; $\hat{R}_c = \frac{\vec{R}_c}{|R|}$

$$\vec{F}_{cf} = \frac{mv_{\parallel}^2}{R_c}\frac{R_c}{|R|}$$

$$\boxed{\vec{F}_{cf} = \frac{mv_{\parallel}^2}{R_c^2}\vec{R}_c} \rightarrow (3)$$

$$\vec{V}_c = \frac{\vec{F}\times\vec{B}}{qB^2} \rightarrow (4)$$

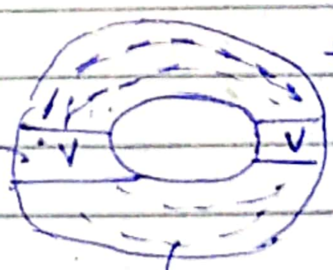
Curvature drift.

putting (3) in (4)

$$\boxed{\vec{V}_c = \frac{mv_{\parallel}^2}{R_c^2}\frac{\vec{R}_c\times\vec{B}}{qB^2}}$$ called curvature drift.

- $\vec{\nabla}\cdot\vec{B} = 0$
- $\vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$
- $\nabla\times B = \mu_0 J$

$V_{\nabla B}$, V_R



$B \propto 1/r$

V_c and V_R is same for curvature drift

For closed path
$\vec{\nabla}\times\vec{B} = 0$

one path has some value other must be changed for closed path.

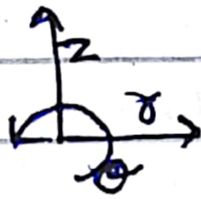
Cylinderical co-ordinate:- $\boxed{\frac{1}{r}\frac{\partial A_z}{\partial \theta}} - \frac{\partial A_\theta}{\partial z}$

Radial component

(ii) $(\vec{\nabla}\times A)_\theta = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}$

(iii) $(\vec{\nabla}\times\vec{A})_z = \boxed{\frac{1}{r}\frac{\partial}{\partial r}(r A_\theta)} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}$

$\vec{\nabla} B \sim \hat{r}$ $\quad B \sim \hat{z}$



$\vec{\nabla}\times\vec{B} = 0$

✓ its z-component.

$$(\vec{\nabla}\times\vec{B})_z = \frac{1}{r}\frac{\partial}{\partial r}(r B_\theta) - \frac{1}{r}\frac{\partial B_r}{\partial \theta} = 0$$

B_r is not varying with θ.

$$\frac{1}{r}\frac{\partial}{\partial r}(r B_\theta) = 0$$

$$\frac{\partial}{\partial r}(r B_\theta) = 0$$

$r B_\theta$ = const.

$B_\theta = 1/r$

Tokamak Magnetic feild in EMT

→ Toroid can be exactly circle.

$B_\theta = \frac{\text{constt}}{r}$, $\vec{V}_{\nabla B} \pm \frac{V_\perp r_\perp}{\partial B^2} \vec{B} \times \vec{\nabla} B$ } seen it is directed in θ.

$$\vec{V_c} + \vec{V_{\nabla B}} = 0$$

$$\vec{\nabla} B_r = \frac{\partial}{\partial r}\left(\frac{\text{constt}}{r}\right) r_z - \frac{\text{const}\,\hat{r}}{r}$$

$$\vec{r} = R_c .$$

$$\vec{\nabla} B_\theta = \frac{-\text{Const}\,\hat{R_c}}{R_c^2}$$

$$\frac{\vec{\nabla} B_\theta}{B_\theta} = \frac{-\text{constt}\,R_c}{R_c^2\ \text{const}^2}$$

$$\vec{\nabla} B_\theta = \frac{-\cancel{\text{const}}\,R_c .\ \cancel{R_c}}{R_c^2\ \cancel{\text{const}}}$$

$$\boxed{\frac{\vec{\nabla} B_\theta}{B_\theta} = \frac{-R_c}{R_c}} \rightarrow (1)$$

→ Plasma will be confined in Tokamak when vector sum of curvature drift and grad. B drift is zero.

So first U have to determine.

$$\boxed{\vec{V_R} + \vec{V_{\nabla B}} = \frac{m V_\parallel^2}{q B^2 R_c^2} \vec{R_c} \times \vec{B} \pm \frac{V_\perp r_\perp}{2B^2} \vec{B} \times}$$

eq (2)

Using ① in ② we get

$$\vec{V}_R + \vec{V}_{\nabla B} = \frac{mV_{\parallel}^2}{qB^2R_c^2} \vec{R}_c \times \vec{B} \pm \frac{V_\perp r_\perp}{2B} \frac{\vec{B} \times \vec{R}_c}{R_c}$$

$$\hat{R}_c = \frac{\vec{R}_c}{|R_c|}$$

$$\vec{V}_B + \vec{V}_{\nabla B} = \frac{mV_{\parallel}^2}{qB^2R_c^2} \vec{R}_c \times \vec{B} \pm \frac{V_\perp r_\perp}{2B} \frac{\vec{R}_c \times \vec{B}}{R_c}$$

$$\frac{\vec{R}_c \times \vec{B}}{R_c^2 B}\left[\frac{mV_{\parallel}^2}{qB} \pm \frac{V_\perp^2}{\omega_c 2}\right]$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$r_\perp = W_\perp / \omega_c$$

$$\omega_c = \frac{\pm qB}{m}$$

$$\boxed{\vec{V}_R + \vec{V}_{\nabla B} = \frac{\vec{R}_c \times \vec{B}}{R_c^2 B}\left[\frac{1}{\omega_c} mV_{\parallel}^2 \pm \frac{V_\perp^2}{W_c 2}\right]}$$ ×

$$\vec{V}_R + \vec{V}_{\nabla B} = \frac{\vec{R}_c \times \vec{B}}{R_c^2 B}\left[\frac{mV_{\parallel}^2}{qB} \pm \frac{V_\perp^2 m}{\pm qB 2}\right]$$

$$\vec{V}_R + \vec{V}_{\nabla B} = \frac{\vec{R}_c \times \vec{B} m}{qR_c^2}\left[V_{\parallel}^2 + \frac{1}{2}V_\perp^2\right] = 0$$

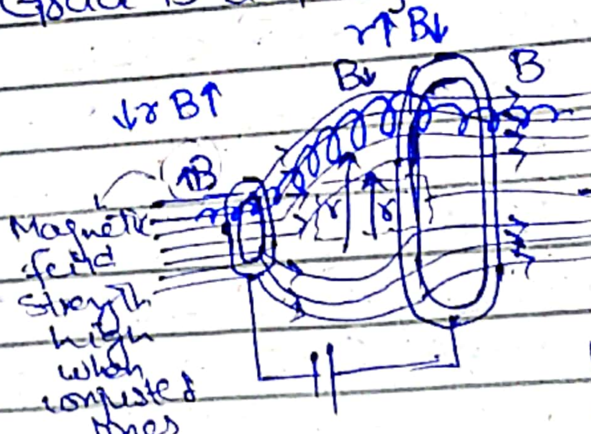
Question.
Confine Tokmak Plasma

Plasma is confined in Tokmak and if it $\neq 0$ the not confined plasma in Tokamak.

→ Tokmak ma grad. B zero) hota ha after that 1/r and solving vector value.

$\nabla \vec{B} \perp \vec{B}$ Grad B drift $\Big\{$ Tokamak.

$\nabla \vec{B} \parallel \vec{B}$



$V = \frac{F \times B}{q B^2}$

$\nabla \times B = 0$

- Magnetic Mirror $\nabla \vec{B} \parallel \vec{B}$, $\nabla \cdot B = 0$
- Tokamak $\nabla \vec{B} \perp \vec{B}$, $\nabla \cdot B = 0$
- $\nabla \times B \neq 0$

$\nabla \vec{B} \parallel \vec{B}$

$\vec{\nabla} \cdot \vec{B} = 0$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}(r B_r)\right) + \frac{\partial B_z}{\partial z} = 0$$

$\vec{B} \sim \hat{z}$

$\nabla \vec{B} \sim \hat{z}$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}(r B_r)\right) = -\frac{\partial B_z}{\partial z}$$

$$\frac{\partial}{\partial r}(r B_r) = -r\frac{\partial B_z}{\partial z}$$

- Integrate w.r.t "r".

$$\int \frac{\partial}{\partial r}(r B_r) = \int -r\frac{\partial B_z}{\partial z}$$

$$r B_r = -\int r \left.\frac{\partial B_z}{\partial z}\right|_{r=0} dr$$

↳ Take point where $r=0$ in figure

$$r B_r = -\frac{\partial B_z}{\partial z}\int r\,dr.$$

$$'' = -\frac{\partial B}{\partial z}\Big|_{r=0}\ \frac{r^2}{2}\Big|_0^r$$

$$\boxed{B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}\Big|_{r=0}}$$

Magnetic Mirror.

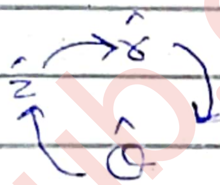
here magnetic mirror r is directly varry while in Tokmak $1/r$.

$$\vec{F} = q(\vec{V}\times\vec{B})$$

Taking its components

Here $B_\theta = 0$
$\vec{B} \to \hat{z}\ \vec{\nabla} B \sim z$
$B_r,\ B_z$

$$\vec{V}\times\vec{B} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ V_r & V_\theta & V_z \\ B_r & \cancel{B_\theta}^{\,0} & B_z \end{vmatrix}$$

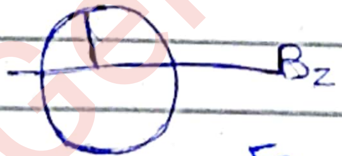


$$F_r = q(V_\theta B_z - V_z \cancel{B_\theta}^{\,0}) = q V_\theta B_z \quad (1)$$

$$F_\theta = -q(V_r B_z - V_z B_r) = -q(\overset{(2)}{V_r B_z} - \overset{(3)}{V_z B_r})$$

$$F_z = q(V_r \cancel{B_\theta}^{\,0} - V_\theta B_r) = -q V_\theta B_r \quad (4)$$

Term (1) and (2) gives rise to the larmar gyration term, the third term



$V_r \perp B$
$V_\theta \perp B$

(edges par force nahi lag raha drift nahi ha bic edges [illegible] fa[illegible] ha while inside [illegible] 3d)

(3) $\overset{F_\theta}{\boxed{+q V_z B_r}}$ vanishes on the axis.

For determination of drift (4) term is of our interest

4 term

$$\boxed{F_z = -q V_\theta B_r} \rightarrow ②$$

$$F_z = +q V_\theta \, r/2 \left.\frac{\partial B_z}{\partial z}\right|_{r=0}$$

$V_\theta = V_\perp$

clockwise then -ve sign.

anticlockwise rotation then +ve sign.

$V_\theta = \mp V_\perp$

$$F_z = \mp q \frac{V_\perp}{2} r \left.\frac{\partial B_z}{\partial z}\right|_{r=0}$$

$$r_L = \frac{V_\perp}{W_c} = \frac{V_\perp}{\pm qB} m$$

$$V_\perp = \pm \frac{qBV_\perp}{m}$$

$$F_z = \frac{\mp q V_\perp^2 m}{2(\pm qB)} \left.\frac{\partial B_z}{\partial z}\right|_{r=0}$$

$$F_z = -\frac{1}{2} \frac{mV_\perp^2}{B} \left.\frac{\partial B_z}{\partial z}\right|_{r=0}$$

Average

$$\boxed{\bar{F}_z = -\frac{1}{2} \frac{mV_\perp^2}{B} \frac{\partial B_z}{\partial z}} \rightarrow eq\ 3$$

Magnetic Moment:-

$\mu = IA$

$$\mu = \frac{q}{t} \pi r_L^2$$

$t = T$

For one cycle

$t = T \quad r = r_L$

$$\mu = \frac{q}{T} \pi r_L^2$$

$$T = 2\pi/\omega$$

$$\mu = \frac{q\omega \pi r_L^2}{2\pi}$$

$$\mu = \frac{q\omega_c v_\perp^2}{2\omega_c^2}$$

$$\mu = \frac{1}{2}\frac{q v_\perp^2}{\omega_c}$$

$$\mu = \frac{q v_\perp^2 m}{2 \pm q B}$$

For ion:-

$$\boxed{\mu \equiv \frac{1}{2}\frac{m v_\perp^2}{B}} \quad (4)$$

→ magnetic moment in plasma.

For electron:-

$$\mu = \frac{m v_\perp^2}{b} \qquad \mu = \frac{q v_\perp^2}{2 q B}$$

Using eq (4) in (3)

$$\vec{F}_z = -\mu \frac{\partial B_z}{\partial z}$$

$$\vec{F}_{\parallel} = -\mu \nabla_{\parallel} B$$

$$m\frac{d\vec{v}_{\parallel}}{dt} = -\mu \vec{\nabla}_{\parallel} B$$

$$m\frac{dv_z}{dt} = -\mu \frac{\partial B_z}{\partial z}$$

$$\frac{dv_z}{dt} = -\mu/m \frac{\partial B_z}{\partial z}$$

$$\frac{dv_{\parallel}}{dt} = -\frac{\mu}{m}\frac{\partial B}{\partial s}$$

$s \rightarrow$ is directed along B.

$V_{\parallel}$ ~ $\frac{\partial B}{\partial s}$ ~ expression.

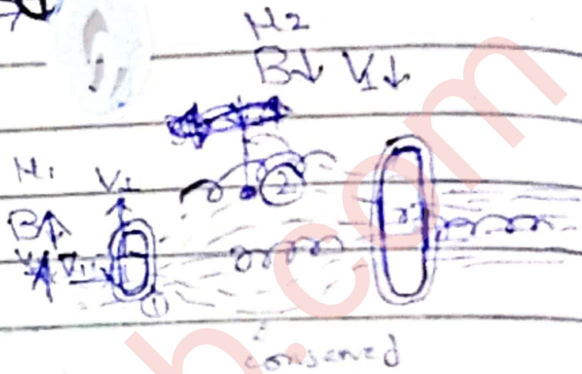
1. Laws:-

Invariance of H

$$\left(\tfrac{1}{2}mv_{\parallel}^2 + \tfrac{1}{2}mv_{\perp}^2\right) = \tfrac{1}{2}mv_{\parallel}^2 + \tfrac{1}{2}mv_{\perp}^2$$

$$\frac{d}{dt}\left(\tfrac{1}{2}mv_{\parallel}^2 + \tfrac{1}{2}mv_{\perp}^2\right) = 0 \rightarrow ①'$$

② Law of conservation of energy.



$$H = \frac{\tfrac{1}{2}mv_{\perp}^2}{B} \rightarrow ②'$$

H is invariant quantity

B↓ H↑ (increased) V⊥↓

B↑ H↓ V⊥↑

$$BH = \tfrac{1}{2}mv_{\perp}^2$$

example
$x \propto yz$
$x \propto y/z$ ↑

In order to conserve the energy when $V_{\perp}$ ↓ $V_{\parallel}$ ↑ vice versa $V_{\perp}$ ↑ $V_{\parallel}$ ↓.

Mathematical proof plasma particles are confined by using $②'$

$$HB = \tfrac{1}{2}mv_{\perp}^2 \rightarrow ③'$$

Using $③'$ in $①'$

$$\frac{d}{dt}\left(\tfrac{1}{2}mv_{\parallel}^2 + HB\right) = 0$$

$$\frac{2mv_{\parallel}}{2}\frac{dv_{\parallel}}{dt} + H\frac{dB}{dt} + B\frac{dH}{dt} = 0$$

$$mv_{\parallel}\frac{dv_{\parallel}}{dt} + H\frac{dB}{dt} + B\frac{dH}{dt} = 0$$

→ a particle having velocity zero (when it parallel) then bounce back inside neither going outside

($V_{\perp}$↓ $V_{\parallel}$↑)

Plasma particles are confined.

$$m v_{\parallel} \frac{dv_{\parallel}}{dt} = -\mu v_{\parallel} \frac{dB}{ds}.$$

$$v = \frac{ds}{dt}$$

$$m v_{\parallel} \frac{dv_{\parallel}}{dt} = -\mu \frac{\cancel{ds}}{dt} \frac{dB}{\cancel{ds}} = -\mu\, dB/dt$$

$$-\mu \frac{\cancel{dB}}{dt} + \mu \frac{\cancel{dB}}{dt} + B \frac{d\mu}{dt} = 0$$

$$B\, d\mu/dt = 0.$$

$$B \neq 0$$

So $d\mu/dt = 0$

μ = invariant or constt.
(Mathematical proved).

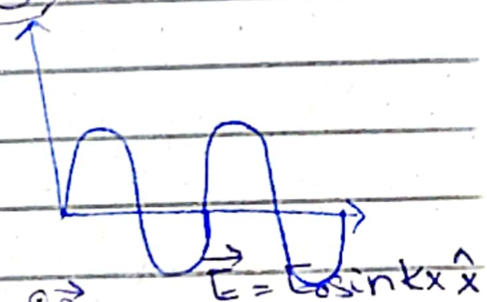
Non-Uniform E-field:-

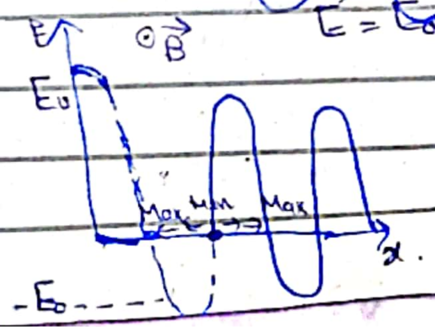
Non-Uniform B-feild.

Let's assume that E-feild is directed along x-direction.

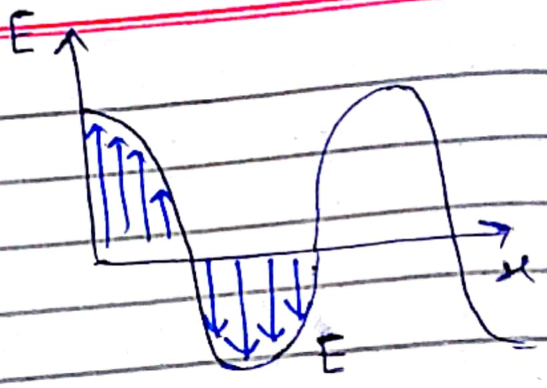
$$\boxed{\vec{E} = E_0 \cos kx\, \hat{x}} \rightarrow (1)$$

Shown in figure 2
while for sin figure 1.

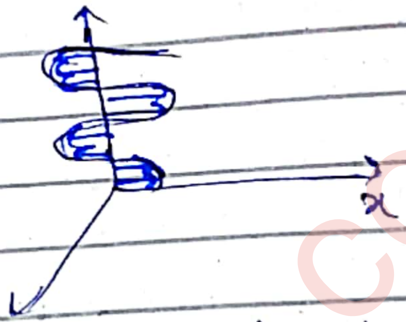




→ $\vec{E} = E_0 \cos kx \,\hat{y}$

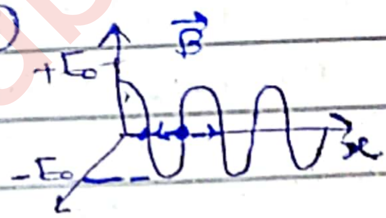


→ $\vec{E} = E_0 \cos ky \,\hat{x}$



B is uniform and directed along $\hat{z}$.

$$m\frac{dv}{dt} = q\left[E(x) + \vec{V} \times \vec{B}\right] \rightarrow (2)$$



$$m\dot{V}_x = q\left[E(x) + V_y B\right] \rightarrow (3)$$

$$m\dot{V}_y = q\left[-V_x B\right] \rightarrow (4)$$

$$\vec{V} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ V_x & V_y & V_z \\ 0 & 0 & B \end{vmatrix}$$

$$= \hat{x}(V_y B - 0) - \hat{y}(V_x B) + \hat{z}($$

$$m\dot{V}_z = 0$$

$$\dot{V}_z = 0$$

$$V_z = \text{const.}$$

(3), (4) Coupled eq so first decouple eq's we get

$$\ddot{V}_x = q/m \left[E\dot{x} + B\dot{V}_y\right]$$

$$\ddot{V}_y = -\frac{q B \dot{V}_x}{m}$$

$$\ddot{V}_x = \frac{q}{m}\left[\dot{E}(x) + B\left(-\frac{q V_x B}{m}\right)\right]$$

$$\ddot{V}_x = \frac{q}{m}\dot{E}(x) - \frac{q^2B^2}{m^2}V_x$$

$$\boxed{\ddot{V}_x = \frac{q}{m}\dot{E}(x) - W_c^2 V_x} \rightarrow (5)$$

$$\ddot{V}_y = -\frac{qB}{m}\left[\frac{q}{m}\left(E(x)+V_yB\right)\right]$$

$$\ddot{V}_y = -\frac{q^2B}{m^2}E(x) - W_c^2 V_y.$$

$\times$ and $\div$ by B.

$$\ddot{V}_y = -\frac{q^2B^2}{m^2 B}E(x) - \omega_c^2 V_y.$$

$$\ddot{V}_y = -\frac{W_c^2}{B}E(x) - \omega_c^2 V_y.$$

* Multi and divide eq (5) with B

$$\ddot{V}_x = \frac{q}{m}\frac{B}{B}\dot{E}(x) - W_c^2 V_x.$$

$$\ddot{V}_x = \pm W_c \frac{\dot{E}(x)}{B} - W_c^2 V_x \rightarrow (6)$$

$\omega_c = \pm \frac{qB}{m}$

$\pm \omega_c = qB/m,$

$$\ddot{V}_y = -W_c^2 V_y - \frac{\omega_c^2}{B}E(x) \rightarrow (7)$$

if here we have fn of y then write eq according to y.

$$x = x_0 + x_L \sin \omega_c t \rightarrow (8)$$

{undisturbed orbit}.

Using eq (8) in (7) we get

$$\ddot{V}_y = -W_c^2 V_y - \frac{W_c^2}{B}E_0 \cos kx$$

$$E(x) = E_0 \cos kx.$$

$$\boxed{\ddot{V}_y = -W_c^2 V_y - \frac{W_c^2}{B}E_0 \cos k\left[x_0 + x_L \sin \omega_c t\right]} \rightarrow (9)$$

$$\cos k[x_0 + r_L \sin\omega_c t] = \cos(kx_0)\cos(k r_L \sin\omega_c t) - \sin(kx_0)\sin(k r_L \sin\omega_c t)$$

$$k r_L <<< 1$$

$$\cos\theta = 1 - \frac{\theta^2}{2} + \dots\dots$$

$$\sin\theta = \theta + \dots\dots$$

$$\cos(k r_L \sin\omega_c t) = 1 - \frac{k^2 r_L^2 \sin^2\omega_c t}{2}$$ neglect other values

$$\sin(k r_L \sin\omega_c t) = k r_L \sin\omega_c t$$

$$\underline{\text{average}}\ \cos k[x_0 + r_L \sin\omega_c t] = \cos kx_0\left[1 - \underline{\frac{k^2 r_L^2 \sin^2\omega_c t}{2}}_{\text{average}}\right] - \sin(kx_0)[k r_L \sin\omega_c t].$$

$\underline{\text{average}}$

$$" = \cos kx_0\left[1 - \frac{k^2 r_L^2}{4}\right] - \cancel{\sin kx_0 (k r_L 0)} \to 0$$

no average so skip

$$\overline{\ddot{V}_y} = -\omega_c^2 \overline{V_y} - \frac{\omega_c^2 E_0 \cos kx_0}{B}\left[1 - \frac{k^2 r_L^2}{4}\right] \to (10)$$

$\sin^2\theta \sim 1/2$

$\sin\theta = 0$

$$\boxed{\overline{\ddot{V}_y} = 0} \to \text{average of } V_y$$

$$\ddot{V}_y = \frac{d^2 V_y}{dt^2} = \frac{d a_y}{dt}$$

$$0 \cancel{\ddot{V}_y} = -\omega_c^2 V_y - \omega_c^2/_B\, E_0 \cos kx_0\left[1 - \frac{k^2 r_L^2}{4}\right]$$

$$V_y = -\frac{\cancel{\omega_c^2} E_0}{\cancel{\omega_c^2} B} \cos kx_0\left[1 - \frac{k^2 r_L^2}{4}\right]$$

$$V_y = -\frac{E_0 \cos kx_0}{B}\left[1 - \frac{k^2 r_L^2}{4}\right]$$

$\overline{V_y} = V_y.$

$$V_y = -\frac{E(x_0)}{B}\left[1 - \frac{k^2 r_L^2}{4}\right] \to (11)$$

Drift always be taken for average value

$\vec{E} \times \vec{B} = E(x)\hat{x} \times B(\hat{z})$

$'' \quad = -E(x)\,\hat{y}$

$\vec{V}_y = -\frac{E(x_0)B}{B^2}\left[1 - \frac{k^2 r_L^2}{4}\right]\hat{y}$,

$\vec{V}_y = \frac{\vec{E} \times \vec{B}}{B^2}\left[1 - \frac{k^2 r_L^2}{4}\right]$

$\vec{V}_E = \frac{\vec{E} \times \vec{B}}{B^2}\left[1 - \frac{k^2 r_L^2}{4}\right]$ $\qquad \therefore \vec{\nabla} = i\vec{k}$

$\nabla \cdot \nabla = i^2 k^2$

$\nabla^2 = -k^2$

$\boxed{\vec{V}_E = \frac{\vec{E} \times \vec{B}}{B^2}\left[1 + \frac{\nabla^2 r_L^2}{4}\right]}$ → Electric feild variation by this term.

$\vec{V}_E = \frac{\vec{E} \times \vec{B}}{B^2}$ if $\nabla = 0$

$\vec{\nabla} \Rightarrow ik$

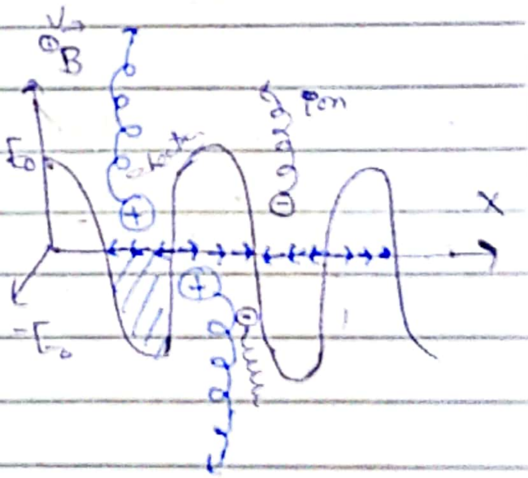
$\vec{E}(x) = E_0 \cos kx\,\hat{x}$

$\frac{\partial E}{\partial x} = -kE_0 \sin kx$

$\partial/\partial x = -k$

$\frac{\partial E}{\partial x} = -kE(x)$.

$\partial E/\partial x = -kE(x)$.



Non-Uniform feilds:-

Time varying E feild　　　　Time varying [B] feild

$\vec{E} = \underbrace{E_0 e^{i\omega t}}_{\text{time varying}} \hat{x}$

ω = freq with which E-feild is varying

+time varying that is increasing with time B is uniform and directed along z-axis:

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ V_x & V_y & V_z \\ 0 & 0 & B \end{vmatrix}$$

$$m\dot{V}_x = q(\vec{E} + V_y B) \rightarrow (2)$$

$$m\dot{V}_y = q(-V_x B) \rightarrow (3)$$

$$\cancel{m\ddot{V}_x = q(\dot{E} - qV_x B^2/m)}$$

$$m\ddot{V}_x = q(\dot{E} + \dot{V}_y B) \rightarrow (4)$$

from (3) $\dot{V}_y = -qV_x B/m \rightarrow (5)$

using (5) in (4)

$$m\ddot{V}_x = q\left(\dot{E} - \frac{qB^2 V_x}{m}\right) \rightarrow (6)$$

$$\ddot{V}_x = \frac{q\dot{E}}{m} - \frac{q^2 B^2}{m^2} V_x$$

$$\omega_c^2 = q^2B^2/m^2$$

$$\ddot{V}_x = q\dot{E}/m - \omega_c^2 V_x \rightarrow (7)$$

$$E = E_0 e^{i\omega t}$$

$$\dot{E} = i\omega E_0 e^{i\omega t} = i\omega E$$

$$\dot{E} = i\omega E \rightarrow (8)$$

Using ⑧ in ⑦

$\ddot{V}_x = q/m \, i\omega E - \omega_c^2 V_x$

$\ddot{V}_x = -\omega_c^2 \left(V_x - \frac{q i \omega E}{m \omega_c \omega_c} \right)$

$\omega_c = \pm qB/m$

$\ddot{V}_x = -\omega_c^2 \left(V_x \mp \frac{q i \omega E}{m q B/m} \right)$

$\ddot{V}_x = -\omega_c^2 \left(V_x \mp \frac{\cancel{q} i \omega E \cancel{m}}{\cancel{m} \cancel{q} B} \right)$

$\ddot{V}_x = -\omega_c^2 \left(V_x \mp \frac{i\omega \tilde{E}}{\omega_c B} \right) \rightarrow ⑨$

→ direction — average ~ times amplitude varies
↳ tilda

(time varying E feild)

$\ddot{V}_x = -\omega_c^2 \left(V_x \mp \frac{i\omega}{\omega_c} \frac{\tilde{E}_x}{B} \right)$

Let us define

$\tilde{V}_p = \pm \frac{i\omega}{\omega_c} \frac{\tilde{E}_x}{B}$ → polarization drift which aries b/c time E field varying

$\ddot{V}_x = -\omega_c^2 (\underbrace{V_x - \tilde{V}_p}_{\tilde{V}_x})$ → ⑩

$m\ddot{V}_y = -q\dot{V}_x B$

$m\ddot{V}_y = -qB\left[\frac{q}{m}(E + V_y B)\right]$.

$'' = -\frac{q^2 B E}{m} - \frac{q^2 B^2 V_y}{m}$

$\ddot{V}_y = \frac{-q^2 B^2 E}{m^2 B} - \frac{q^2 B^2 V_y}{m^2}$

$$\ddot{V}_y = -\omega_c^2 E/B - \omega_c^2 V_y$$

$$\ddot{V}_y = -\omega_c^2 (V_y + E/B)$$

$$\ddot{V}_y = -\omega_c^2 (V_y + \tilde{E}/B) \rightarrow (11)$$

$$\vec{V}_E = \vec{E} \times \vec{B} / B^2$$

time varying $\Rightarrow \tilde{V}_E = \tilde{E} \times \vec{B}/B^2$

$$\vec{V}_E = \frac{\tilde{E} B \hat{x} \times \hat{z}}{B^2} = \frac{-\tilde{E}\hat{y}}{B^2}$$

magnitude: $\tilde{V}_E = -\tilde{E}/B$

$\Rightarrow (11)$ $\ddot{V}_y = -\omega_c^2 (V_y - \tilde{V}_E) \rightarrow (12)$

→ $\ddot{V}_x$ ~~gives~~ ~~The~~ time varying electric drift. electric feild drift but this time varying E feild drift

$\ddot{V}_y$ not gives time derivation B of E is in eq (11) so, it is giving time varying electric drift

$$V_x = V_\perp e^{i\omega_c t} + V_P$$

$$V_y = \pm i V_\perp e^{i\omega_c t} + V_E$$

solution of eq (10)

solution of eq (12)

if these two equations are satisfy then we can have solution of (10) and (12)

Now we double derivative

$\dot{V}_x = i\omega_c V_\perp e^{i\omega_c t} + \tilde{V}_p$

$\ddot{V}_x = (i\omega_c)(i\omega_c) V_\perp e^{i\omega_c t} + \ddot{V}_p \Big\} \rightarrow (13)$

Insecting (13) in (10)

$-\omega_c^2 V_\perp e^{i\omega_c t} + \ddot{V}_p = \omega_c^2 (V_\perp e^{i\omega_c t} - \tilde{V}_p) + \tilde{V}_p$

$\ddot{V}_p = 0$ ~~which is not satisfied.~~

$\ddot{V}_x = -\omega_c^2 V_\perp e^{i\omega_c t} + \ddot{V}_p$

$$\tilde{V}_p = \pm \frac{i\omega}{\omega_c} \frac{\tilde{E}}{B}.$$

$$\dot{\tilde{V}}_p = \pm i\omega_c / \omega_c \; \dot{\tilde{E}}/B$$

$$\ddot{\tilde{V}}_p = \pm i\omega_c / \omega_c \; \ddot{E}/B$$

$$\tilde{E} = E_0 e^{i\omega_c t}$$

$$\dot{\tilde{E}} = i\omega E_0 e^{i\omega t}$$

$$\ddot{\tilde{E}} = i^2\omega^2 E_0 e^{i\omega t}$$

$$\text{"} = -\omega^2 E_0 e^{i\omega t}$$

$$\ddot{\tilde{V}}_p = \pm i\omega_c / \omega_c \left(\frac{-\omega^2 E_0 e^{i\omega t}}{B} \right)$$

$$\text{"} = \pm i\omega_c / \omega_c \left(E_0 e^{i\omega t} / B \right)$$

and $\omega \rightarrow$ low magnetic feild and we values are high so this approches to zero

$\omega_c = \pm qB/m$ $\quad B\uparrow\uparrow \quad \omega_c \uparrow\uparrow \quad \omega\downarrow$

$\boxed{\omega \ll \omega_c}$ $\quad B\uparrow\uparrow$

$\ddot{\tilde{V}}_p \Rightarrow 0$

If this is the solution of eq so we can apply approximation.

eq (10) $\Rightarrow \ddot{V}_x = -\omega_c^2 V_x + \omega_c^2 \tilde{V}_p - \omega_c^2 \tilde{V}_p$

$\dot{\tilde{V}}_p = \pm \dfrac{i\omega\, i\omega \tilde{E}}{\omega_c B}$

$'' = \pm i\omega i\omega i\omega \tilde{E}/\omega_c B$

$\dddot{\tilde{V}}_p = + i^2 \omega^2 \tilde{V}_p$

$'' = -\omega^2 \tilde{V}_p$

eq (10)' $= V_x = V_\perp e^{i\omega_c t} + \tilde{V}_p$

$V_x = i\omega_c V_\perp e^{i\omega_c t} + \dot{\tilde{V}}_p$

$\dot{V}_x = i\omega_c^2 V_\perp e^{i\omega_c t} + \ddot{\tilde{V}}_p$

~~Now eq (10)'~~

$-V_\perp \omega_c^2 e^{i\omega_c t} + \ddot{\tilde{V}}_p = -\omega_c^2 V_\perp e^{i\omega_c t} + \tilde{V}_p + \omega_c^2 \tilde{V}_p - \omega^2 \tilde{V}_p$

$-V_\perp \omega_c^2 \cancel{e^{i\omega_c t}} + \cancel{\ddot{\tilde{V}}_p} = -\omega_c^2 V_\perp \cancel{e^{i\omega_c t}} + \cancel{\tilde{V}_p} + \omega_c^2 \tilde{V}_p - \omega^2 \tilde{V}_p - \omega^2 \tilde{V}_p$

$0 = 0$

Satisfied

So sol:-

$V_x = V_\perp e^{i\omega_c t} + \tilde{V}_p \longrightarrow$ solution

$\ddot{V}_x = -\omega_c^2 V_x + (\omega_c^2 - \omega^2)\tilde{V}_p$

$\boxed{\omega_c >> \omega}$ or $\boxed{\omega << \omega_c}$

highly magnefied plasma.

eq (10) and (12) are similar

$$V_y = \pm i V_d e^{i\omega_c t} + \tilde{E}$$

$$\ddot{V}_y = -\omega_c^2 V_y + (\omega_c^2 - \omega^2)\tilde{F}_E$$

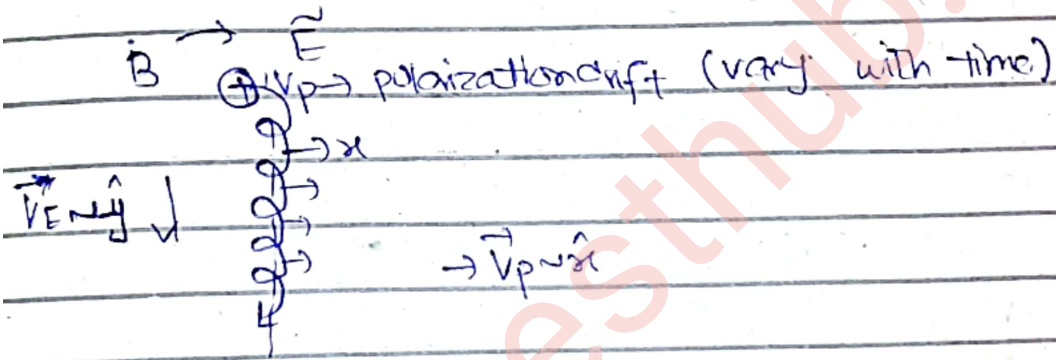
{Polarization drift}

$$V_p = \pm \frac{i\omega}{\omega_c} \tilde{E}/B$$

$$E = E_0 e^{i\omega t}$$

$$d/dt \rightarrow i\omega \qquad \dot{E} = i\omega E$$

$$\boxed{V_p = \pm \frac{1}{\omega_c B} \, dE/dt}$$ General expression for the polarization



We calculate the current density and This current density happens when electron and ion are in opposite behaviour

$$J = (en_i V_{pi} - en_e V_{pe})$$

lecture 15

Ch # 2

Time Varying B. feild:-

if B feild is varying with time then we know that varying B. feild with time produces an E-feild

$$\partial\vec{B}/\partial t = -\vec{\nabla} \times \vec{E}.$$

So presence of E-feild will produce drift in the system by shifting the guiding centre.

drift ~~causes due~~ to shift in guiding ~~centre~~ centre and guiding ~~centre~~ happens due to workdo...

$$\vec{\nabla} \times \vec{E} = -\vec{B}$$

$$m \frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) \rightarrow (1)$$

Taking dot product of eq (1) with $\vec{v}$.

$$m \cdot \vec{v} \cdot \frac{d\vec{v}}{dt} = q \vec{v} \cdot \vec{E} + q \, \cancel{\vec{v} \cdot \vec{v} \times \vec{B}}^{\,0}$$

$$m \frac{d}{dt}(\vec{v} \cdot \vec{v}) = q \vec{v} \cdot \vec{E}$$

$$\vec{v} \cdot \vec{v} = v_\perp^2$$ (here is component teclar)

$$m \frac{d}{dt} v_\perp^2 = q \vec{v}_\perp \cdot \vec{E}$$

$$v_\perp = d\vec{L}/dt$$

$$m \frac{d}{dt} v_\perp^2 = q \frac{d\vec{L}}{dt} \cdot \vec{E}$$

$$d/dt \left(\tfrac{1}{2} m v_\perp^2\right) = q \vec{E} \, dl/dt \rightarrow (2)$$

$$d/dt \left(\tfrac{1}{2} m v_\perp^2\right) = \tfrac{1}{\cancel{2}} m \cancel{2} m v_\perp dv_\perp/dt$$

The change in one gyration is obtain over one period by integrating eq (2) with dt

$$\tfrac{1}{2} m v_\perp^2 = q \vec{E} \cdot \frac{d\vec{l}}{dt}$$

$$\int d/dt \; \tfrac{1}{2} m v_\perp^2 \, dt = \int q \vec{E} \cdot \frac{d\vec{L}}{dt} \cancel{dt}$$

$$\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = \int q\,\vec{E}\,d\vec{l} \rightarrow (3)$$

line integral

Stokes theorem

$$\oint \vec{E}\,d\vec{l} = \int_s (\vec{\nabla}\times\vec{E}).ds$$

using stokes theorem in eq (3)

$$\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = q\int \vec{\nabla}\times\vec{E}\cdot ds$$

surface integral

$s \rightarrow$ surface enclosed by Larmour radius.

$$\vec{\nabla}\times\vec{E} = -\dot{\vec{B}}$$

$$\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = -q\int_s \dot{\vec{B}}.ds$$

$s \rightarrow$ is the surface enclosed by Larmor radius.

Change in energy results when we have time varying B feild.

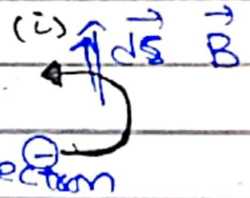
$$\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = -q\int \dot{\vec{B}}.ds$$

$$= -q\int \dot{B}\,ds\cos\theta$$

$\cos\theta = 1$

$\rightarrow \theta = 0 \longrightarrow$ then $ds = \pi r_L^2$

$$\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = -q\,\dot{\vec{B}}\int ds.$$

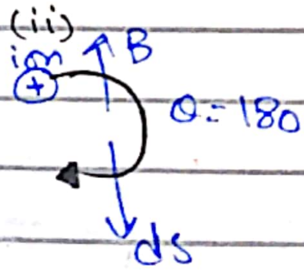


$$\boxed{\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = -q\,\dot{B}\pi r_L^2}$$

when

$\rightarrow \theta = 180^\circ$

$\cos\theta = -1$



$$\boxed{\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = +q\,\dot{B}\pi r_L^2.}$$

$$\boxed{\delta\left(\tfrac{1}{2}mv_{\perp}^2\right) = \pm q\,\dot{B}\pi r_L^2} \rightarrow (4)$$

$r_L = \frac{V_\perp}{\omega_c} = \frac{V_\perp m}{\pm qB}$

Using (5) in (4)

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \pm q\dot{B}\,\pi\,\frac{V_\perp^2}{\omega_c\,\omega_c}$$

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \pm q\dot{B}\,\pi\,\frac{V_\perp^2 m}{\pm qB\,\omega_c}$$

X and ÷ by 2

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \frac{2\,\tfrac{1}{2}mV_\perp^2\,\pi\dot{B}}{B\,\omega_c}$$

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \frac{\tfrac{1}{2}mV_\perp^2}{B}\,\frac{2\pi\dot{B}}{\omega_c} \rightarrow (6)$$

$$\mu = \tfrac{1}{2}mV_\perp^2/B \rightarrow (7)$$

using (7) in (6)

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \mu\,\frac{2\pi\dot{B}}{\omega_c}$$

$\dot{B} = \partial B/\partial t = f_c B$

$\partial/\partial t = f_c$

$\dot{B} = \partial B/\partial t = f_c\,\delta B$

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \mu\,\frac{2\pi f_c B}{\omega_c}$$

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \mu\,\frac{\boxed{2\pi f_c}\,\delta B}{\omega_c}$$

$$\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \mu\,\frac{\cancel{\omega_c}\,\delta B}{\cancel{\omega_c}}$$

$$\boxed{\delta\left(\tfrac{1}{2}mV_\perp^2\right) = \mu\,\delta B}$$

$\delta(\frac{1}{2}mv_\perp^2) = \mu\,\delta B$. time varying B. feild so in time varying B feild μ is constent

as : $\frac{1}{2}mv_\perp^2 = \mu B$.

$$\delta(\mu B) = \mu\,\delta B$$

$$\cancel{\mu\,\delta B} + B\,\delta\mu = \cancel{\mu\,\delta B}$$

$$B\,\delta\mu = 0$$

as $B \neq 0$ $\quad \delta\mu = 0$

μ is constant or μ is invariant also in the case of time varying B. feild

$$\Phi = \vec{B}\cdot\vec{A}$$

$$\Phi = BA\cos\theta$$

$$\cos\theta = 1$$

$$\Phi = BA$$

$$\Phi = B\pi r_L^2 = \frac{B\pi v_\perp^2}{\omega_c^2}$$

$$\Phi = \frac{2B\pi\, v_\perp^2\, m\; m}{2\, q^2 B}$$

$$\boxed{\Phi = \frac{2\pi m\mu}{q^2}}$$

→ Suppose if 5 lines passing Through big circle Then 5 can also be seen passing small circle.

* Through Lamor orbit magnetic flux is const. b/c μ is const or invariant.