(1) How can we express the magnitude of a vector?

Ans: Symbolically, the magnitude of a vector can be represented by light face letter e.g A, d, r etc. Graphically, the magnitude of a vector can be measured by length of a vector according to selected scale.

(2) What is meant by Null vector?

Ans: A vector whose magnitude is zero and has an arbitrary direction is called Null vector. It is represented by $\vec{O}$.

We can obtain the null vector by adding a vector into its negative vector.

$$\vec{A}+\left(-\vec{A}\right)=0$$

(3) If force of magnitude 20N makes an angle of 30° with x – axis then find its y – components?

Ans: F = 20N

$\theta = 30^{\circ}$

F_y =?

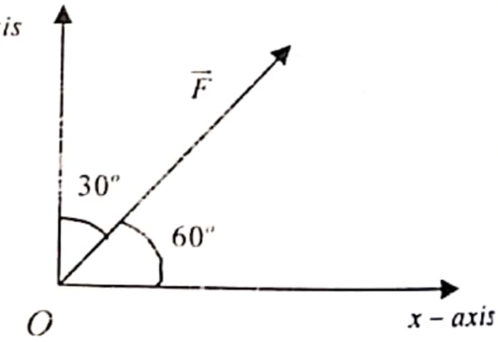
$F_y = F \sin\theta$

$= 20 \sin 30^{\circ}$

$= 20\left(\frac{1}{2}\right)$

$F_y = 10N$

(4) If force $\vec{F}$ of magnitude 10N makes an angle of 30° with y – axis then find its x – component.



Ans: F = 10N

Angle of $\vec{F}$ with x-axis

$\theta = 90^{\circ} - 30^{\circ} = 60^{\circ}$

F_x =?

$F_x = F\cos\theta$

$= 10 \cos (60^{\circ})$

$= 10\left(\frac{1}{2}\right)$

$F_x = 5N$

(5) How can we add the number of vectors $\vec{A}, \vec{B}, \vec{C}$.................. by rectangular components method.

Ans: To determine the resultant, we have to find its magnitude and direction so

$$R = \sqrt{Rx^2 + Ry^2}$$

$$R = \sqrt{(Ax + Bx + Cx + ...)^2 + (Ay + By + Cy + ...)^2}$$

Direction

$$\theta = \tan^{-1}\left(\frac{Ry}{Rx}\right)$$

$$= \tan^{-1}\left(\frac{Ay + By + Cy + ...}{Ax + Bx + Cx + ...}\right)$$

(6) Prove that $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} + \hat{j} - 5\hat{k}$ are mutually perpendicular.

Ans: $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{B} = 4\hat{i} + \hat{j} - 5\hat{k}$

$\vec{A}.\vec{B} = (2\hat{i} - 3\hat{j} + \hat{k}).(4\hat{i} + \hat{j} - 5\hat{k})$

$\vec{A}.\vec{B} = (2)(4) + (-3)(1) + (1)(-5)$

$= 8 - 3 - 5$

$= 0$

Since dot product of two vectors $\vec{A}$ & $\vec{B}$ is equal to zero. So they are perpendicular to each other.

(7) If two vectors $\vec{F_1}$ and $\vec{F_2}$ lie in yz – plane. Then what will be the orientation of $\vec{F_1} \times \vec{F_2}$.

Ans: By using right hand rule the direction of $\vec{F_1} \times \vec{F_2}$ is perpendicular to yz -plane i.e along x – axis

(8) Show that the self dot product of vector $\vec{A}$ is equal to the square of its magnitude.

Ans: The dot product of a vector with itself is called self dot product.

Let $\vec{A}$ be given vector then.

$\vec{A}.\vec{A} = AA\cos\theta \qquad \theta = 0''$

= AA cos (0)

= A^2

(9) What is the geometrical significance of cross product of two vectors.

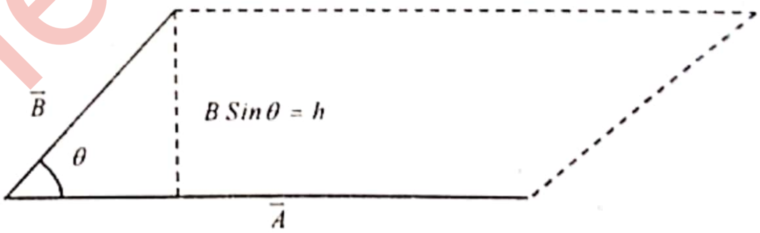
Ans: Magnitude of $\vec{A} \times \vec{B}$ is equal to the area of the parallelogram formed with $\vec{A}$ and $\vec{B}$ as two adjacent sides.

Area of parallelogram = (length)(height)

$= (A)(B\sin\theta)$

$= AB\sin\theta$

$= |\vec{A} \times \vec{B}|$



(10) Write the factors on which the torque depends.

Ans: Torque depends upon three factors.

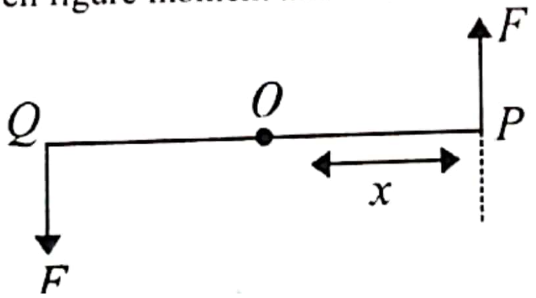
(i) Applied force $\vec{F}$

(ii) Vector $\vec{r}$ from pivot point to point of application of force

(ii) Angle between $\vec{r}$ and $\vec{F}$.

(11) Define the moment arm of a couple.

Ans: The perpendicular distance between the line of action of forces of couple is called moment arm. In the given figure moment arm = $OP = x$



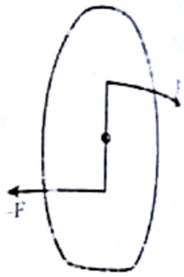
Torque is rotational analogy of force which produces the linear acceleration in a body. The torque acting on a body produces angular acceleration.

(?) What is the rotational analog of force?

Ans: Torque is rotational analogy of force which produces the linear acceleration in a body. The torque acting on a body produces angular acceleration.

(13) Why we need second condition of equilibrium even though the first condition of equilibrium is satisfied for the complete equilibrium?

Ans: When the forces do not act on the same point on the body as shown in figure. Then first condition would be satisfied. But the forces can produce the turning effect. i.e net torque would not be zero. Therefore, so we have to fulfill the second condition to obtain the complete equilibrium.



(14) If $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ and $\vec{b}=\hat{i}+\hat{k}$ then how could they be made parallel to each other?

Ans: Addition of vectors $\hat{i}+\alpha\hat{j}+\beta\hat{k}=u\hat{i}+u\hat{j}$,

Let $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a}=u\vec{b} \Rightarrow \hat{i}+\alpha\hat{j}+\beta\hat{k}=u(\hat{i}+\hat{j})$,

$\Rightarrow 1=u, \alpha=o, \beta=u$ Hence $\alpha=o, \beta=u$ makes $\vec{a}$ parallel to $\vec{b}$.

(15) The resultant of $\hat{a}$ and $\hat{b}$ inclined at $60°$ is what percentage of their maximum resultant?

Ans: If $\vec{R}$ is resultant of $\hat{a}$ and $\hat{b}$ then $R^2=|\hat{a}|^2+|\hat{b}|^2+2|\hat{a}||\hat{b}|\cos 60°$.

Now $R_{max}=2$ is $\frac{R}{R_{max}}=\frac{\sqrt{3}}{2}=0.86$,

$R^2=1+1+1, R=\sqrt{3}, \rightarrow$ Now $R_{max}=2$, So $\frac{R}{R_{max}}=\frac{\sqrt{3}}{2}=0.86$, 50 R is 86% of R_{max}.

(16) If two vectors are parallel and anti parallel what will be their resultant vector?

Ans: If two vectors are parallel, then the resultants is maximum and have the magnitude equal to the sum of the magnitudes of the given parallel vectors. If two vectors are anti parallel, the resultant is minimum and have the magnitude equal to the difference of the magnitudes of the given anti parallel vectors.

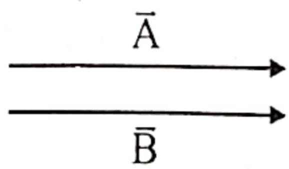
(17) If we have two non-zero vectors $\vec{A}$ and $\vec{B}$ then under what condition the dot product of two vectors will be maximum.

Ans: The dot product of two vectors will be maximum when the angle between them is zero.

$\vec{A}.\vec{B}=AB\cos\theta$

$\vec{A}.\vec{B}=AB\cos(0)$

$\vec{A}.\vec{B}=AB$(maximum value)



(18) What is dot product? Write the formula of K.E in terms of self dot product of velocity vectors.

Ans: The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity, defined as $\vec{A}.\vec{B}=AB\cos\theta$

As kinetic energy of object of mass m and speed v is given by the relation

$K.E=\frac{1}{2}mv^2$

As we know square of magnitude of vector is equal to its self dot product. So

$v^2 = \vec{v}.\vec{v}$

$K.E = \frac{1}{2} m(\vec{v}.\vec{v})$

(19) What does $\frac{\vec{A}\times\vec{B}}{AB\sin\theta}$ represent?

Ans: $\frac{\vec{A}\times\vec{B}}{AB\sin\theta}$ represent the unit vector. Which shows the direction of $\vec{A}\times\vec{B}$.

By definition

$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$

$\hat{n} = \frac{\vec{A}\times\vec{B}}{AB\sin\theta}$

It is unit vector perpendicular to plane containing $\vec{A}$ *and* $\vec{B}$.

(20) What are concurrent and coplanar forces.

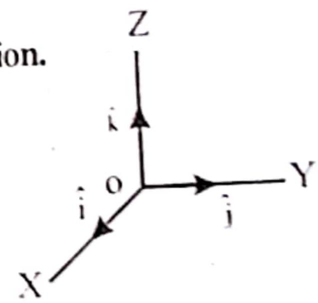
Ans: Concurrent Forces

The force which passes through one point are called concurrent forces.

Coplanar Forces

The forces lying in the same plane are called coplanar forces.

(21) What is the angle between unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$. What are their orientation.



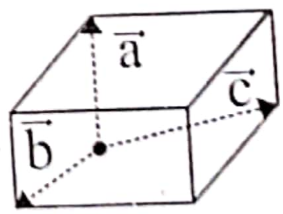
Ans: The unit vectors $\hat{i}, \hat{j}$ *and* $\hat{k}$ are mutually perpendicular.

So the angle between any two given unit vectors is 90°.

The unit vectors $\hat{i}, \hat{j}$ *and* $\hat{k}$ re usually along x-axis,

y – axis and z – axis respectively.

(22) If $\bar{a}, \bar{b}, \bar{c}$ are adjacent sides of a unit cube then what is $\bar{a}.\bar{b}\times\bar{c}$?



Ans: Let us consider unit cube as shown. Hence $\bar{b}\times\bar{c}$ is parallel to $\bar{a}$. So

$\bar{a}.(\bar{b}\times\bar{c}) = \bar{a}.(1)(1)(\sin 90°)\hat{a}$

$= a\,\hat{a}.\hat{a}\,; \because \sin 90° = 1$

$= a\,; \because \hat{a}.\hat{a} = 1$

$\bar{a}.\bar{b}\times\bar{c} = 1 \because \bar{a}$ is a unit vector.

(23) With the help of diagram, show that $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$.

Ans: Consider two vectors $\vec{A}$ & $\vec{B}$ making an angle θ with each other. Direction of product vector is obtained by right hand rule.

Rotate the first vector $\vec{A}$ into $\vec{B}$ through the smaller of two possible angles. This rotation is represented by curling the fingers of stretched right hand placed on the first vector $\vec{A}$, then thumb represents the direction of vector product. The direction of $\vec{A}$ x $\vec{B}$ will be vertically upward as shown in the fig (a).