

Assignment 1:-

SSP II

(d)

NUMERICALS

① A parallel plate condenser has capacitance  $2\mu\text{f}$ . The dielectric has permittivity  $\epsilon_r = 100$ .

(a) For an applied voltage of  $1000\text{V}$  find energy stored in condenser, as well as in polarizing in dielectric. (b) A field strength  $E$  applied to dielectric. Show stored energy in polarized atom equal to  $\frac{1}{2} \alpha E^2$ , where  $\alpha$  is permittivity?

Sols-

(a)  $C = 2\mu\text{F}$ ,  $\epsilon_r = 100$ ,  $V = 1000\text{ Volt}$

Energy stored in capacitor

$$W = \frac{1}{2} CV^2$$

$$= \frac{1}{2} (2 \times 10^{-6}) (10^6)$$

$$W = 1 \text{ joule.}$$

\*Capacitance of capacitor when dielectric is removed

$$C_0 = \frac{C}{\epsilon_r} = \frac{2}{100}$$

$$C_0 = 0.02\mu\text{f}$$

energy stored in capacitor without dielect

$$W = C_0 V^2 / 2$$

$$W_0 = C_0 V^2 / 2$$

$$C_0 = 0.02$$

$$W_0 = \frac{(0.02 \times 10^{-6}) (10^6)^2}{2}$$

$$W_0 = 0.01 \text{ joule}$$

Energy stored in dielectric

$$W' = W - W_0 = 1 - 0.01$$

$$W' = 0.99 \text{ Joule}$$

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(b) Energy stored in polarizing dielectric is

$$W = \frac{1}{2} [C - C_0] V^2$$

$$= \frac{1}{2} \left[ \frac{A \epsilon_0 \epsilon_r}{d} - \frac{A \epsilon_0}{d} \right] V^2$$

$$W' = \epsilon_0 \left[ \frac{A}{2} \right] \frac{V^2}{d} (\epsilon_r - 1)$$

$$\text{Polarization} = P = E \epsilon_0 (\epsilon_r - 1)$$

$$\epsilon_r - 1 = \frac{P}{E \epsilon_0}$$

$$W' = \left[ \frac{A \epsilon_0 V^2}{2d} \right] \frac{P}{E \epsilon_0}$$

$$= \left[ \frac{A V^2}{2d} \right] \frac{P}{E}$$

Energy stored for unit volume due to Polarization

is :-  $\frac{W'}{\text{Volume}} = \frac{W'}{Ad} = \left( \frac{1}{Ad} \right) \left[ \frac{A V^2}{2d} \right] \left[ \frac{P}{E} \right]$

$$= \frac{1}{2} \left[ \frac{V^2}{d^2} \right] \left[ \frac{P}{E} \right]$$

$$= \frac{P E^2}{2 \cdot E} = \frac{1}{2} P E$$

But  $P = N \sigma E$

Thus  $\frac{W'}{\text{Volume}} = \frac{N \sigma E^2}{2}$

hence energy stored in polarized atom

is  $\frac{1}{2} \sigma E^2$

② Two parallel plates have equal and opposite charges and are separated by dielectric 5mm thick of dielectric constant 3.

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If electric density in dielectric is  $10^6 \text{ V/m}$ . Calculate  
(a) free charge per unit area on conducting plate.  
(b) the bound charge per unit on surface of dielectric. (c) the polarization  $P$  in dielectric.  
(d) the displacement in dielectric (e) energy density in dielectric?

Soln -

Thickness = 5mm

Dielectric constant = 3

Electric density =  $10^6 \text{ V/m}$

Free charge per unit area =  $\sigma$

As  $E = \frac{\sigma}{K\epsilon_0}$   $\epsilon_0 = \text{permittivity of free space}$

$$\sigma = E K \epsilon_0$$

$$\sigma = (10^6 \text{ V/m}) \times 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \times 3$$

$$\sigma = 26.55 \times 10^{-6} \text{ C/m}^2$$

(b) Bound charge per unit area on surface of dielectric =  $\sigma_B$

$$\sigma_B = \sigma \left(1 - \frac{1}{K}\right)$$

$$\sigma_B = (26.5 \times 10^{-6}) \left(1 - \frac{1}{3}\right)$$

$$\sigma_B = 17.766 \times 10^{-6} \text{ C/m}^2$$

(c) Polarization  $P$  in dielectric

$$P = \epsilon_0 (\epsilon_r - 1) E = P = \epsilon_0 (K - 1) E$$

$$P = 8.85 \times 10^{-12} (3 - 1) 10^6$$

$$P = 17.7 \times 10^{-6} \text{ C/m}^2$$

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d) displacement

$$D = E \epsilon_0 + P$$

$$D = (10^6) (8.85 \times 10^{-12}) + 17.7 \times 10^{-6}$$

$$D = 26.55 \text{ C/m}^2$$

e) Energy =  $U = \frac{1}{2} k \epsilon_0 E^2$

$$U = \frac{1}{2} (3) (8.85 \times 10^{-12}) (10^6)^2$$

$$U = 13.275 \text{ J/m}^3$$

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3) If all molecular dipoles in 1cm radius water drop are pointed in same direction, Calculate the intensity of polarization. Dipole moment of water molecule  $6 \times 10^{-30} \text{ C.m}$ ?

Sol:-

Molecular Mass of water = 18 gm

$\frac{18 \text{ gm}}{10^3}$  of water contains  $6.023 \times 10^{23}$  mol  
 $\frac{18 \text{ m}^3}{10^3}$  will contain  $\frac{6.02 \times 10^{26}}{18}$  mol. (water molecules)

Volume of water drop =  $V = \frac{4}{3} \pi r^3$

$$V = \frac{4}{3} (3.14) (10^{-2})^3 \text{ m}^3$$

$$V = 4.187 \times 10^{-6} \text{ m}^3$$

For No. of molecules in drop  $N$  can be find by using formula.

$$\rightarrow N = \frac{6.023 \times 10^{26} \times 4 \times 3.14 \times 10^{-6}}{18 \times 3}$$

$$N = 1.4 \times 10^{20} \text{ m}^{-3}$$

$\rightarrow$  Polarization  $P = N \mu$

$$P = 1.4 \times 10^{20} \text{ m}^{-3} \times 6 \times 10^{-30}$$

$$P = 8.4 \times 10^{-10} \text{ C/m}^2$$

4) Assuming Polarizability of  $\text{K}_2$  atom is  $2.18 \times 10^{-40} \text{ Fm}^2$ . Calculate dielectric constant of  $\text{K}_2$  at  $0^\circ \text{C}$  and 1 atmosphere?

Sol:-  $\alpha_{\text{K}_2} = 2.18 \times 10^{-40} \text{ Fm}^2$

$$\epsilon = \epsilon_0 \epsilon_r$$

$$T = 0^\circ \text{C}$$

$$P = 1 \text{ atm}$$

$$\epsilon_r - 1 = \frac{1}{3\epsilon_0} \sum N_i \alpha_i$$

$$\epsilon_r + 2$$

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$$\epsilon_0 - 1 = \frac{N\alpha}{3\epsilon_0} (\epsilon_0 + 2)$$

$$\epsilon_0 = \frac{N\alpha \epsilon_0}{3\epsilon_0} + \frac{2N\alpha}{3\epsilon_0}$$

$$\epsilon_0 - \frac{N\alpha \epsilon_0}{3\epsilon_0} = \frac{2N\alpha}{3\epsilon_0} + 1$$

$$\left( \frac{1 - N\alpha}{3\epsilon_0} \right) \epsilon_0 = \frac{2N\alpha}{3\epsilon_0} + 1$$

$$\epsilon_0 = \frac{2N\alpha}{3\epsilon_0} + 1 \div \frac{1 - N\alpha}{3\epsilon_0}$$

$$PV = nRT$$

$$P = 1 \text{ atm}$$

$$\frac{P}{KT} = \frac{N}{V} = N$$

$$\frac{101325}{(273)(1.38 \times 10^{-23})} = N$$

$$N = 2.69 \times 10^{25} \text{ atoms/m}^3$$

$$\frac{N\alpha}{3\epsilon_0} = 2.2 \times 10^{-4}$$

$$\epsilon_0 = \frac{(2 \times 2.2 \times 10^{-4} + 1)}{1 - 2.2 \times 10^{-4}}$$

$$\epsilon_0 = 1.00066$$

$$\epsilon = 8.85 \times 10^{-12}$$

5) If a dielectric is introduced b/w the plates of parallel capacitor, show induced charge varies with dielectric as.

$$\epsilon_0 = \left( 1 - \frac{\sigma'}{\sigma} \right)^{-1} \text{ where } \sigma \text{ is surface density?}$$

Sol:-

Electric field b/w the plates of a parallel plate capacitor is

$$E = \sigma / \epsilon_0$$

in presence of dielectric

$$E = \frac{\sigma}{\epsilon_0 \epsilon_r}$$

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The induced charge  $\sigma'$  is given as

$$\sigma' = \epsilon_0 (E_0 - E)$$

$$\sigma' = \epsilon_0 \left( \frac{\sigma}{\epsilon_0} - \frac{\sigma}{\epsilon_0 \epsilon_r} \right)$$

$$\sigma' = \epsilon_0 \frac{\sigma}{\epsilon_0} \left( 1 - \frac{1}{\epsilon_r} \right)$$

$$\sigma' = \sigma \left( 1 - \frac{1}{\epsilon_r} \right)$$

$$\frac{\sigma'}{\sigma} = \left( 1 - \frac{1}{\epsilon_r} \right)$$

$$\frac{1}{\epsilon_r} = 1 - \frac{\sigma'}{\sigma}$$

$$\boxed{\epsilon_r = \left( 1 - \frac{\sigma'}{\sigma} \right)^{-1}}$$

$$\epsilon_r = 1 + \frac{N\alpha}{3\epsilon_0}$$

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6) The Polarizability of neon gas  $0.35 \times 10^{-40} \text{ fm}^2$ .  
If gas contains  $2.7 \times 10^{25}$  atoms/m<sup>3</sup> at 0°C and  
one atmosphere pressure, Calculate its relative  
dielectric constant. Also solve  $\epsilon_r$  of Krypton  
having  $\sigma_e = 3.54 \times 10^{-40} \text{ fm}^2$ .

Sol:-

$$\epsilon_r = 1 + \frac{N\sigma_e}{\epsilon_0}$$

Formula

Same as  
previous  
questions.

7) An elemental dielectric material has  $\epsilon_r = 12$   
and it contains  $5 \times 10^{28}$  atoms/m<sup>3</sup>. Calculate its  
electronic polarizability assuming Lorentz field?

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Sol:-

$$P = N \alpha_0 E$$

$$\alpha_0 = \frac{P}{NE} \rightarrow (1)$$

$$P = (\epsilon_r - 1) \epsilon_0 E \rightarrow (2)$$

$$\alpha_0 = \frac{(\epsilon_r - 1) \epsilon_0 E}{NE}$$

$$\alpha_0 = \frac{8.85 \times 10^{-12} (12 - 1)}{5 \times 10^{28}}$$

$$\alpha_0 = 1448 \times 10^{-40} \text{ Fm}^2$$

Q. A sphere of radius R carries a polarization  $P(r) = kr$  where k is constant and r is centre. Find field outside and inside of sphere?

Sol:- Inside:-  $\oint E \cdot da = Q_{enc} / \epsilon_0 \rightarrow (1)$

$$Q_{enc} = \frac{4}{3} \pi r^3 \rho$$

$$E (4\pi r^2) = \frac{4/3 \pi r^3 \rho}{\epsilon_0} \rightarrow (2)$$

$$E = \frac{\rho r}{3 \epsilon_0} \hat{r} = \frac{r(-3k)}{3 \epsilon_0} \hat{r}$$

$$E = \frac{-rk}{\epsilon_0} \hat{r}$$

Outside:-  $\oint E \cdot da = Q_{enc} / \epsilon_0$

$$Q_{enc} = 0$$

$$E = 0 \hat{r}$$

10 A solid dielectric material has  $5 \times 10^{28}$  atoms/m<sup>3</sup>. If polarizability is  $3.6 \times 10^{-40}$  Fm<sup>2</sup>. Calculate the Lorentz field normalized with respect to external field i.e.  $E_i/E$ ?

Soln-  $N = 5 \times 10^{28}$  atoms/m<sup>3</sup>  
 $\alpha = 3.6 \times 10^{-40}$  Fm<sup>2</sup>.

We know Lorentz field, in the internal field  
 $E_i = E + \frac{P}{3\epsilon_0}$

$P = N\alpha E_i$

$E_i = E + \frac{N\alpha E_i}{3\epsilon_0}$

$E_i - \frac{N\alpha E_i}{3\epsilon_0} = E$

$E_i \left(1 - \frac{N\alpha}{3\epsilon_0}\right) = E$

$\frac{E_i}{E} = \frac{1}{\left(1 - \frac{N\alpha}{3\epsilon_0}\right)}$

" =  $\frac{1}{\left(1 - \frac{5 \times 10^{28} \times 3.6 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}}\right)}$

" = 1.6777

$E_i = E + \frac{N\alpha E}{3\epsilon_0}$

$E_i = E \left(1 + \frac{N\alpha}{3\epsilon_0}\right)$

$E_i/E = 1 + \frac{N\alpha}{3\epsilon_0}$

" =  $1 + \frac{(5 \times 10^{28})(3.6 \times 10^{-40})}{3(8.85 \times 10^{-12})}$

" = 1 + 0.6777

" = 1.6777

11) Three identical atoms in a string are subjected to a homogeneous electric field  $E$  volt/m along the line joining their centres. If polarizability is  $2.5 \times 10^{-40} \text{ Fm}^2$  and centre spacing is  $0.3 \text{ mm}$ . Find  $E_i/E$  at position of centre of atom.

Sol:- The internal field line joining the centre of atom is

$$E_i = E + E_s$$

where  $E_s = \frac{1}{4\pi\epsilon_0} \left[ \frac{4\pi\alpha}{a^3} \right]$

due to each atom 'a' is centre to centre space

$$4\pi\alpha = \alpha E_i$$

$$E_i = E + \frac{\alpha E_i}{4\pi\epsilon_0 a^3}$$

in case of two central atom, Thus

$$E_i \left[ 1 - \frac{\alpha}{4\pi\epsilon_0 a^3} \right] = E$$

$$\frac{E_i}{E} = \frac{1}{\left[ 1 - \frac{\alpha}{4\pi\epsilon_0 a^3} \right]}$$

$$a = 3 \times 10^{-3}$$

$$\therefore \frac{\alpha}{4\pi\epsilon_0 a^3} = \frac{2.5 \times 10^{-40}}{4\pi \times 8.8 \times 10^{-12} \times 27 \times 10^{-9}} = 0.33$$

$$\frac{E_i}{E} = \frac{1}{[1 - 0.33]}$$

$$\boxed{\frac{E_i}{E} = 1.5}$$

12) Suppose the field inside a large piece of dielectric is  $E_0$  so the electric displacement is equal to  $D_0 = \epsilon_0 E_0 + P$ .

(a) Now a small spherical cavity is hollowed out of material. Find the field at centre of cavity in terms of  $E_0$  and  $P$ . Also find  $D_0$  and  $P$ ? (b) Do same for long needle shaped cavity running parallel to  $P$ ? (c) Do same for water shaped cavity perpendicular to  $P$ ?

Sol: - Consider the field inside a large piece of dielectric is  $E_0$

Consider the dielectric displacement is  $D_0 = \epsilon_0 E_0 + P$

The displacement at centre of cavity in terms of  $D_0$  and  $P$

$$D = \epsilon_0 E \rightarrow (1) \quad \because \epsilon_0 = \text{relative Permittivity}$$

The  $E_0$  plus the field of spherically equally polarized with polarization will make the E.F within the hollowed hole.

$$\vec{E} = E_0 - \left( -\frac{P}{3\epsilon_0} \right) = E_0 + \frac{P}{3\epsilon_0} \rightarrow (2)$$

Substitute in eq (1)

$$D = \epsilon_0 E_0 + \frac{P}{3\epsilon_0}$$

$$\rightarrow D_0 = P + \frac{P}{3}$$

$$D_0 = -\frac{2}{3}P$$

→ value of displacement at centre of cavity in terms of  $D_0$  and  $P$ .

(b) The thin needle may be modeled as a very thin cylinder, acting as a dipole, with dipole moment antiparallel to  $P$ . So,

$$\vec{E} \approx \vec{E}_0$$

Substitute  $\vec{E}_0$  for  $\vec{E}$  in eq (1)

$$D = \epsilon_0 \vec{E}_0 = \vec{D}_0 - P$$

So value for long needle shaped cavity running parallel to  $P$  is

$$\begin{aligned} D &= D_0 - P \\ \vec{E} &= \frac{D}{\epsilon_0} = \frac{D_0 - P}{\epsilon_0} = \frac{D_0}{\epsilon_0} - \frac{(-P)}{\epsilon_0} \\ &= \vec{E}_0 + \frac{P}{\epsilon_0} \end{aligned}$$

Substituting  $\vec{E}_0 + \frac{P}{\epsilon_0}$  for  $\vec{E}$

$$D = \epsilon_0 \left( \vec{E}_0 + \frac{P}{\epsilon_0} \right)$$

$$= D_0$$

So value of same for a thin water shaped cavity  $\perp$  to  $P$  is  $\vec{D} = \vec{D}_0$

14) The ~~center~~ <sup>electronic</sup> of two identical atoms of polarizability of Ar atom is  $\alpha_e = 1.7 \times 10^{-40} \text{ fm}^2$ . What is dielectric constant  $\epsilon_r$  of solid Ar below (84K) if density is  $1.8 \text{ g/cm}^3$  and atomic mass is  $39.95$ ?

Soln - Molar mass =  $39.95 \text{ g mol}^{-1}$

$$N = \frac{N_A d}{M_{\text{at}}}$$

$$N = \frac{(6.02 \times 10^{23} \text{ mol}^{-1}) (1.8 \text{ g cm}^{-3})}{39.95 \text{ g mol}^{-1}}$$

$$N = 2.71 \times 10^{22} \text{ cm}^{-3}$$

$$\alpha_e = 1.7 \times 10^{-40} \text{ fm}^2$$

$$\epsilon_0 = 1 + \frac{N \alpha_e}{\epsilon_0}$$

$$\epsilon_0 = \frac{(2.71 \times 10^{22}) (1.7 \times 10^{-40})}{8.85 \times 10^{-12}}$$

$$\epsilon_0 = 1.55$$

Clausius Mossotti eq:-

$$\epsilon_r = \frac{1 + N \alpha_e / \epsilon_0}{1 - \frac{2}{3} N \alpha_e / \epsilon_0}$$

$$\epsilon_r = 1.87$$

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13) The center of two identical atoms of polarizability  $\alpha = 2 \times 10^{-40} \text{ Fm}^2$  are separated at distance  $5 \times 10^{-10} \text{ m}$ . A homogeneous electric field is applied in direction to the line joining the centers of two atoms. Calculate b/w  $E_i$  and  $E$ ?

Sol<sup>n</sup> -

$$\alpha = 2 \times 10^{-40} \text{ Fm}^2$$

$$d = 5 \times 10^{-10} \text{ m}$$

$$E_i = E + \frac{1.2\mu}{\pi d^3 \epsilon_0} \quad \mu = \alpha E$$

$$E_i/E = 1 + \frac{1.2\alpha E}{\pi d^3 \epsilon_0}$$

$$= 1 + \frac{1.2 \times 2 \times 10^{-40}}{2.14 \times 9.85 \times 10^{-32} \times (5 \times 10^{-10})^3}$$

$$E_i/E = 1.069$$

"OR"

$$E_i = E + \frac{\mu_{ind}}{4\pi\epsilon_0 a^3} \rightarrow \textcircled{1}$$

$$\mu = \alpha E$$

$$E_i = E + \frac{\alpha E_i}{4\pi\epsilon_0 a^3}$$

$$E_i - \frac{\alpha E_i}{4\pi\epsilon_0 a^3} = E$$

$$E_i \left(1 - \frac{\alpha}{4\pi\epsilon_0 a^3}\right) = E$$

$$E_i = E \frac{1}{\left(1 - \frac{\alpha}{4\pi\epsilon_0 a^3}\right)}$$

$$E_i/E = \frac{1}{1 - \frac{\alpha}{4\pi\epsilon_0 a^3}}$$

$$= \frac{1}{1 - \frac{(2 \times 10^{-40})}{4\pi\epsilon_0 (5 \times 10^{-10})^3}}$$

$$E_i/E = 1.069$$

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15) Find the polarizability of  $\text{CO}_2$  if its susceptibility is  $0.985 \times 10^{-3}$ , Density of Carbon dioxide is  $1.977 \text{ kg/m}^3$ .

Sol:-  $\chi_e = 0.98 \times 10^{-3}$

$\rho = 1.977 \text{ kg/m}^3$

$\alpha = p$

Clausius Mossotti eq.

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

$$\alpha = \frac{3\epsilon_0 (\epsilon_r - 1)}{N (\epsilon_r + 2)}$$

$\epsilon_r = 1 + \chi_e$

" =  $1 + 0.98 \times 10^{-3}$

" =  $1.000985$

$n = m/M$

$nNA = N_i = \frac{mNa}{M}$

$$\frac{nNA}{V} = \frac{N_i}{V} = \frac{mNa}{MV}$$

$$N = \frac{m}{V} \cdot \frac{NA}{M} = \rho \frac{NA}{M}$$

$M = 12 + 2 \times 16$

" =  $12 + 2 \times 16$

" =  $44 \text{ g/mol}$

$$N = \frac{1.977 \times 6.02 \times 10^{23}}{44 \times 10^{-3}}$$

$$N = 2.7 \times 10^{25} / \text{m}^3$$

$$\alpha = \frac{3(1.000985)}{2.7 \times 10^{25}} \left( \frac{1.000985 - 1}{1.000985 + 2} \right)$$

$$\alpha = 3.227 \times 10^{-40} \text{ Fm}^2$$

16) The dielectric constant of helium measure at  $0^\circ\text{C}$  and atmosphere  $\epsilon_r = 1.000684$ .



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under these conditions, gas contains  $2.7 \times 10^{25}$  atoms/m<sup>3</sup>.  
 Calculate radius of electron cloud (atomic radius).  
 Also calculate the displacement  $x$  when a helium atom subjected to field  $10^6$  volt/m?

Sol: -

$$\epsilon_0(\epsilon_r - 1) = N\sigma_e$$

$$N(4\pi\epsilon_0 r_0^3) = \epsilon_0(\epsilon_r - 1)$$

$$r_0^3 = \frac{\epsilon_0(\epsilon_r - 1)}{N4\pi\epsilon_0}$$

$$r_0^3 = 1.0000684$$

$$4\pi \times 2.7 \times 10^{25}$$

$$r_0 = 5.87 \times 10^{-11} \text{ metre.}$$

$$r_0 = 0.0587 \text{ nm (atomic radius)}$$

Displacement: -

$$x = \frac{4\pi\epsilon_0 E r_0^3}{Ze}$$

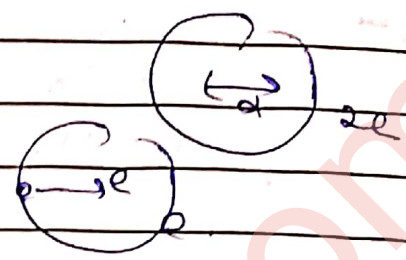
$$x = \frac{4\pi\epsilon_0 \times 10^6 \times (0.0587 \times 10^{-9})^3}{2 \times 1.6 \times 10^{-19}}$$

$$x = 7.03 \times 10^{-17} \text{ m}$$

Q17: Calculate the induced dipole moment per unit volume (density of Polarization) of He gas if placed in field  $6 \times 10^5$  V/m. The atomic Polarizability of helium is  $0.18 \times 10^{-40}$  Fm<sup>2</sup> and concentration of helium atom is  $2.6 \times 10^{25}$ /m<sup>3</sup>. Also calculate separation of positive and negative charges in each atom?

Sol:-  $P = \frac{P}{V}$  Induced dipole moment

$E_{local} = 6 \times 10^5 \text{ V/m}$   
 $\alpha = 0.18 \times 10^{-40} \text{ Fm}^2$   
 $N = 2.6 \times 10^{25} \text{ Fm}^2$   
 $d = ?$



$P = \alpha E_{local} = qd$

$P = N \alpha E_{local}$

$P = (2.6 \times 10^{25})(6 \times 10^5) \times 0.18 \times 10^{-40}$

$P = 2.808 \times 10^{-10} \text{ C/m}^2$

$\frac{\alpha E_{local}}{q} = d$

$d = (0.18 \times 10^{-40})(6 \times 10^5) / 1.602 \times 10^{-19}$

$d = 6.74 \times 10^{-11} \text{ m}$

OR  
 $P = Nq$   
 $P = Ncd$   
 $d = \frac{P}{Nc}$

18) The polarizability of  $\text{NH}_3$  molecules in gaseous state, from the measurement of dielectric constant is found to be  $2.5 \times 10^{-39} \text{ Fm}^2$  and  $2 \times 10^{-29} \text{ Fm}^2$  at temp 300K and 400K respectively. Calculate the contribution to polarizability because of deformation and contribution of permanent dipole moment of each temp.

Sol. -

According to Langevin-Debyers theory, the total polarizability :-

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$\alpha = \alpha_1$  (due to deformation) +  $\alpha_2$  (due to permanent dipole)

$\therefore \alpha = \alpha_1 + \frac{N_0^2}{3k_0T}$

$\alpha = \alpha_1 + \beta/T$

with  $2.5 \times 10^{-39}$  at 300K and  $2 \times 10^{-39}$  at 400K.

$2.5 \times 10^{-39} = \alpha_1 + \frac{\beta}{300}$  → (1)

$2 \times 10^{-39} = \alpha_1 - \beta/400$  → (2)

$\frac{\beta}{300} - \frac{\beta}{400} = 2.5 \times 10^{-39} - 2 \times 10^{-39}$

" =  $0.5 \times 10^{-39}$

$\beta(400 - 300) = 0.5 \times 10^{-39} \times 700$

$100\beta = 12 \times 10^4 \times 0.5 \times 10^{-39}$

$\beta = 6 \times 10^{-37}$

Substitute in eq (1)

$\alpha_1 = 2.5 \times 10^{-39} - \frac{6 \times 10^{-37}}{300}$

$\alpha_1 = 0.5 \times 10^{-39} \text{ Fm}^2$

Similarly: -

$\alpha_2 = \beta/T$  with  $\beta = 6 \times 10^{-37}$   $T = 300$

" " " "  $T = 400$

$\alpha_2 = \frac{6 \times 10^{-37}}{300}$

$\alpha_2 = 2 \times 10^{-39} \text{ Fm}^2$

$$d_2 = \frac{6 \times 10^{-37}}{400}$$

$$d_2 = 1.5 \times 10^{-39} \text{ Fm}^2$$

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An atom of oxygen on being polarized produces a dipole moment of  $0.5 \times 10^{-22} \text{ C-m}$ . If distance of centre of ~~gravity~~ negative charge cloud from the nucleus be  $4 \times 10^{-17} \text{ m}$  calculate polarizability of oxygen atoms.

Sol:-

In equilibrium position both Lorentz force and coulomb interaction will be equal.

$$(8e) E = \frac{(8e)(8e)}{4\pi \epsilon_0 d^2}$$

$$E = \frac{8e}{4\pi \epsilon_0 d^2} \quad \text{--- (1)}$$

$$E = \frac{8 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (4 \times 10^{-17})^2}$$

$$E = 2.6 \times 10^{34} \text{ volt / metre.}$$

Dipole moment

$$M = \alpha E$$

$$\alpha = M / E$$

$$\alpha = \frac{0.5 \times 10^{-22}}{2.6 \times 10^{34}}$$

$$2.6 \times 10^{34}$$

$$\alpha = 1.9 \times 10^{-45} \text{ Fm}^2$$

20) Two water molecules each having dipole moment  $6.2 \times 10^{-30}$  C.m. point on same direction along the line joining their centres. Calculate the potential energy due to dipole interaction with their centres at  $3.1 \times 10^{-10}$  m apart.

Sol: -

Let  $\mu_1$  and  $\mu_2$  be dipole moments of two dipoles. We may consider first dipole to be placed in field produced by second.

$$W = -(\mu_1 E)$$

where  $E$  is field produced by second dipole

$$W = - \left[ \mu_1 \times \frac{1}{4\pi\epsilon_0 r^3} \left\{ 3\mu_2 \times r - \mu_2 \right\} \right]$$

$$= \frac{1}{4\pi\epsilon_0 r^3} [\mu_1 \mu_2 - 3\mu_1 \mu_2]$$

$$W = \frac{1}{4\pi\epsilon_0 (3.1 \times 10^{-10})^3} \left[ (6.2 \times 10^{-30})^2 - 3(6.2 \times 10^{-30})^2 \right]$$

$$W = -0.0232 \times 10^{-19} \text{ Joule.}$$

$$W = -0.0145 \text{ eV.}$$

21) Calculate the individual dipole moment of molecule of Carbon tetrachloride with following data. Find also average electron displacement. Relative permittivity

$\epsilon_r = 1.24$  Density =  $1.6 \times 10^3 \text{ kg/m}^3$   
Molecular weight = 156. Field  $10^7 \text{ volt/m}$   
Date: / /

Sol: -

$$\text{Molecular Density} = N = \frac{N_A \times \rho}{M_A}$$

$$N = \frac{6.02 \times 10^{23} \times 1.6 \times 10^3}{156}$$

$$N = 6.17 \times 10^{27} \text{ molecules/m}^3$$

$$P = E \epsilon_0 (\epsilon_r - 1)$$

$$\mu_0 = \frac{P}{N} = \frac{E \epsilon_0 (\epsilon_r - 1)}{N}$$

$$\mu_0 = \frac{10^7 \times 8.85 \times 10^{-12} (1.24)}{6.17 \times 10^{27}}$$

$$\mu_0 = 1.77 \times 10^{-32} \text{ C.m.}$$

Disp bent = Since 74 electrons in e.c.d.y so

$$\mu_0 = Ze(\lambda)$$

$$\lambda = \frac{\mu_0}{Ze}$$

$$\lambda = \frac{1.77 \times 10^{-32}}{74 \times 1.6 \times 10^{-19}}$$

$$\lambda = 1.5 \times 10^{-15} \text{ m}$$