Date:

SSP-II

Assignment no 1

"Exam Preparation Questions"

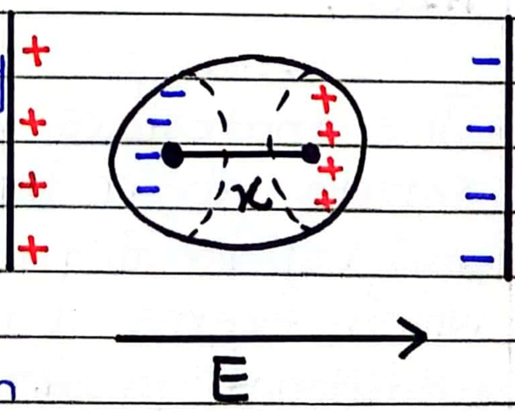
Q1) What is meant by polarization mechanisms in dielectrics? Discuss different types of mechanisms and explain their temperature dependence.

ANS In dielectrics, polarization mechanisms refer to the ways in which the material responds to an external electric field by rearranging it's dipoles.

The four types of polarization mechanisms are as follows.

(i) **Electronic/Atomic Polarization** occurs in materials with bound electrons, such as insulators.

The displacement of +ively charged nucleus and -ively charged electrons of an atom in opposite direction on application of Electric Field is known as Electronic polarization.



→ It is independent of temperature.

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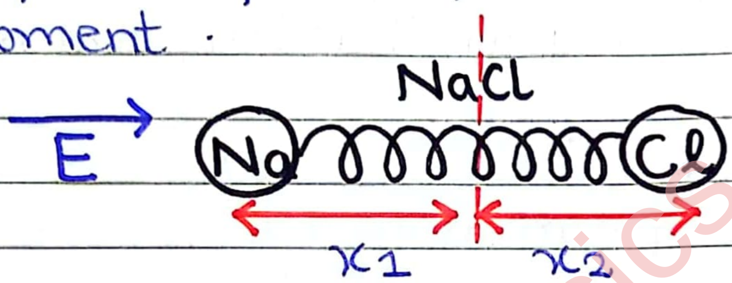
(ii) **Ionic Polarization:-** occurs in ionic solids such as NaCl, KBr, LiBr.

When a field is applied to a molecule, +ive and −ive atoms are displaced in opposite direction until ionic binding force stops the process, thus increasing the dipole moment.



→ **Temperature dependence:-**
Ionic polarization tends to decrease with increasing temperature due to enhanced thermal motion leading to disordered arrangement of ions.

(iii) **Dipolar / Orientational Polarization:-** occurs in materials having dipole moment like H_2O.

The materials with permanent dipoles rotate about their axis of symmetry and try to allign with the applied field which exerts a torque on them. This additional polarization effect is known as dipolar polarization.

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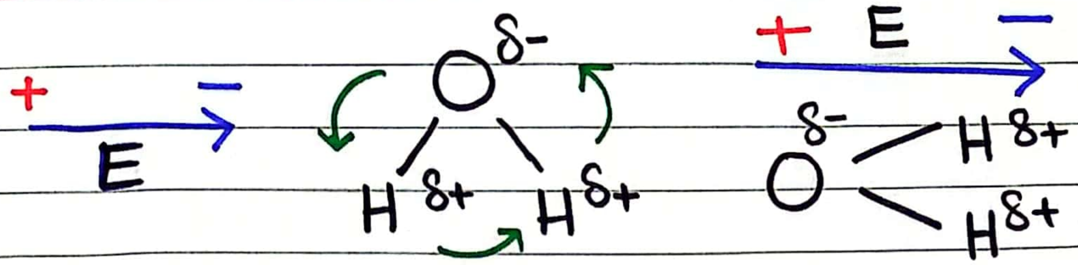
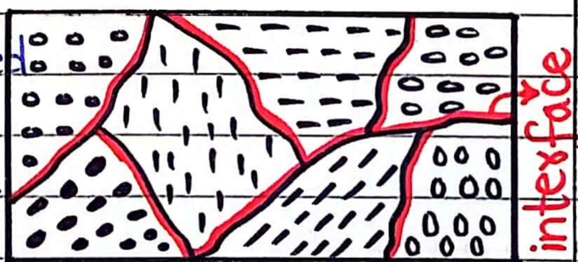


→ **Temperature dependence:-** of dipolar polarization varies depending on nature of the molecules. Generally, it decreases with the increase of temperature.

(iv) **Space Charge Polarization:-** occurs in materials with mobile charge carriers, such as semi-conductors.

→ It occurs due to accumulation of charges at the electrodes or at the interface in multi-phase solids.

In the presence of applied field. Mobile positive and negative ions migrate towards the +ive and -ive electrode to an appropriate distance giving rise to redistribution of charges but they remain in the dielectric material.



Multiphase Solid

→ It occurs when the rate of charge accumulation is different from the rate of charge removal. It is not significant in dielectrics.

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→ **Temperature dependence**:- of space charge polarization can be complex as it is influenced by factors such as charge carriers mobility.

Q2) Explain polarization in dielectrics. Arrive at a relation between the dielectric constant and atomic polarizability.

ANS **Polarization** is a process of producing electric dipoles inside the dielectric by application of an externally applied E.Field.

$P \propto E$ ∴ More E.F, More Polarization

$P = \alpha E$ ∴ α = polarizability

(i) → $\boxed{P = N\alpha E}$ ∴ for N no. of dipoles

Polarizability is defined as polarization per unit applied field.

$$\alpha = \frac{P}{E}$$

→ Electric flux density is given by,

$$D = \varepsilon_0 E + P \qquad \because \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$\varepsilon E = \varepsilon_0 E + P$$

$$P = \varepsilon E - \varepsilon_0 E$$

$$P = E(\varepsilon - \varepsilon_0)$$

$$= E(\varepsilon_0 \varepsilon_r - \varepsilon_0)$$

$$\boxed{P = \varepsilon_0 E(\varepsilon_r - 1)} \rightarrow \text{(ii)}$$

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→ From (i) and (ii),

$$N \alpha \cancel{E} = \varepsilon_0 \cancel{E}(\varepsilon_r - 1)$$

$$\boxed{\alpha = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}} \rightarrow \text{(iii)}$$

→ Eq (iii) gives us relation between 'α' polarizability and 'ε_r' dielectric constant

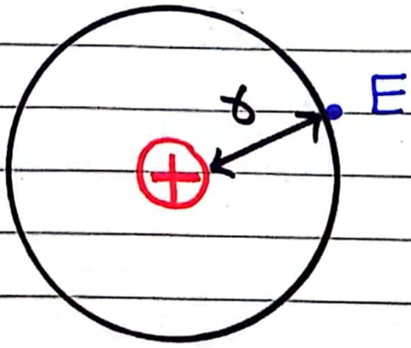
Q3) Get the relation between the flux density '**D**' and intensity of electric field '**E**'.

Electric Flux Density / Displacement "D" is defined as number of electric field lines passing through unit area.

$$D = \frac{q}{\text{Area}}$$

∴ no. of field lines are numerically equal to q charge.

$$D = \frac{q}{4\pi r^2}$$



Also,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{D}{\varepsilon_0}$$

$$\boxed{D = \varepsilon_0 E}$$ (For free space)

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→ **More Medium:-**

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

∴ ε = permitivity of medium.
∴ ε_0 = permitivity of free space.

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

∴ ε_r = relative permitivity

→ Electric Field is given by,

$$E = \frac{q}{A\varepsilon_0\varepsilon_r} = \left(\frac{q}{4\pi r^2}\right)\frac{1}{\varepsilon_0\varepsilon_r} = \frac{D}{\varepsilon_0\varepsilon_r}$$

$$\boxed{D = \varepsilon_0\varepsilon_r E}$$ For Medium

Q4) Show that $P = E\varepsilon_0(\varepsilon_r - 1)$ where P is the electric polarization.

→ Electric Displacement 'D' for dielectric medium is given by,

$$D = \varepsilon_0\varepsilon_r E \quad \rightarrow (i)$$

→ In terms of polarization, it is,

$$D = \frac{\varepsilon_0 q}{A\varepsilon_0\varepsilon_r} + P$$

$$D = \varepsilon_0 E + P \quad \rightarrow (ii)$$

⇒ From (i) and (ii),

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$$\varepsilon_0 \varepsilon_r E = \varepsilon_0 E + P$$

$$\varepsilon_0 \varepsilon_r E - \varepsilon_0 E = P$$

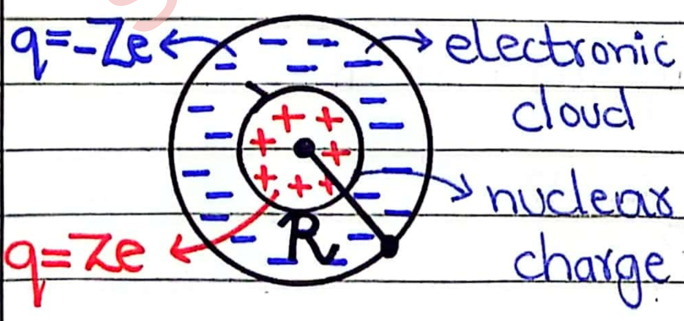
$$P = \varepsilon_0 E(\varepsilon_r - 1)$$ Hence Proved.

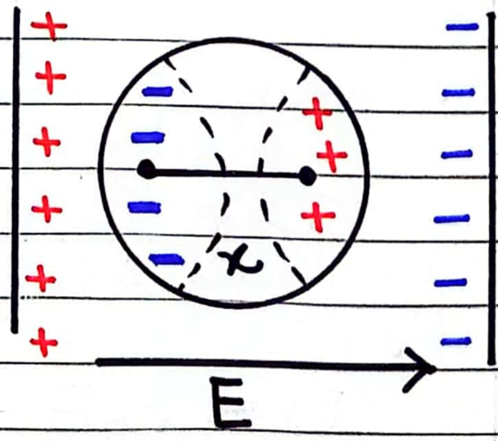
Q5) Explain electronic polarization and orientational polarization in dielectrics.

(i) Electronic Polarization (α_e) occurs in materials with bound electrons, such as insulators.

→ The displacement of positively charged nucleus and negatively charged electrons of an atom in opposite direction on application of Electric Field is known as electronic polarization.

→ On application of E.F, electron cloud around the nucleus shifts towards the positive end of the field.





→ Two types of forces acts here

(a) Lorentz force.

(b) Coulombic force.

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(a) Lorentz Force acts to move the atom away from the centre.

$$L.F = qE$$

$$\boxed{L.F = (Ze)E}$$

$\therefore$ Charge density $(\rho) = \frac{q}{V} = \frac{Ze}{\frac{4\pi R^3}{3}}$

(b) Coulomb's Force pulls the atom towards the centre.

$$C.F = \frac{Kq_1 q_2}{r^2}$$

$\therefore q_1 =$ charge of nucleus

$\therefore q_2 =$ charge enclosed by sphere of radius x.

$$q_2 = \rho V = \left(\frac{Ze}{\frac{4\cancel{\pi}R^3}{\cancel{3}}}\right)\left(\frac{4\cancel{\pi}x^3}{\cancel{3}}\right)$$

$$\boxed{q_2 = \frac{Zex^3}{R^3}}$$

$$C.F = \frac{(Ze)}{x^2}\left(\frac{Zex^3}{R^3}\right)K$$

$$\boxed{C.F = \frac{Z^2e^2x}{4\pi\varepsilon_0 R^3}}$$

→ Both forces cancels out each other and equillibrium is maintained

$$L.F = C.F$$

$$\cancel{Z}\cancel{e}E = \frac{Z^{\cancel{2}}e^{\cancel{2}}x}{4\pi\varepsilon_0 R^3}$$

$$\boxed{\frac{4\pi\varepsilon_0 R^3 E}{Ze} = x}$$

'x' is the displacement through which the charges moved by applying Electric Field.

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Dipole Moment (μ) would be formed within the atom by application of external Electric Field.

$$\mu = qx$$

$$\mu = (\cancel{Ze})\left(\frac{4\pi\varepsilon_0 R^3 E}{\cancel{Ze}}\right)$$

$$\mu = (4\pi\varepsilon_0 R^3)E$$

$$\mu_e = \alpha_e E$$

$\therefore \alpha_e$ = polarizability

$\therefore \alpha_e = 4\pi\varepsilon_0 R^3$

For N dipoles:- $P = N\mu_e$

$$\boxed{P = N\alpha_e E}$$

Also,

$$P = \varepsilon_0 E(\varepsilon_r - 1)$$

$$N\alpha_e \cancel{E} = \varepsilon_0 \cancel{E}(\varepsilon_r - 1)$$

$$\boxed{\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}}$$

This is electronic polarizabity expression in terms of relative permitivity.

From the expression we can see that there is no temperature dependence.

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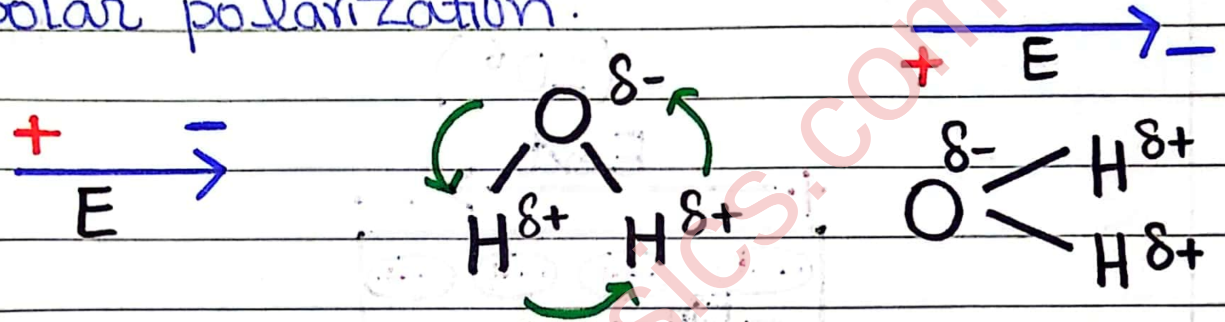
(ii) **Orientational Polarization** (α_d) occurs in materials having dipole moment like H_2O.

→ The materials with permanent dipoles rotate about their axis of symmetry and try to allign with the applied field which exerts a torque in them. This additional polarizational effect is known as orientational or dipolar polarization.



→ It only occurs in polar molecules. With Electronic and Ionic polarization, externally applied field is balanced by elastic binding forces but for orientational polarization, no such force exists.

→ Dipolar polarization's expression is given by,

$$P_d = \frac{N\mu_d^2 E}{3KT}$$

Also,

$$P = N\alpha_d E$$

$$\cancel{N}\alpha_d \cancel{E} = \frac{\cancel{N}\mu_d^2 \cancel{E}}{3KT}$$

$$\boxed{\alpha_d = \frac{\mu^2 d}{3KT}}$$

∴ α_d is temp dependent

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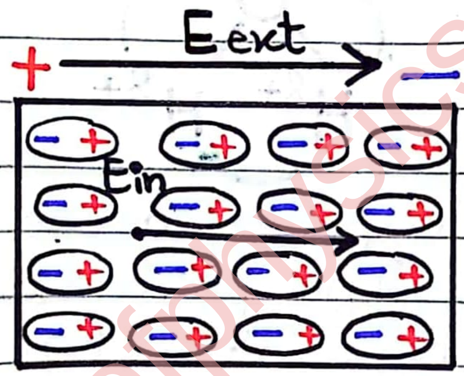
Q6) Explain local field in a solid dielectric. Obtain Clausis-Mosotti formula relating microscopic polarizabilities (α_e) and macroscopic dielectric constant (ε)

ANS **Local Field:-** is a microscopic field (10^{-6}) acting on an atom from inside the material.

It is different from the externally applied macroscopic field (10^{6}).



When an external electric field is applied to a dielectric, it induces polarization in the material, alligning the dipoles in same direction to the field.

The local Field 'E_{loc}' refers to the effective electric field experienced by indivisual polarizable entities (atoms).

$$E_{loc} = E_0 + E_1 + E_2 + E_3 .$$

- E_0 — Polarizing field
- E_1 — Depolarizing field
- E_2 — Lorentz Field
- E_3 — Field inside lorentz cavity

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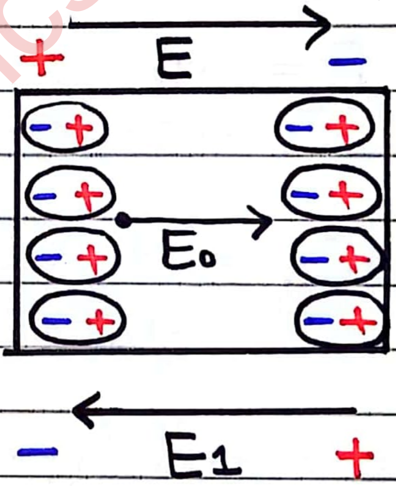
(i) **Polarizing Field (E_o)** is that portion of externally applied field which is used to polarize the atoms.

If we have a dielectric, the displacement vector would be,

$$D = \varepsilon_o E + P$$

If the dielectric consists of single atom only, then ($E = E_o$). All the external field would work to polarize the atom.

$$D = \varepsilon_o E_o$$



Now,

$$D = \varepsilon_o E + P$$

$$\varepsilon_o E_o = \varepsilon_o E + P$$

(1) → $\boxed{E_o = E + \frac{P}{\varepsilon_o}}$

where,

∴ E = applied field

∴ P/ε_o = field generated due to polarization

(ii) **Depolarizing Field (E_1)** is a field opposite to that of externally applied field.

$\boxed{E_1 = -\frac{P}{\varepsilon_o}}$ → (2)

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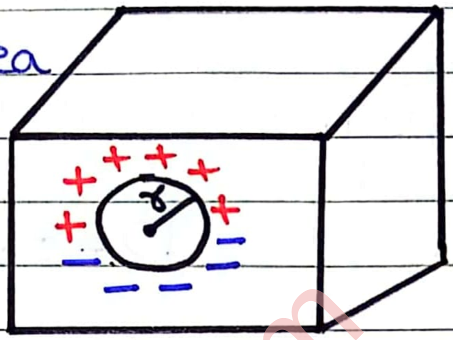
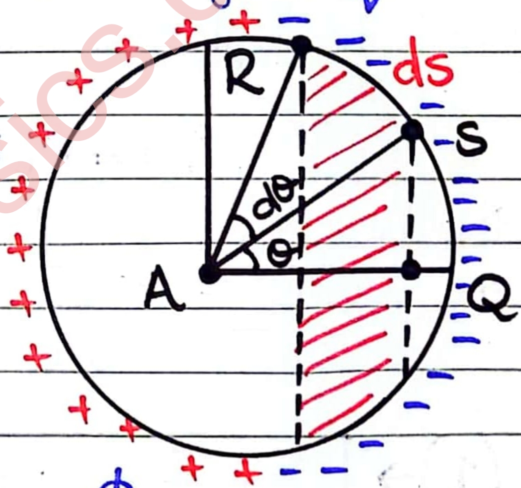
(iii) **Lorentz Field (E_2)** is the field due to the charges on surface of the cavity (spherical)

Let us consider small area ds on the surface of sphere between θ and $\theta + d\theta$ as shown in fig



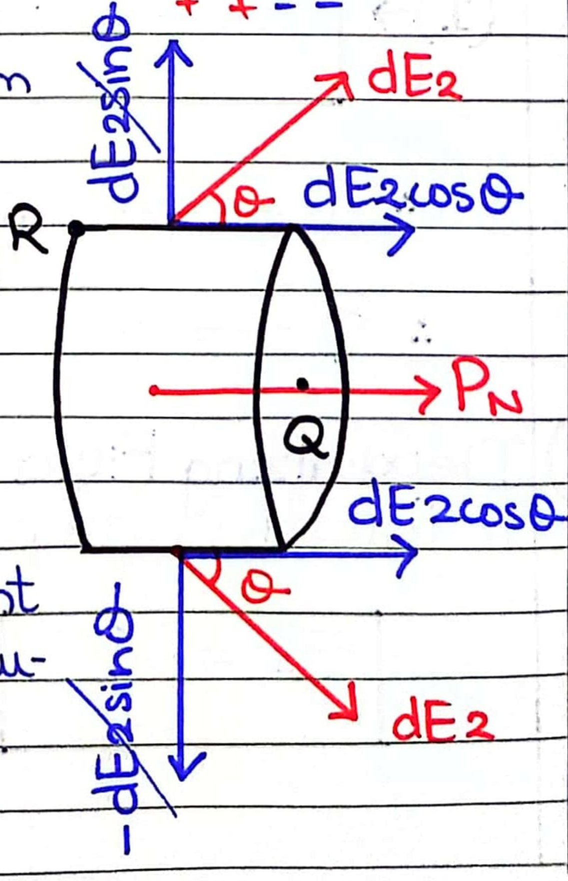
Let dq be the charge on ds. The electric field intensity at A due to charge 'dq' is given by,



(1) $\rightarrow$ $E_2 = \dfrac{q}{4\pi\varepsilon_0 r^2}$

(2) $\rightarrow$ $dE_2 = \dfrac{dq \cos\theta}{4\pi\varepsilon_0 r^2}$

The second figure is zoom in version of spherical cavity, and we are only concerned with a small portion 'ds' of that cavity shown by the third figure.



Let P_N be the component of polarization perpendicular to ds,

$P_N = P\cos\theta$

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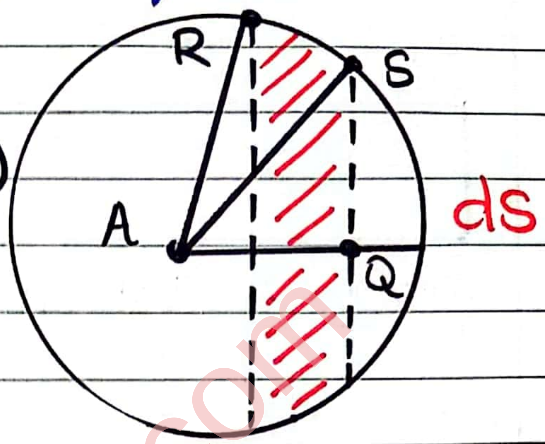
Polarization is basically charge upon area,

$$P_N = P\cos\theta = \frac{dq}{ds} \quad , \quad dq = P\cos\theta \, ds \rightarrow (3)$$

→ Put (3) in (2),

$$dE_2 = \frac{P\cos^2\theta \, ds}{4\pi\varepsilon_0 r^2} \rightarrow (4)$$

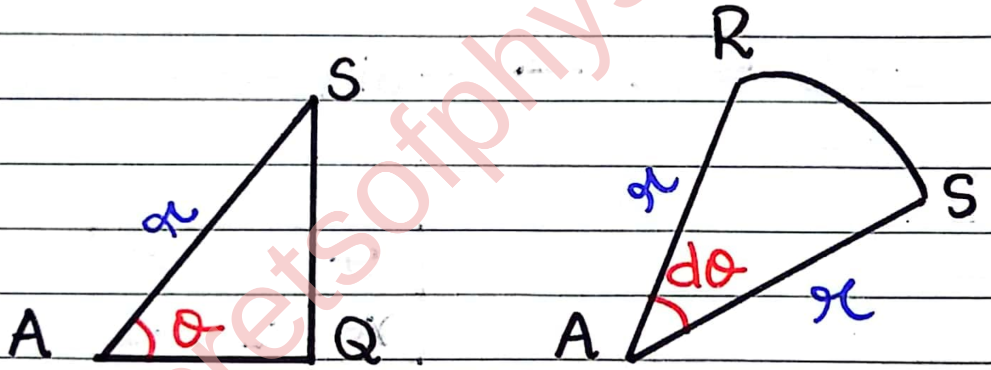


→ To find ds,

Area = Circumference of ring x thickness

$$= [(2\pi) \times \text{radius}] \times [\text{arc length}]$$

$$= 2\pi(QS) \times (RS)$$



$$\sin\theta = \frac{Perp}{Hyp}$$

$$\text{Arc} = \text{Angle} \times \text{radius}$$

$$RS = d\theta \times r$$

$$\sin\theta = \frac{QS}{AS}$$

$$QS = r\sin\theta$$

So,

$$\text{Area}(ds) = 2\pi r\sin\theta \, (r\,d\theta)$$

$$ds = 2\pi r^2 \sin\theta \, d\theta \rightarrow (5)$$

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→ Put (5) in (4),

$$dE_2 = \frac{P\cos^2\theta \, (\cancel{2}\cancel{\pi}\cancel{r^2}\sin\theta \, d\theta)}{\cancel{4}\cancel{\pi}\varepsilon_0 \cancel{r^2}}$$

$$\boxed{dE_2 = \frac{P\cos^2\theta \sin\theta \, d\theta}{2\varepsilon_0}}$$

This is electric field due to small patch ds. To find for the whole cavity, we must integrate it.

$$\int dE_2 = \frac{P}{2\varepsilon_0}\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta$$

Put $\cos\theta = x \rightarrow -\sin\theta \, d\theta = dx$

$$E_2 = \frac{P}{2\varepsilon_0}\int_1^{-1} x^2(-dx)$$

θ	0	π
x	1	-1

$$E_2 = \frac{-P}{2\varepsilon_0}\left(\frac{x^3}{3}\Big|_1^{-1}\right)$$

$$E_2 = \frac{-P}{6\varepsilon_0}\left[(-1)^3 - (1)^3\right]$$

$$E_2 = \frac{-P}{6\varepsilon_0}(-1-1)$$

$$\boxed{E_2 = \frac{P}{3\varepsilon_0}} \rightarrow (3)$$

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(iv) E_3 is the field due to charges inside cavity

$\boxed{E_3 = 0}$ (for cubical structures)

$E_3 \neq 0$ (for non-cubical structures)

⇒ The dielectric slab is a cubical structure and spherical (imaginary) cavity is a sphere. Both are symmetrical surfaces. That's why the electric fields of the dipoles cancels out each other giving a net of zero.

Total Local Field :-

$$E_{Loc} = E_0 + E_1 + E_2 + E_3$$

$$E_{Loc} = \left(E + \cancel{\frac{P}{\varepsilon_0}}\right) + \left(-\cancel{\frac{P}{\varepsilon_0}}\right) + \left(\frac{P}{3\varepsilon_0}\right) + 0$$

$$\boxed{E_{Loc} = E + \frac{P}{3\varepsilon_0}} \rightarrow (4)$$

"CLAUSIS MOSOTTI RELATION"

This relation relates macroscopic dielectric constant with micoroscopic polarizability.

Derivation:- Starting with the polarization formula,

$$\vec{P} = N\alpha E$$

$$P = N(\alpha_0 + \alpha_i + \alpha_e) E_{loc}$$

→ For this derivation, we are considering elements with electronic polarizabilities only ignoring ionic and orientational ones.

$$\alpha_i = \alpha_o = 0 \quad ; \quad \alpha = \alpha_e$$

$$\vec{P} = N\alpha_e \vec{E}_{loc} \qquad (\text{put } 4)$$

$$\vec{P} = N\alpha_e \left(\vec{E} + \frac{\vec{P}}{3\varepsilon_0} \right)$$

$$\vec{P} = N\alpha_e \vec{E} + N\alpha_e \frac{\vec{P}}{3\varepsilon_0}$$

$$\vec{P} - \frac{\vec{P} N\alpha_e}{3\varepsilon_0} = N\alpha_e \vec{E}$$

$$\boxed{\vec{P}\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right) = N\alpha_e \vec{E}} \rightarrow (5)$$

→ Displacement vector:-

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$$

$$\vec{P} = \varepsilon_0 \varepsilon_r \vec{E} - \varepsilon_0 \vec{E}$$

$$\boxed{\vec{P} = \varepsilon_0 (\varepsilon_r - 1) \vec{E}} \rightarrow (6)$$

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→ Put (6) in (5),

$$\varepsilon_0(\varepsilon_r-1)\cancel{\vec{E}} = \frac{N\alpha_e\cancel{\vec{E}}}{\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}$$

$$\boxed{\varepsilon_0(\varepsilon_r-1) = \frac{N\alpha_e}{\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}} \rightarrow (7)$$

→ Adding $3\varepsilon_0$ on both sides,

$$\varepsilon_0\varepsilon_r - \varepsilon_0 + 3\varepsilon_0 = \frac{N\alpha_e}{1-\frac{N\alpha_e}{3\varepsilon_0}} + 3\varepsilon_0$$

$$\varepsilon_0\varepsilon_r + 2\varepsilon_0 = \frac{N\alpha_e + 3\varepsilon_0\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}{1-\frac{N\alpha_e}{3\varepsilon_0}}$$

$$\varepsilon_0\varepsilon_r + 2\varepsilon_0 = \frac{\cancel{N\alpha_e} + 3\varepsilon_0 - \cancel{N\alpha_e}}{1-\frac{N\alpha_e}{3\varepsilon_0}}$$

$$\boxed{\varepsilon_0(\varepsilon_r+2) = \frac{3\varepsilon_0}{1-\frac{N\alpha_e}{3\varepsilon_0}}} \rightarrow (8)$$

→ Dividing eq (7) by (8),

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$$\frac{\cancel{\varepsilon_0}(\varepsilon_r-1)}{\cancel{\varepsilon_0}(\varepsilon_r+2)} = \frac{N\alpha_e}{\cancel{\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}} \div \frac{3\varepsilon_0}{\cancel{\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}}$$

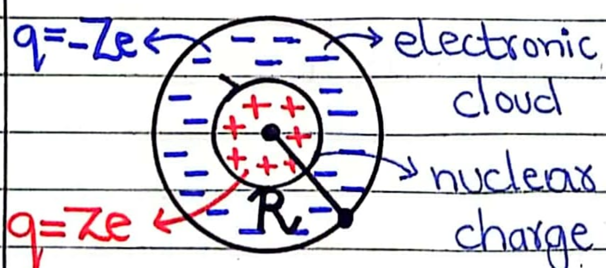
$$\boxed{\frac{(\varepsilon_r-1)}{(\varepsilon_r+2)} = \frac{N\alpha_e}{3\varepsilon_0}}$$ Clausis Mosotti eq

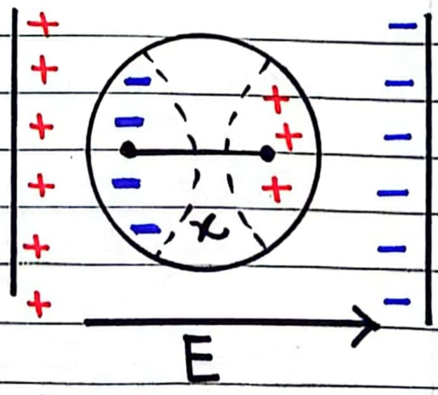
Q7) Explain electronic polarization in atoms and obtain expression for electronic polarizability in terms of radius of atom.

(i) **Electronic Polarization (α_e)** occurs in materials with bound electrons, such as insulators.

→ The displacement of positively charged nucleus and negatively charged electrons of an atom in opposite direction on application of Electric Field is known as electronic polarization.

→ On application of E.F, electron cloud around the nucleus shifts towards the positive end of the field.





→ Two types of forces acts here
(a) Lorentz force.
(b) Coulombic force.

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(a) Lorentz Force acts to move the atom away from the centre.

$$L.F = qE$$

$$\boxed{L.F = (Ze)E}$$

$\therefore$ Charge density (ρ) $= \frac{q}{V} = \frac{Ze}{\frac{4\pi R^3}{3}}$

(b) Coulomb's Force pulls the atom towards the centre.

$$C.F = \frac{Kq_1q_2}{r^2}$$

$\therefore q_1 =$ charge of nucleus

$\therefore q_2 =$ charge enclosed by sphere of radius x.

$$C.F = \frac{(Ze)}{x^2}\left(\frac{Zex^3}{R^3}\right)K$$

$$q_2 = \rho V$$

$$= \left(\frac{Ze}{\frac{4\cancel{\pi}R^3}{\cancel{3}}}\right)\left(\frac{4\cancel{\pi}x^3}{\cancel{3}}\right)$$

$$\boxed{C.F = \frac{Z^2e^2x}{4\pi\varepsilon_0 R^3}}$$

$$\boxed{q_2 = \frac{Zex^3}{R^3}}$$

→ Both forces cancels out each other and equillibrium is maintained

$$L.F = C.F$$

$$\cancel{Ze}E = \frac{Z^{\cancel{2}}e^{\cancel{2}}x}{4\pi\varepsilon_0 R^3}$$

$$\boxed{\frac{4\pi\varepsilon_0 R^3 E}{Ze} = x}$$

'x' is the displacement through which the charges moved by applying Electric Field.

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Dipole Moment (μ) would be formed within the atom by application of external Electric Field.

$$\mu = qx$$

$$\mu = (\cancel{Z}e)\left(\frac{4\pi\varepsilon_0 R^3 E}{\cancel{Ze}}\right)$$

$$\mu = (4\pi\varepsilon_0 R^3)E$$

$$\mu_e = \alpha_e E$$

$\therefore \alpha_e$ = polarizability

$\therefore \alpha_e = 4\pi\varepsilon_0 R^3$

Q8) Show that dielectric constant of polar molecule varies inversely as first power of temp.

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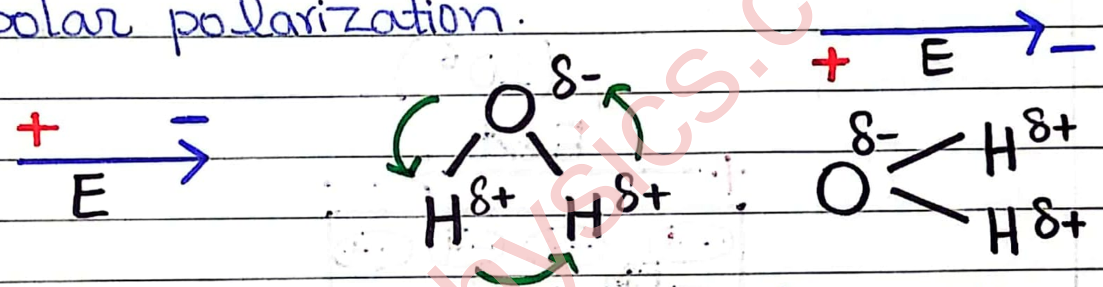
(ii) **Orientational Polarization** (α_d) occurs in materials having dipole moment like H_2O.

→ The materials with permanent dipoles rotate about their axis of symmetry and try to allign with the applied field which exerts a torque in them. This additional polarizational effect is known as orientational or dipolar polarization.



→ It only occurs in polar molecules. With Electronic and Ionic polarization, externally applied field is balanced by elastic binding forces but for orientational polarization, no such force exists.

→ Dipolar polarization's expression is given by,

$$P_d = \frac{N\mu_d^2 E}{3KT}$$

Also,

$$P = N\alpha_d E$$

$$\cancel{N}\alpha_d \cancel{E} = \frac{\cancel{N}\mu_d^2 \cancel{E}}{3KT}$$

$$\boxed{\alpha_d = \frac{\mu_d^2}{3KT}}$$

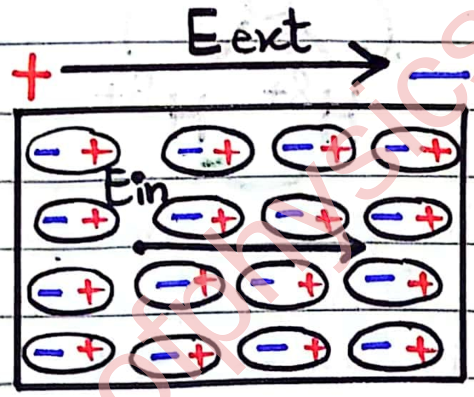
∴ α_d is temp dependent

Hence Proved.

Q8 (b) What is meant by local field in dielectric and how it is calculated for cubical structures.

ANS **Local Field:-** is a microscopic field (10^{-6}) acting on an atom from inside the material.

It is different from the externally applied macroscopic field (10^{6}).



When an external electric field is applied to a dielectric, it induces polarization in the material, alligning the dipoles in same direction to the field.

The local Field 'E_{loc}' refers to the effective electric field experienced by indivisual polarizable entities (atoms).

$$E_{loc} = E_0 + E_1 + E_2 + E_3$$

- E_0 → Polarizing field
- E_1 → Depolarizing field
- E_2 → Lorentz Field
- E_3 → Field inside lorentz cavity

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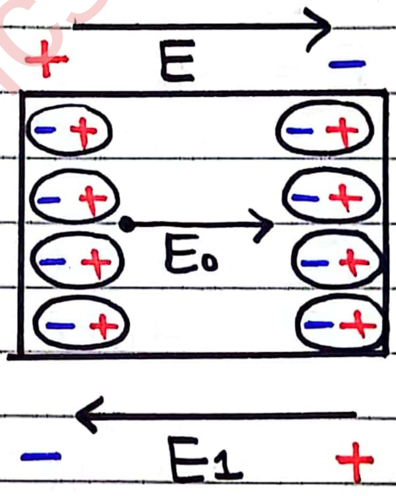
(i) **Polarizing Field (E_o)** is that portion of externally applied field which is used to polarize the atoms.

If we have a dielectric, the displacement vector would be,

$$D = \varepsilon_o E + P$$

If the dielectric consists of single atom only, then $(E = E_o)$. All the external field would work to polarize the atom.

$$D = \varepsilon_o E_o$$



Now,

$$D = \varepsilon_o E + P$$

$$\varepsilon_o E_o = \varepsilon_o E + P$$

(1) → $$\boxed{E_o = E + \frac{P}{\varepsilon_o}}$$

where,

∴ E = applied field

∴ P/ε_o = field generated due to polarization

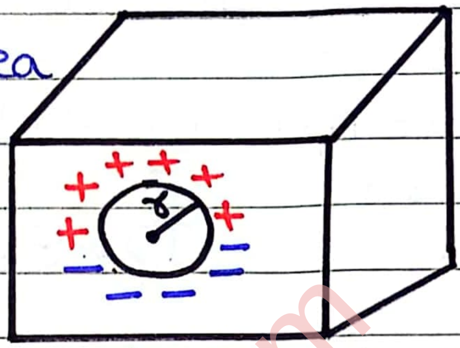
(ii) **Depolarizing Field (E_1)** is a field opposite to that of externally applied field.

$$\boxed{E_1 = -\frac{P}{\varepsilon_o}} \rightarrow (2)$$

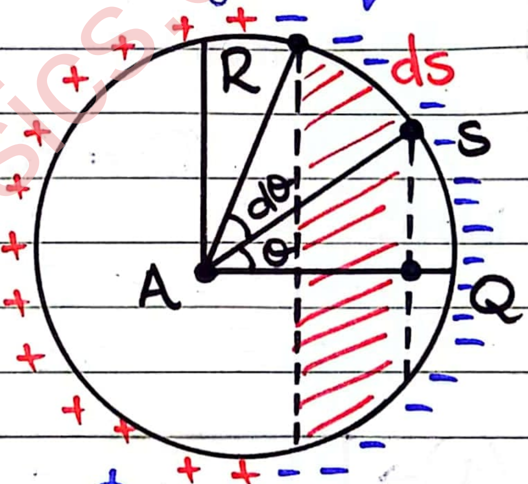
Date:

(iii) **Lorentz Field (E_2)** is the field due to the charges on surface of the cavity (spherical)

Let us consider small area ds on the surface of sphere between θ and $\theta + d\theta$ as shown in fig



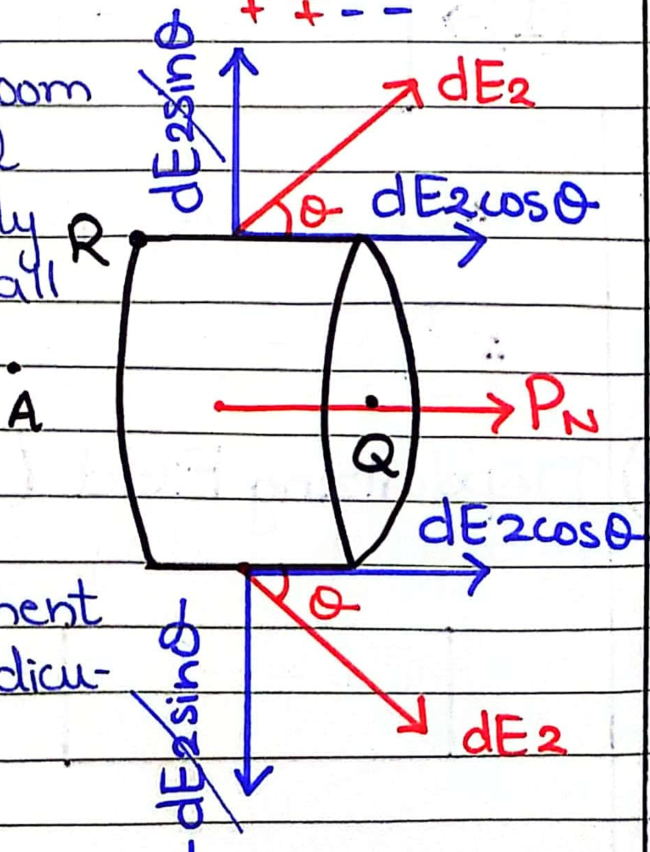
Let dq be the charge on ds. The electric field intensity at A due to charge 'dq' is given by,



(1) → $$E_2 = \frac{q}{4\pi\varepsilon_0 r^2}$$

(2) → $$dE_2 = \frac{dq \cos\theta}{4\pi\varepsilon_0 r^2}$$

The second figure is zoom in version of spherical cavity, and we are only concerned with a small portion 'ds' of that cavity shown by the third figure.



Let P_N be the component of polarization perpendicular to ds,

$$P_N = P\cos\theta$$

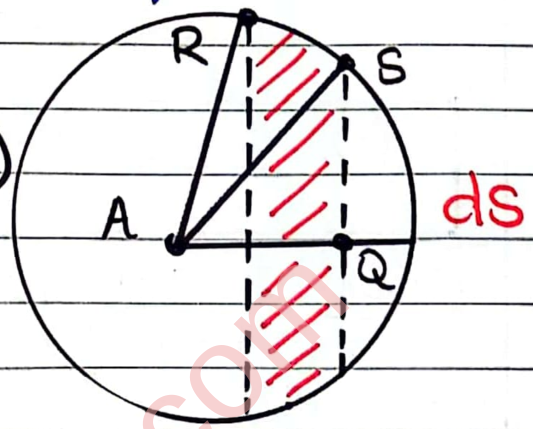
Date:

Polarization is basically charge upon area,

$$P_N = P\cos\theta = \frac{dq}{ds} \quad , \quad dq = P\cos\theta \, ds \rightarrow (3)$$

→ Put (3) in (2),

$$dE_2 = \frac{P\cos^2\theta \, ds}{4\pi\varepsilon_0 r^2} \rightarrow (4)$$

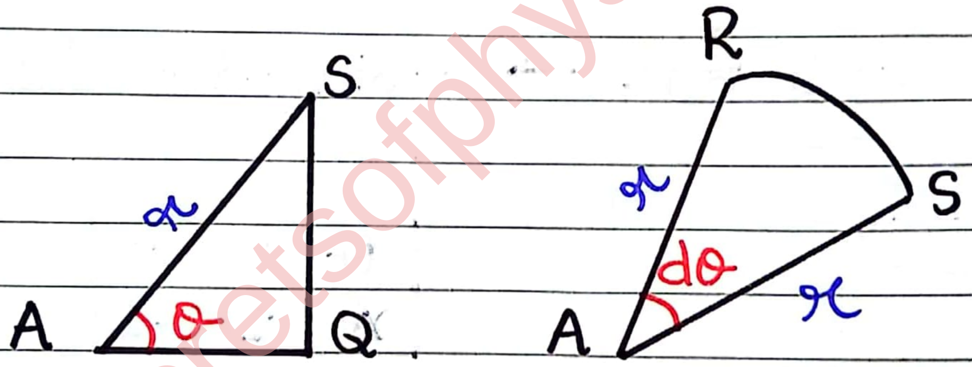


→ To find ds,

Area = Circumference of ring x thickness

$$= [(2\pi) \times \text{radius}] \times [\text{arc length}]$$

$$= 2\pi(QS) \times (RS)$$



$$\sin\theta = \frac{\text{Perp}}{\text{Hyp}}$$

Arc = Angle x radius

$$RS = d\theta \times r$$

$$\sin\theta = \frac{QS}{AS}$$

$$QS = r\sin\theta$$

So,

$$\text{Area } (ds) = 2\pi r\sin\theta \, (r\, d\theta)$$

$$ds = 2\pi r^2 \sin\theta \, d\theta \rightarrow (5)$$

Date:

→ Put (5) in (4),

$$dE_2 = \frac{P\cos^2\theta \, (2\cancel{\pi} \cancel{r^2} \sin\theta \, d\theta)}{4\cancel{\pi}\varepsilon_0 \cancel{r^2}}$$

$$\boxed{dE_2 = \frac{P\cos^2\theta \sin\theta \, d\theta}{2\varepsilon_0}}$$

This is electric field due to small patch ds. To find for the whole cavity, we must integrate it.

$$\int dE_2 = \frac{P}{2\varepsilon_0} \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta$$

Put $\cos\theta = x \rightarrow -\sin\theta \, d\theta = dx$

$$E_2 = \frac{P}{2\varepsilon_0} \int_1^{-1} x^2 (-dx)$$

θ	0	π
x	1	-1

$$E_2 = \frac{-P}{2\varepsilon_0} \left(\frac{x^3}{3} \Big|_1^{-1} \right)$$

$$E_2 = \frac{-P}{6\varepsilon_0} \left[(-1)^3 - (1)^3 \right]$$

$$E_2 = \frac{-P}{6\varepsilon_0} (-1-1)$$

$$\boxed{E_2 = \frac{P}{3\varepsilon_0}} \rightarrow (3)$$

Date:

(iv) E_3 is the field due to charges inside cavity

$\boxed{E_3 = 0}$ (for cubical structures)

$E_3 \neq 0$ (for non-cubical structures)

⇒ The dielectric slab is a cubical structure and spherical (imaginary) cavity is a sphere. Both are symmetrical surfaces. That's why the electric fields of the dipoles cancels out each other giving a net of zero.

Total Local Field :-

$$E_{Loc} = E_0 + E_1 + E_2 + E_3$$

$$E_{Loc} = \left(E + \cancel{\frac{P}{\varepsilon_0}}\right) + \left(\cancel{-\frac{P}{\varepsilon_0}}\right) + \left(\frac{P}{3\varepsilon_0}\right) + 0$$

$$\boxed{E_{Loc} = E + \frac{P}{3\varepsilon_0}} \rightarrow (4)$$

Q9) How Clausis Mosotti relation is used in predicting the dielectric constant of solids.

"CLAUSIS MOSOTTI RELATION"

Date:

→ For this derivation, we are considering elements with electronic polarizabilities only ignoring ionic and orientational ones,

$$\alpha_i = \alpha_o = 0 \quad ; \quad \alpha = \alpha_e$$

$$\vec{P} = N\alpha_e \vec{E}_{loc} \qquad \text{(put 4)}$$

$$\vec{P} = N\alpha_e\left(\vec{E} + \frac{\vec{P}}{3\varepsilon_0}\right)$$

$$\vec{P} = N\alpha_e\vec{E} + N\alpha_e\frac{\vec{P}}{3\varepsilon_0}$$

$$\vec{P} - \frac{\vec{P}N\alpha_e}{3\varepsilon_0} = N\alpha_e\vec{E}$$

$$\boxed{\vec{P}\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right) = N\alpha_e\vec{E}} \rightarrow (5)$$

→ Displacement vector:-

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

$$\vec{P} = \vec{D} - \varepsilon_0\vec{E}$$

$$\vec{P} = \varepsilon_0\varepsilon_r\vec{E} - \varepsilon_0\vec{E}$$

$$\boxed{\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}} \rightarrow (6)$$

Date:

→ Put (6) in (5),

$$\varepsilon_0(\varepsilon_r-1)\cancel{\vec{E}} = \frac{N\alpha_e \cancel{\vec{E}}}{\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}$$

$$\boxed{\varepsilon_0(\varepsilon_r-1) = \frac{N\alpha_e}{\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}} \rightarrow (7)$$

→ Adding $3\varepsilon_0$ on both sides,

$$\varepsilon_0\varepsilon_r - \varepsilon_0 + 3\varepsilon_0 = \frac{N\alpha_e}{1-\frac{N\alpha_e}{3\varepsilon_0}} + 3\varepsilon_0$$

$$\varepsilon_0\varepsilon_r + 2\varepsilon_0 = \frac{N\alpha_e + 3\varepsilon_0\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)}{1-\frac{N\alpha_e}{3\varepsilon_0}}$$

$$\varepsilon_0\varepsilon_r + 2\varepsilon_0 = \frac{\cancel{N\alpha_e} + 3\varepsilon_0 - \cancel{N\alpha_e}}{1-\frac{N\alpha_e}{3\varepsilon_0}}$$

$$\boxed{\varepsilon_0(\varepsilon_r+2) = \frac{3\varepsilon_0}{1-\frac{N\alpha_e}{3\varepsilon_0}}} \rightarrow (8)$$

→ Dividing eq (7) by (8),

Date. ____________

$$\frac{\cancel{\varepsilon_0}(\varepsilon_r - 1)}{\cancel{\varepsilon_0}(\varepsilon_r + 2)} = \frac{N\alpha_e}{\left(1 - \cancel{\frac{N\alpha_e}{3\varepsilon_0}}\right)} \div \frac{3\varepsilon_0}{\left(1 - \cancel{\frac{N\alpha_e}{3\varepsilon_0}}\right)}$$

$$\boxed{\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}}$$ Clausis Mosotti eq

Q10) Define "Polarization Density". Obtain an expression for Lorentz Field in dielectric material.

Polarization Density :- denoted by P, is the measure of density of electric dipole moments in a material (dielectric).

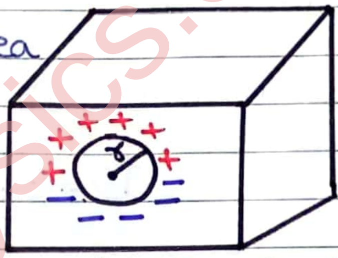
It represents the electric dipole moments per unit volume.

$$P = \frac{\mu}{V}$$

∴ μ = dipole moment
∴ V = volume

(iii) **Lorentz Field** (E_2) is the field due to the charges on surface of the cavity (spherical)

Let us consider small area ds on the surface of sphere between θ and $\theta + d\theta$ as shown in fig



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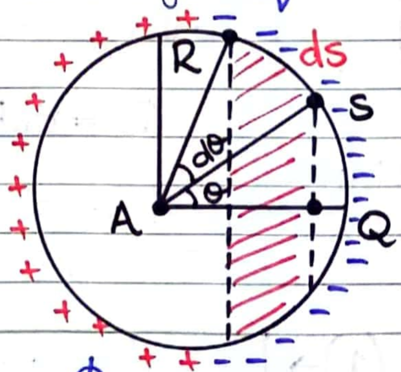

(1) ⟶ $$E_2 = \frac{q}{4\pi\varepsilon_0 r^2}$$

(2) ⟶ $$dE_2 = \frac{dq\cos\theta}{4\pi\varepsilon_0 r^2}$$

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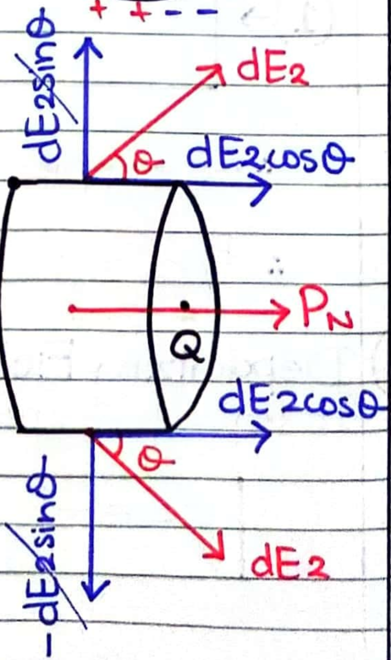

Let P_N be the component of polarization perpendicular to ds,

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$$P_N = P\cos\theta = \frac{dq}{ds} \quad , \quad dq = P\cos\theta\, ds \rightarrow (3)$$

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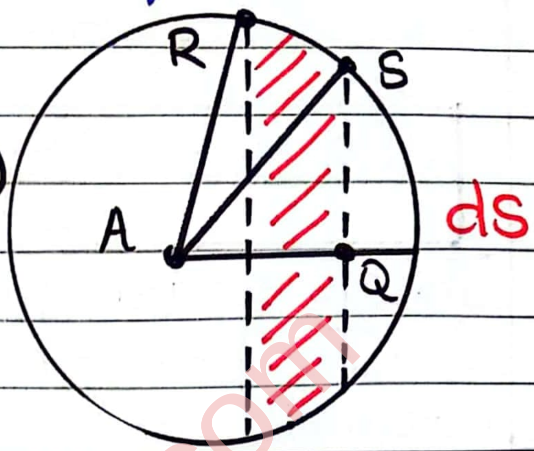
$$dE_z = \frac{P\cos^2\theta\, ds}{4\pi\varepsilon_0 r^2} \rightarrow (4)$$



→ To find ds,

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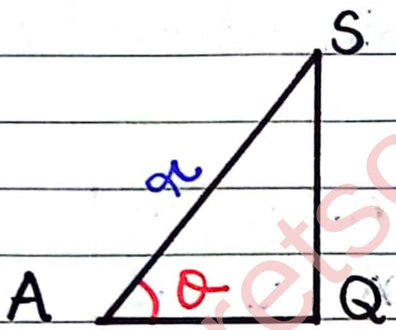
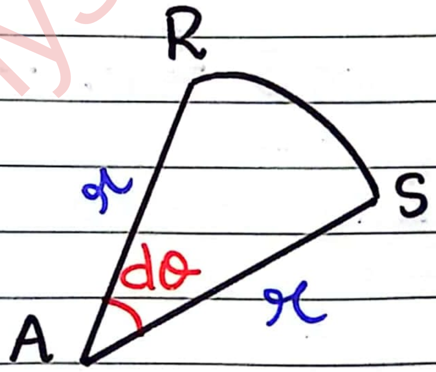
$= [(2\pi) \times \text{radius}] \times [\text{arc length}]$

$= 2\pi(QS) \times (RS)$





$$\sin\theta = \frac{Perp}{Hyp}$$

Arc = Angle x radius

$$RS = d\theta \times r$$

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So,

$$\text{Area}\,(ds) = 2\pi r\sin\theta\,(r\,d\theta)$$

$$ds = 2\pi r^2 \sin\theta\, d\theta \rightarrow (5)$$

Date:

→ Put (5) in (4),

$$dE_2 = \frac{P\cos^2\theta\ (\cancel{2}\cancel{\pi}\cancel{r^2}\sin\theta\, d\theta)}{\cancel{4}\cancel{\pi}\varepsilon_0 \cancel{r^2}}$$

$$\boxed{dE_2 = \frac{P\cos^2\theta \sin\theta\, d\theta}{2\varepsilon_0}}$$

This is electric field due to small patch ds. To find for the whole cavity, we must integrate it.

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$$E_2 = \frac{-P}{6\varepsilon_0}\left[(-1)^3-(1)^3\right]$$

$$E_2 = \frac{-P}{6\varepsilon_0}(-1-1)$$

$$\boxed{E_2 = \frac{P}{3\varepsilon_0}} \rightarrow (3)$$