ASSIGNMENT 04

(1) The resonance absorption exhibited by a medium shows a single absorption line at 6000Å (in vacuum). When a beam of light of this wavelength travels through 2.5 cm in the medium. The intensity of light drops 1/e of its initial value. Calculate the maximum value of imaginary part of the index of refraction?

Solution:-

We can use the formula for intensity of light after travelling through a medium with absorption

$$I = I_0 e^{-2\alpha x} \rightarrow (1)$$

where I is the intensity of light travelling through medium; I_0 is intial intensity; α is absorption coffient; x is distance traveled through medium.

We have given intensity drops to $1/e$ of its intial value so we have

$$I/I_0 = 1/e. \rightarrow (2)$$

Subsitute eq ① in ②

$1/e = 2\alpha x$

Taking log on above eq.

$\ln(1/e) = \ln(e^{-2\alpha x})$

$-1 = -2\alpha x$

$\alpha = 1/2x \rightarrow ③$

The imaginary part of index of refraction Im(n) is related to absorption cofficent (α) through formula.

$$\alpha = \frac{4\pi k}{\lambda} \rightarrow ④$$

k is imaginary part of index of refraction ; λ is wavelength.

Subsitute ③ in ④

$$\frac{1}{2x} = \frac{4\pi k}{\lambda} \rightarrow ⑤$$

Rearrange eq ⑤

$$k = \frac{\lambda}{8\pi x} \rightarrow ⑥$$

Subsitute values in eq ⑥

$$k = \frac{6000 \times 10^{-10}}{8 \times 3.14 \times 2.5 \times 10^{-2}}$$

$$k = \frac{6000 \times 10^{-10}}{20\pi \times 10^{-2}}$$

$$k = \frac{3}{10\pi} \times 10^{-8}$$

$$k = \frac{3 \times 10^{-9}}{\pi}$$

$$\boxed{k = 9.554 \times 10^{-10}}$$

maximum value for imaginary part of index of refraction.

(2) The complex index of refraction of a metal for infrared radiation ($\omega\tau \ll 1$) expressed as $\sqrt{\epsilon(\omega)} = (n + ik) = \sqrt{1 + 4\pi i \sigma_0/\omega}$.

where σ_0 is the electrical conductivity for static fields. Check that σ_0 has unit s^{-1} in esu. Assuming $\sigma_0 \gg \omega$ (leading $n = k$) show that reflectance of metal approximately equals. $[(1 + 2\omega/\pi\sigma_0)^{1/2}]$.

Dielectric properties responco to Electric field and responco of dielectric of EF is equal to the refractive index of optical properties.

Here infrared radiation expressed as $\sqrt{\varepsilon(\omega)} = (n + ik) = \sqrt{1 + 4\pi i \sigma_0/\omega}$. $\{\omega\tau \ll 1\}$.

where σ_0 is electrical conductivity

Let assume $\sigma_0 \gg \omega$, $n = k$ so,

In this case we consider $\omega \ll \omega_p$,

$|\varepsilon_0| \ll 4\pi\sigma/\omega$ $\tilde{n} \simeq \tilde{k}$ Thus we obtain

$$\varepsilon(\omega) \simeq \frac{4\pi i \sigma}{\omega} \simeq \frac{4\pi i n e^2 \tau}{\omega m^*} \simeq i\varepsilon_2(\omega) \cong$$

$$2i\tilde{n}\tilde{k} \simeq 2i\tilde{k}^2.$$

This gives extinction cofficient $\hat{k}(\omega)$:-

$$\hat{k}(\omega) = \sqrt{\frac{2\pi n e^2 \tau}{m^* \omega}}$$

and absorption cofficient becomes:-

$$a_{abs}(\omega) = \frac{2\omega \hat{k}(\omega)}{c} = \sqrt{\frac{8\pi\omega n e^2 \tau}{m^* c^2}}$$

For this limit $a_{abs}(\omega)$ is proportional to $\sqrt{\omega}$. Usually the convenient observable for metals is the reflectivity. In limit appropiate for metals $\hat{n} = \hat{k}$ and both $\hat{n}$ an $\hat{k}$ are large. We thus have

$$R = \frac{(\hat{n}-1)^2 + \hat{k}^2}{(\hat{n}+1)^2 + \hat{k}^2} = \frac{\hat{n}^2 - 2\hat{n} + 1 + \hat{k}^2}{\hat{n}^2 + 2\hat{n} + 1 + \hat{k}^2}$$

$$" = 1 - \frac{4\hat{n}}{\hat{n}^2 + \hat{k}^2 + 2\hat{n} + 1}$$

$$R \simeq 1 - \frac{4\hat{n}}{\hat{n}^2 + \hat{k}^2} \simeq 1 - 2/\hat{n}.$$

For condition $\hat{n} \simeq \hat{k} >> 1$ we obtain

$$\hat{n}(\omega) \simeq \sqrt{\frac{2\pi me^2\tau}{m^*\omega}}$$

so that reflectivity goes as :-

$$R(\omega) \simeq 1 - 2\sqrt{\frac{m^*\omega}{2\pi ne^2\tau}}$$

so it approximately equal to reflectance of metals.

$$\left[\left(1 + 2\omega/\pi\sigma_0\right)^{1/2}\right]$$

③ Discuss the classical theory of response of a solid to an oscillating electro-magnetic field and obtain an expression for the dielectric function in terms of plasma frequency?

Solution:-

The classical theory of response of solid to an oscillating frequency at electromagnetic field is described by the Drude model. This model treats the electrons in solid as a free electron gas that can respond to the external electric field. The responce of electrons to field is characterized by motion under influence of field and collisions with atoms in the lattice.

In presence of an oscillating field $E(t) = E_0 e^{-i\omega t}$, the equation of motion for electrons in Drude model can be written as

$$\frac{m d^2 x}{dt^2} + m\gamma \frac{dx}{dt} + m\omega_p^2 r = -eE(t)$$

where m is effective mass of electron; γ is damping cofficent; ω_p is plasma frequency; r is position vector; e is elementary charge; $E(t)$ is electric field.

Assuming the solution of equation has form $r(t) = r_0 e^{-i\omega t}$ where r_0 is complex amplitude, we can find dielectric function $\epsilon(\omega)$ as ratio induced Polarization P in electric feild E.

$$\varepsilon(\omega) = 1 + \frac{Ne^2}{m} \frac{1}{\omega_p^2 - \omega^2 - i\gamma\omega}$$

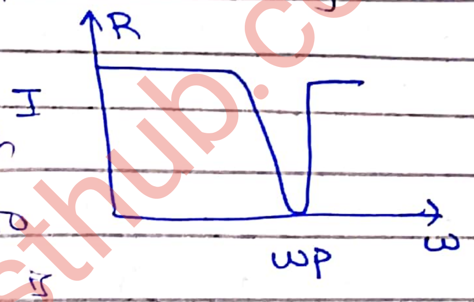
where n is density of electron.

So, expression for plasma frequency ω_p as:-

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - i\gamma\omega}$$

This is the classical expression for dielectric function in term of plasma frequency. It describes response of external field. The imaginary part $\varepsilon(\omega)$ represents the absorption of electromagnetic material.

"A charateristic frequency at which material changes to dielectric response is called Plasma frequency".



(4) Drive an expression for optical properties of metal electron concentration and constant relaxation τ. How are the real and imaginary part ε_1 and ε_2 related to conductivity?

Solution:-

To derive expression for complex dielectric (ε) for metal concentration on time τ. we start with Drude model equation:-

$$m\frac{d^2x}{dt^2} + m\gamma\frac{dx}{dt} + m\omega_p^2 x = -eE(t)$$

Assuming the solution of this equation

has form $r(t) = r_0 e^{-i\omega t}$ where r_0 is complex amplitude, we can find Polarization as:-

$$P = -Ner$$

where N is number density

The complex dielectric ε is defined as ratio of induced Polarization to electric field E :-

$$\varepsilon = 1 + P/E$$

Subsitute

$P = -Ner$ and $E = E_0 e^{-i\omega t}$ we get

$$\varepsilon = 1 - \frac{Ne^2 x}{m\omega_p^2 x + m\gamma \frac{dx}{dt} + m\frac{d^2x}{dt^2}}$$

$$\varepsilon = 1 - \frac{Ne^2}{m} \cdot \frac{1}{\omega_p^2 - \omega^2 - i\gamma\omega}$$

Above expression is for metal in terms of ω_p, angular frequency and damping cofficent γ.

Now, let's express ε in terms of real and imaginary an real part

$$\varepsilon = \varepsilon_1 + i\varepsilon_2$$

$$\varepsilon_1 = 1 - \frac{Ne^2}{m} \frac{\omega_p^2 - \omega^2}{(\omega_p^2 - \omega^2)^2 + (\gamma\omega)^2}$$

$$\varepsilon_2 = \frac{Ne^2}{m} \frac{\gamma\omega}{(\omega_p^2 - \omega^2)^2 + (\gamma\omega)^2}$$

Now relate it with conductivity

$$\sigma = \frac{\varepsilon_2 \omega}{4\pi}$$

Therefore we have

$$\varepsilon_2 = \frac{4\pi\sigma}{\omega}.$$

At simple optical property we have

N = n + ik

complex → N, real part → n, imaginary part → k

5) Discuss free carrier absorption in the case of metals and semiconductors?

Solution:-

FREE CARRIER ABSORPTION FOR SEMICONDUCTORS

:- For free carrier absorption we use relation for complex dielectric

$$\varepsilon(\omega) = \varepsilon_1(\omega) + \varepsilon_2(\omega).$$

$$\varepsilon(\omega) = \varepsilon_0 + \frac{4\pi i\sigma}{\omega}$$

The electronic polarizability is related to frequency dependent electrical conductivity by frequency dependent Drude term

$$\sigma = \frac{ne^2\tau}{m^*(1 - i\omega\tau)}$$

The plasma freq as

$$\omega_p^2 = \frac{4\pi ne^2}{m^*\varepsilon_0}$$

For frequency of semiconductors it is true generally $\omega\tau >> 1$. we can write:-

$$\varepsilon(\omega) = \varepsilon_0 + \frac{4\pi i n e^2 \tau(1 + i\omega\tau)}{m^*\omega[1 + (\omega\tau)^2]}$$

$$= \varepsilon_0 + \frac{i\varepsilon_0\omega_p^2\tau(1 + i\omega\tau)}{\omega[1 + (\omega\tau)^2]}.$$

or for $\omega\tau >> 1$

$$\varepsilon(\omega) = \varepsilon_0 + \frac{i\varepsilon_0\omega_p^2\tau^2}{\omega^3\tau^3} - \frac{\varepsilon_0\omega_p^2}{\omega^2}$$

So real part of dielectric is:-

$$\varepsilon_1(\omega) = \hat{n}^2(\omega) - \hat{k}^2(\omega) \simeq \varepsilon_0$$

Now imaginary part for semiconductor

$$\varepsilon_2(\omega) = 2\hat{n}(\omega)\hat{k}(\omega) \simeq 2\sqrt{\varepsilon_0}\,\hat{k}(\omega)$$

$$= \frac{\varepsilon_0 \omega_p^2 \tau^2}{\omega^3 \tau^3}$$

which is small, since $\omega_p \ll \omega$. Thus absorption coefficent can be written as:-

$$\alpha_{abs}(\omega) = \frac{2\omega\hat{k}(\omega)}{c}$$

$$" \simeq \frac{2\omega}{c} \frac{\varepsilon_0 \omega_p^2}{2\sqrt{\varepsilon_0}\,\omega^3 \tau}$$

$$\alpha_{abs}(\omega) = \frac{\sqrt{\varepsilon_0}\,\omega_p^2}{c\omega^2 \tau}.$$

The dependence of plasma frequency on carrier concentration is really visible from these data.

FREE CARRIER CONCENTRATION FOR METALS:-

The typical limits for metals are somewhat different than semiconductors In particular we consider here $\omega\tau \ll 1$, $\omega \ll \omega_p$ $|\varepsilon_0| \ll 4\pi\sigma/\omega$ so that $\hat{n} \simeq \hat{k}$. Thus we obtain

$$\varepsilon(\omega) \simeq \frac{4\pi i\sigma}{\omega} \simeq \frac{4\pi i n e^2 \tau}{\omega m^*} \simeq i\varepsilon_2(\omega)$$

$$\equiv 2i\hat{n}\hat{k} \simeq 2i\hat{k}^2.$$

This gives us for extinction cofficent $\hat{k}(\omega)$.

$$\hat{k}(\omega) = \sqrt{\frac{2\pi n e^2 \tau}{m^* \omega}}.$$

So absorption coefficent becomes:-

$$\alpha_{abs}(\omega) = \frac{2\omega\hat{k}(\omega)}{c} = \sqrt{\frac{8\pi\omega n e^2 \tau}{m^* c^2}}$$

In limit appropriate for metals, $\hat{n} = \hat{k}$ both are large so we have

$$R = \frac{(\hat{n}-1)^2 + \hat{k}^2}{(\hat{n}+1)^2 + \hat{k}^2} = \frac{\hat{n}^2 - 2\hat{n} + 1 + \hat{k}^2}{\hat{n}^2 + 2\hat{n} + 1 + \hat{k}^2}$$

$$= 1 - \frac{4\hat{n}}{\hat{n}^2 + \hat{k}^2 + 2\hat{n} + 1}$$

$$R \simeq 1 - \frac{4\hat{n}}{\hat{n}^2 + \hat{k}^2} \simeq 1 - \frac{2}{\hat{n}}.$$

Condition $\hat{n} \simeq \hat{k} \gg 1$ we obtain

$$\hat{n}(\omega) = \sqrt{\frac{2\pi m e^2 \tau}{m^* \omega}}$$

$$R(\omega) \simeq 1 - 2\sqrt{\frac{m^* \omega}{2\pi n e^2 \tau}}$$

This is also known as Hagen-Rubens relation.

(6) Discuss
(i) Direct Inter-band Transitions.
(ii) Indirect Inter-band Transitions.

(i) **DIRECT INTER-BAND TRANSITIONS**

In direct inter-band transitions, an electron is excited from the valence band to conduction band by absorbing a photon and the electron's momentum $\hat{k}$ vector is conserved during the transition. This means electron's energy and momentum are simultaneously changed, and the transition occurs vertically between the bands. Direct transitions occur at specific points in the Brillouin zone, such as Γ point, and are typically observed in direct bandgap materials like GaAs, InP and CdTe.

(iii) **Indirect Inter band transitions:-**

In Indirect inter-band transitions, an electron is excited from valence band to the conduction band but electron's momentum is not conserved and transition occurs horizontally between the bands its moment is not vertically. Indirect transitions occurs at different points in Brillouin zone such as Γ and X points and typically observed in Si, Ge, GaP.

→ In Solid - State Physics, understanding direct and indirect inter-band transitions is crucial for phenomenas as:-

- Optical absorption and emission.
- Electrical conductivity and semiconductivity
- Thermal excitation and relaxation processes.
- Electronic band structure and material properties.

This distinction between direct and in-direct transitions is important for behaviour of solids and designing materials with specific properties for various applications.

$$\alpha = \frac{(\hbar\omega - E_g)^{1/2}}{\hbar\omega}$$

direct allowed gap

$$\alpha = \frac{(\hbar\omega - E_0)^{3/2}}{\hbar\omega}$$

Direct forbidden gap

$$(\alpha\hbar\omega)^2 = \hbar\omega - E_g$$

$$y = mx + c^2$$

$$\alpha = \frac{\hbar\omega - E_g}{\hbar\omega}$$

$n = \frac{1}{2}, 2, \frac{2}{3}, \frac{5}{2}$