

# NUCLEAR PHYSICS

→ characteristics of nucleus. is studied.

\* HRK vol 2 → Introductory of Nuclear Physics  
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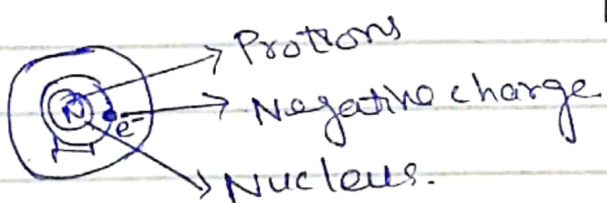
## Nuclear Physics.

↓  
Macroscopic objects.  
(Classical Physics)

Atom  $\sim 10^{-10}$  m

↓  
Microscopic objects.  
(Modern Physics)

Charge on the nucleus is positive



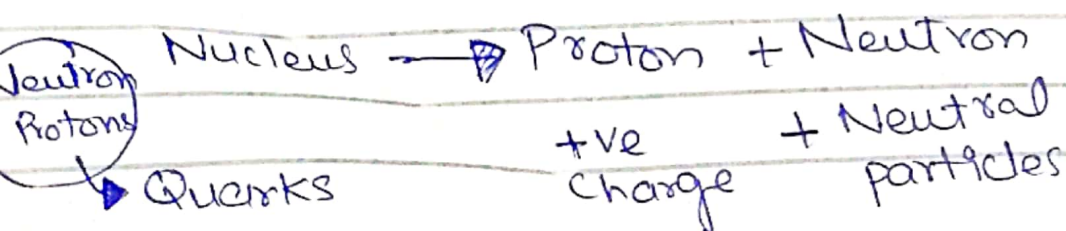
Size of 1 Nucleus. =  $10^{-15}$  m = 1 fm <sup>femto meter.</sup>

→ Black Body Radiation was experiment we switched to Q.M (from CM)

→ Physics after 19 century.

Classical  
Light behave as wave (continuous)

Modern  
Light behave as Particle (Discrete) Photon ( $E = hf$ )





• Nucleus  $\rightarrow$  Proton + Neutron

• Nucleons

[are the particles which are present in the nucleus]

$\rightarrow$  Strong Nuclear forces are present in the nucleus

$$p = h/\lambda$$

$$\lambda = \frac{h}{mv}$$

$\rightarrow$  QM effects are present in microscopic region

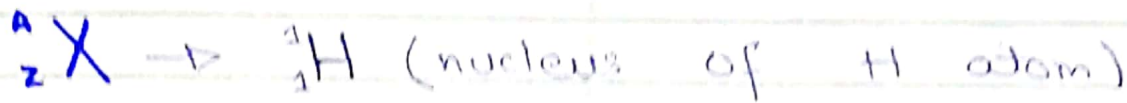
Z is the no. of Protons.

A is the no. of Protons + no. of Neutrons.

$$N = A - Z$$

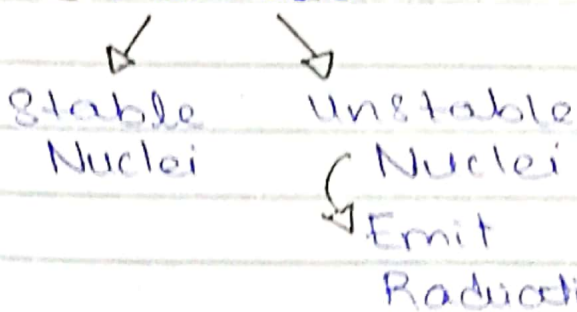
no. of neutrons

Nomenclature:



**ISOTOPES:** are created by change in nucleus (nuclear properties changes)

• **Nucleus**



(Nucleus is unstable by nature)

• If  $Z \geq 82$  emits Radiation.  
(Rutherford discovered nucleus)

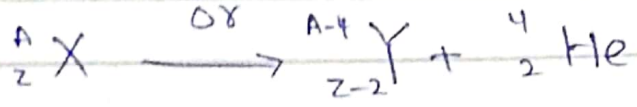
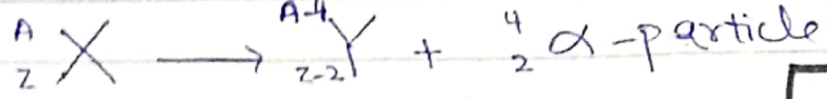
Radioactive radiations

→ Radiations:  $\alpha$ -radiation  
 $\beta$ -radiation  
 $\gamma$ -radiation.

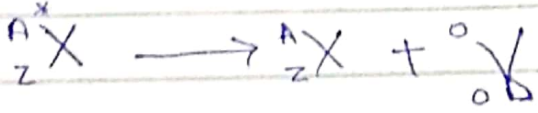
Parent Emit rad → daughter nuclei → Stable (if not emit rad)  
 Unstable (if emit rad)

$\alpha$ -radiations:

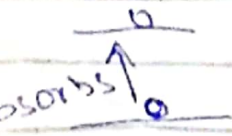
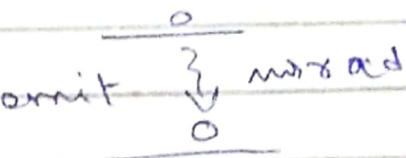
(Helium assemblies  $\alpha$ -radiations)



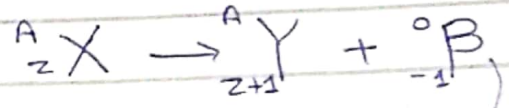
Gamma-Radiations:



Excited State → to → Ground state



$\beta$ -Radiations



assembles the  $e^-$  (but not actually  $e^-$ )

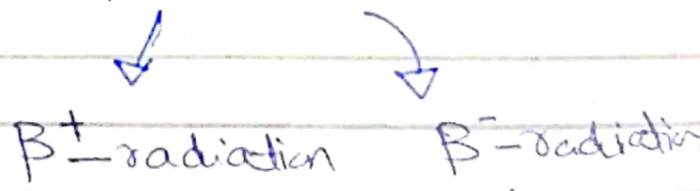
It can be possible.



$\beta^-$  → negative Decay →  $e^-$  (resembles the position)

$\beta^+$  → positive Decay → positron Decay

$\beta$ -radiation



emitted during nuclear radiation.

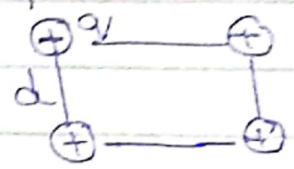
# → NUCLEAR PROPERTIES :

Nucleus is a mysterious object whose properties are much more difficult to characterize than the other macroscopic object. Nuclear characteristics can be described by

- 1) Electric Charge.
- 2) Radius of Nucleus.
- 3) Mass.
- 4) Binding Energy.
- 5) Angular Momentum →  $\vec{P} = m\vec{v}, \vec{L} = \vec{r} \times \vec{p}$
- 6) Parity
- 7) Magnetic dipole and Electric quadrupole moments.

All are **static** properties of nucleus (b/c nucleus is stable)

whereas **dynamic** properties of nucleus includes decay / or radiation and Nuclear Reaction.

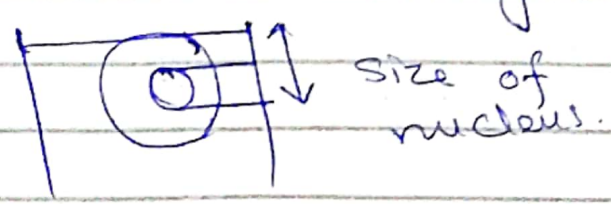


(Fusion & fission) → (Nuclear force strongest force in nature)

## Nuclear Radius:

is determined by the experiment.

- (i) High energy electron scattering
- (ii) Muonic X-ray  $\sim 10^{-10}$  m
- (iii) Optical and X-ray shift.



→ Intensity of deflection tells us about charge concentration.

→ Through coulomb interaction of a charged particle beam with nucleus will determine the distribution of charge present on the nucleus.

{ Rutherford scattering,  $\alpha$ -decay and X-rays scattering }

→ Radiations used to study nucleus properties of very small wavelength.

Range of wavelength  $\lambda \leq 10\text{fm}$   
and  $E = hc/\lambda$   $\lambda \downarrow$  high energy radiations

Required for  
study of nucleus  
beam  $\approx 100\text{MeV}$

No. of nucleons  $\approx$  Constant  
Volume. factor

① Nucleons have a constant distribution out to the surface.

② No. of nucleons per unit volume is roughly constant.

$$\frac{A}{\frac{4}{3}\pi R^3} \approx \text{Constant.}$$

$$A \approx \text{constant } \frac{4}{3}\pi R^3$$

$$A^{1/3} = \text{constant } R.$$

$$R \sim 1/\text{const} A^{2/3}$$

$$R \sim A^{2/3}$$

As  $A \uparrow$  also Radius  $\uparrow$ . no of nucleons  $\uparrow$

$$R = R_0 A^{2/3}$$

where  $R_0 = 1.2 \text{ fm}$ .

$$R = R_0 A^{2/3}$$

Find rad of Hydrogen ?

$$R_H = 1.2 \text{ fm}$$

rad of Helium.

$$R_{He} = R_0 (4)^{2/3}$$

$$R_{He} = (1.2)(4)^{2/3} \text{ fm}$$

## Nuclear Mass:

Mass of nucleus

$$m_{\text{nucleus}} = Zm_p + Nm_n$$

$\swarrow$  mass of single proton  $\searrow$  mass of single neutron

if  $Z = 3$  ,  $N = 2$

$$m_{\text{nucleus}} = 3m_p + 2m_n$$

$$1.6 \times 10^{-27} \text{ kg} = 5 \times 1.6 \times 10^{-27} \text{ kg}$$

$$m_n = 5 \times 1.6 \times 10^{-27} \text{ kg}$$

# NUCLEAR PROPERTIES

## Static Properties

Nuclei is stable  
(No Reactions)  
if - than able  
to measure properties.

## Dynamic Properties.

Nuclei is unstable

Nucleus Radiations  
and  
Nucleos Reactions.

Electric charge on Nucleus.  
Mass of Nucleus.  
Radius of Nucleus.

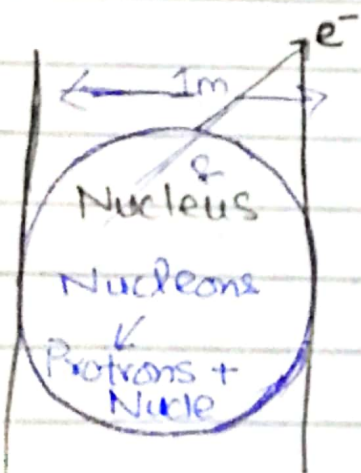
## Nuclear Reactions

Fusion + Fission.

$$Z \quad 1e = 1.6 \times 10^{-19} \text{ C}$$

\* Coloumb force  
repulsion present  
inside while outside  
coloumb force of  
attraction.

\* When Z number  
increase charge  
on nucleus  
increases.



Lighter nuclei  $\rightarrow$  Heavier Nucleus  
 $Z=1$   $U=$   $Z=92$

Charge on nucleus =  $Ze$   
 $= 92 \times 1.6 \times 10^{-19} \text{ C} =$  \_\_\_\_\_

\* Nucleons & charge on nucleus, hence binding energy increases.

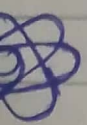
\*  $A = \text{No of protons} + \text{No of Neutrons}$

$A = Z + N$   
 $N = A - Z$

$\rightarrow$  For finding the no of neutrons, we have to find

$M_{\text{nucleus}} = Zm_p + Nm_n$   
 $m_p = m_n = 1.67 \times 10^{-27} \text{ kg}$

proton and multiply with corresponding mass?



$R = R_0 A^{1/3}$   
 $R_0 = 1.2 \text{ fm}$

\*  $\otimes$   $\lambda$  shorter than wave scale.

- \* Angular Momentum
- \* Parity
- \* Magnetic dipole
- \* Electric quadrupole moment

\*  $\otimes$  Electron can't be in nucleus if electron waves in nucleus than uncertainty principle can't be speed of light approaches at particles spot which is in fact

can't be inside

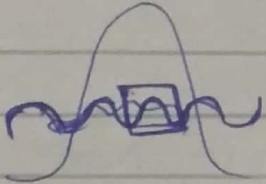


$$E = hf$$

$$E = \frac{hc}{\lambda}$$

$$\lambda \downarrow = E \uparrow$$

Radiations energy  $\approx 100 \text{ MeV}$



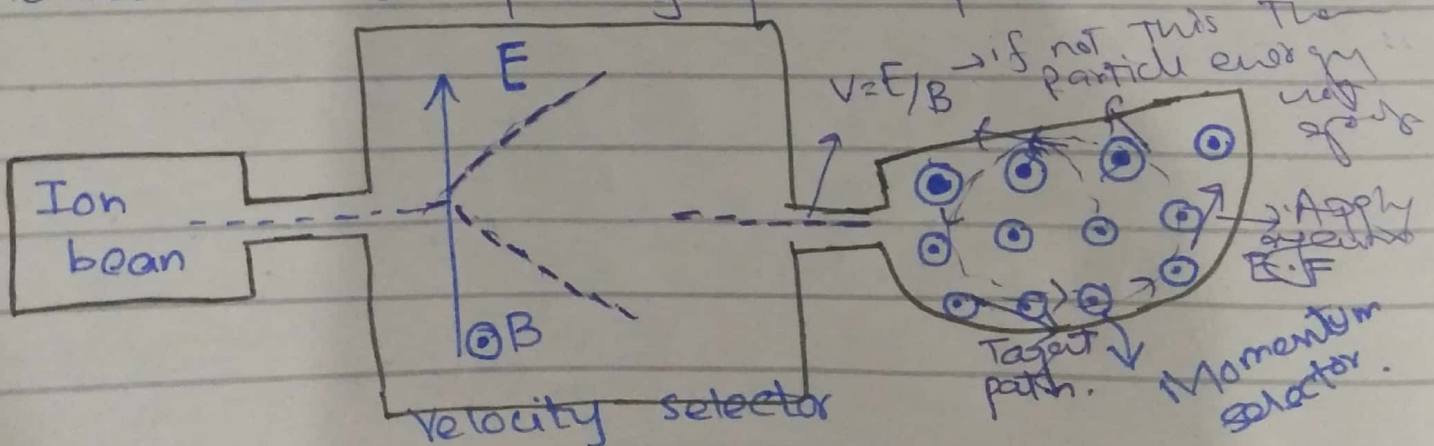
## Mass Spectrometer:

⊗ Mass spectrometry is the first technique of high precision available to the experimentalists to determine some of the static properties of Nucleus.

⊗ Nucleus properties are determined from nucleus isotopes to determine nucleus abundances and this nucleus mass.

Neutron no charge then nucleus changes

⊗ Mass spectroscopy (are injected with mixture of different isotopes) that are separated due to different masses on a photographic plate



\* E & B are perpendicular to each other

$$\vec{F}_e = q\vec{E}$$

Mass spectroscopes begin with an ion source which produces a beam of ionized atoms or molecules. or material under study is

$\vec{F}_e = q\vec{E} \rightarrow \textcircled{1}$   
 $\vec{F}_B = q(\vec{v} \times \vec{B}) \rightarrow \textcircled{2}$   
 $" = qvB \sin 90^\circ \rightarrow \textcircled{3}$

Net force on the incident ion beam is zero when  $qvB = qE$  (compare  $\textcircled{1}$  and  $\textcircled{3}$ )

$V = E/B$

Repeat after below line

bombarded with electrons to produce the ions or ions can be formed as a result of spark discharge between electrode.

① Ions emerging from source have broad range of source velocity  
 ② Velocity selector we apply E and B field  $\perp$  to each other

$$\vec{F}_e = q\vec{E}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}_0$$

$$\vec{F}_e = qvB_0 \sin 90$$

$$\vec{F}_e = qvB_0$$

Ion beam will move undeflected only if the net force on the ion particle is zero

$$\frac{eE}{F_{net}} = eVB_0$$

$$F_{net} = \vec{F}_e + \vec{F}_B$$

$$|\vec{F}_e| = |\vec{F}_B|$$

$$v = E/B_0 \rightarrow (1)$$

Final element is momentum selector which is electrically a uniform B field that bend the beam with circular path with radius  $r$  determined by momentum.

$$F_e = F_B$$

$$\frac{mv^2}{r} = qvB \sin 90^\circ$$

$$\frac{mv}{r} = qB \longrightarrow r = \frac{mv}{qB} \rightarrow (3)$$

$$mv = qBr$$

$$v = r\omega \rightarrow (2)$$

$$m r \omega = q B r$$

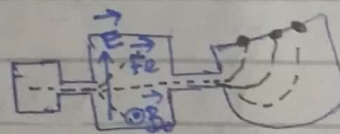
$$m \omega = q B$$

putting (1) in (3) eq

$$r = \frac{m E}{q B_0 B}$$

$$\text{If } B_0 = B$$

$$r = \frac{m E}{q B^2}$$



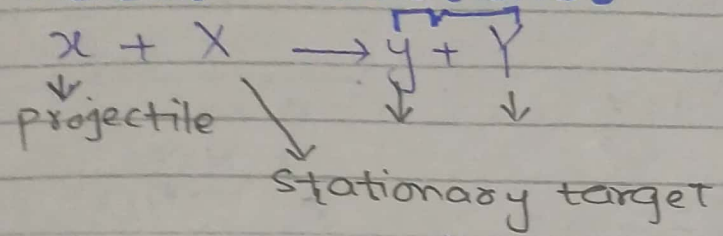
Mass spectrometer is a device which different nuclei on bases of their masses.

$q$ ,  $E$  and  $B$  are determined through experimental

Nucleus dynamic and static properties

setup Abundance more  $r\hat{I}$  than  $m\hat{I}$

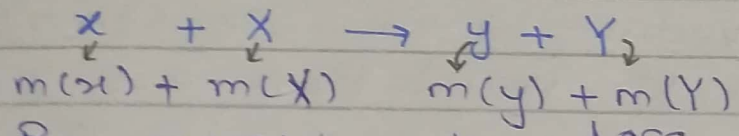
### \* Nuclear Reactions :-



Projectile initiate any nuclear reaction by hitting nuclei.

When we are studying the Nuclear Decays Projectile in that case is not required.

Masses for each in reaction:



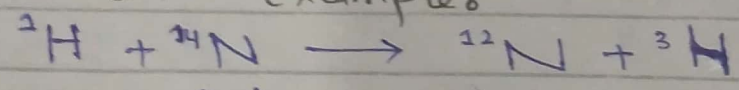
Strongest force is nature use Nucleus while their reaction large amount of abundance and energy is released.

By measuring the difference of masses in any nuclear reaction we find the Q value for reaction.

$$Q = \Delta m c^2$$

$$Q = \{m(x) + m(X)\} - \{m(y) + m(Y)\} c^2$$

Reaction Example:



$$m({}^1\text{H}) = 1.007825 \text{ u}$$

$$m({}^{14}\text{N}) = 14.003074 \text{ u}$$

→  $u = \text{Atomic mass unit}$

$$m({}^3\text{H}) = 3.016049u$$

Mass can be defined in terms of energy in nuclear physics.

$$1u = \frac{931 \text{ MeV}}{c^2}$$

$$Q = -22.1355 \pm 0.0010 \text{ MeV}$$

↓  
-ve means energy released must be smaller than original value.

$$E = mc^2$$

$u = \text{MeV}$   
 $\frac{u}{c^2} = \text{MeV}/c^2$   
Energy unit  
 $\frac{u}{c^2}$  by  $c^2$  we get mass unit

→ if mass or  $Q$  value given with some error then result must have an error value ( $\pm$ )

$$\frac{Q}{c^2} = \{m(\alpha) + m(X)\} - \{m(Y) + m(\gamma)\}$$

$$\frac{Q}{c^2} = m(\alpha) + m(X) - m(Y) - m(\gamma)$$

$$m(Y) = m(\alpha) + m(X) - m(\gamma) - \frac{Q}{c^2}$$

$$m(Y) =$$

$$\frac{931 \text{ MeV}}{c^2} + \frac{931 \text{ MeV}}{c^2} - \frac{931 \text{ MeV}}{c^2} - \left( \frac{-22.1355 \text{ MeV} \pm 0.0010 \text{ MeV}}{c^2} \right)$$

\* not divide  $c^2$  with error and  $Q$  term so that all units will be in same and it will not be complex

\* In this course don't put value of  $c^2$  in any numerical ~~as~~ as it is eliminated by other quantities

$$E = \Delta mc^2$$

Mass spectrometry is used when we have stable nuclei whereas  $E = \Delta mc^2$  is used in all Nuclear Reactions. Either stable or unstable nuclei.

\* Sometimes the end product nuclei is unstable so it converts to other radioactive nuclei.

### ■ Nuclear Binding Energy:

\* In atom we have nucleus with  $e^-$  revolve around nucleus. Atom is bounded because of Coulomb force of Attraction

$$* m_n = Zm_p + (A-Z)m_n$$

Proton  
Neutrons

If you know  $m_n$  then simply  $E = m_n c^2$  this  $E$  is Energy of nucleus with which nucleons are binded with each other.

$$m_N \neq Z m_p + (A-Z) m_n.$$

This is not equal because proton has repulsive force.

$$m_N c^2 = (Z m_p + (A-Z) m_n) c^2 + \sum_{i=1}^Z B_i \rightarrow \textcircled{1}$$

\* No of proton increase then  $\sum_{i=1}^Z$  increases.

Binding energy required to bind proton together

\* For heavier nuclei  $B_i$  increases as compared with the lighter nuclei b/c of the increase in Z-number.

eq ① is indicating that.

The B.E of the nucleus is difference in masses of a Nucleus  ${}^A_Z X_N$  and its constituents Z (proton no) & N neutron.

$$B.E = [Z m_p + N m_n - m[X_N]] c^2$$

$$\hookrightarrow B.E \approx 10 - 100 \text{ keV.}$$

For Hydrogen;

$$Z = 1$$

$$N = 0$$

$$\begin{aligned} \therefore m_p &= 1.67 \times 10^{-27} \text{ kg} \\ \therefore 1u &= 1.67 \times 10^{-27} \\ \therefore 1u &= 931 \frac{\text{MeV}}{c^2} \end{aligned}$$

$$B.E = [1u - 1.007825u] c^2 - [(0.007825) 931 \frac{\text{MeV}}{c^2}] c^2$$

$$B.E = 7.285075 \text{ MeV}$$

$$B.E = 7285.075 \text{ keV}$$

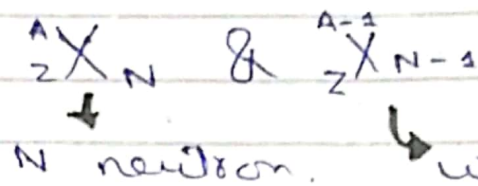
### Neutron & Proton Separation Energy:-

If we have to remove one proton from one



one nucleus how much energy is required? This energy is called Neutron & proton Separation Energy. If we want to take out a neutron energy must be required which must be greater than the B.E energy of the nucleus.

B.E b/w



Binding Energy of  ${}^A_Z X_N = [Zm_p + Nm_n - m({}^A_Z X_N)]c^2$  (1)

" " "  ${}^{A-1}_Z X_{N-1} = [Zm_p + (N-1)m_n - m({}^{A-1}_Z X_{N-1})]c^2$  (2)

$S_n$  is the energy required to remove the neutron from the nucleus

$S_n = B.E({}^A_Z X_N) - B.E({}^{A-1}_Z X_{N-1})$  (3)

using eq (1), (2) ~~in~~ (3)

$S_n = [Zm_p + Nm_n - m({}^A_Z X_N) - Zm_p - (N-1)m_n + m({}^{A-1}_Z X_{N-1})]c^2$

$\therefore p \sim {}^1_2 \text{He}$

$S_n = [-m({}^A_Z X_N) + m_n + m({}^{A-1}_Z X_{N-1})]c^2$

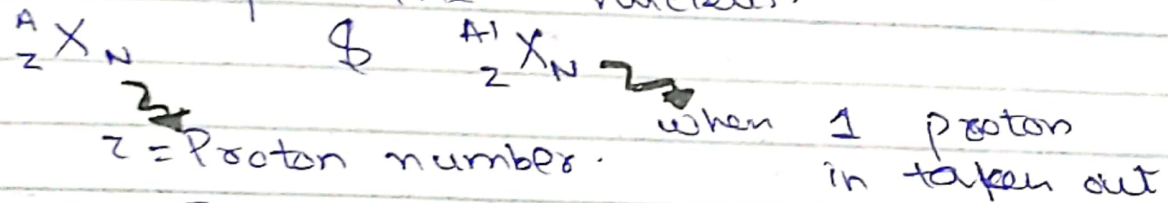
To remove one neutron we need so amount of energy.

To remove 2 or 3 number of neutron for 3 neutron.

$$S_n = \left[ -m \left( {}^A_Z X_N \right) + 3m_n + m \left( {}^{A-3}_Z X_{N-3} \right) \right] c^2$$

### Proton Separation Energy:-

If we want to take out a proton energy must required  $S_p$  must be greater than B.E of the nucleus.



$$B.E \left( {}^A_Z X_N \right) = \left[ Zm_p + Nm_n - m \left( {}^A_Z X_N \right) \right] c^2$$

$$B.E \left( {}^{A-1}_{Z-1} X_{N-1} \right) = \left[ (Z-1)m_p + Nm_n - m \left( {}^{A-1}_{Z-1} X_{N-1} \right) \right] c^2$$

$S_p =$  Energy required to remove the proton from the nucleus.


$$S_p = B.E \left( {}^A_Z X_N \right) - B.E \left( {}^{A-1}_{Z-1} X_{N-1} \right)$$

$$S_p = Zm_p + Nm_n - m \left( {}^A_Z X_N \right) - (Z-1)m_p - Nm_n + m \left( {}^{A-1}_{Z-1} X_{N-1} \right) \Big] c^2$$

$$S_p = \left[ -m \left( {}^A_Z X_N \right) + m_p + m \left( {}^{A-1}_{Z-1} X_{N-1} \right) \right] c^2$$

## Quantum theory of Angular Momentum:

The orbital quantum number  $l$  play a very important role. In atomic physics it serve to label different electron wave functions that describes electron wave functions that describe the spatial behaviour of wave function.

In classical physics the angular momentum of a particle moving with linear momentum is  $\vec{l} = \vec{r} \times \vec{p}$    $\vec{p} = mv$

In quantum mechanics we evaluate the expectation values of angular momentum by:-

$$\langle f \rangle = \int \psi^* f \psi dx$$

We first consider the magnitude of the angular momentum  $l$  for this purpose. For this purpose it is simplest to

calculate  $\langle L^2 \rangle$

$$\langle L^2 \rangle = \int \psi^* L^2 \psi d\tau$$

$$\hat{P}_x = -i\hbar \frac{\partial}{\partial x} \quad ; \quad \hat{P}_y = i\hbar \frac{\partial}{\partial y} \quad ; \quad \hat{P}_z = i\hbar \frac{\partial}{\partial z}$$

Evaluate the cross product

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\hat{L}_x = ? \quad ; \quad \hat{L}_y = ? \quad ; \quad \hat{L}_z = ?$$

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$\begin{aligned} \hat{L}_x &= y p_z - z p_y \\ \hat{L}_y &= z p_x - x p_z \\ \hat{L}_z &= x p_y - y p_x \end{aligned}$$

Total angular momentum

$$\langle L^2 \rangle = \langle \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 \rangle$$

$$\psi = R(r) Y_{lm}(\theta, \phi)$$

$\downarrow$                        $\downarrow$   
 Radial part / linear part      Angular part

quote after that line

When a particle move in central potential then its wave function will not have linear dependence it only have angular dependent

The magnitude of angular momentum is fixed

$$\langle L^2 \rangle = \hbar^2 l(l+1)$$

Angular momentum describe by orbital quantum number  $l$  is used to find the state of the particle.

\* Nucleus only have spin motion

\* Electrons, Atom have spin and orbital motion

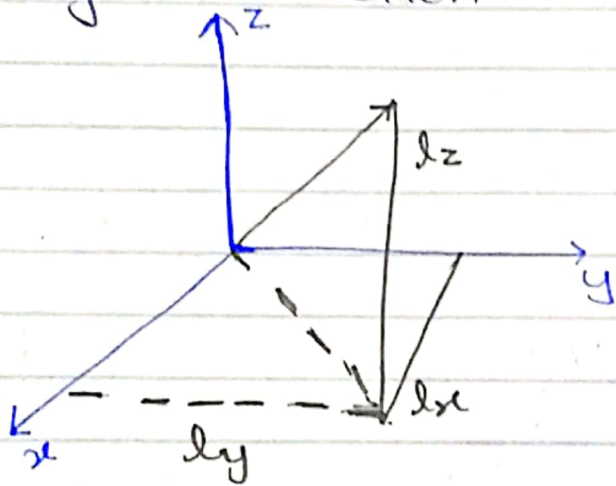
$l=0$  [state - s]

$l=1$  [p-state]

$l=2$  [d-state]

∴  $l = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$   
state = s      p      d      f      g      h      i

• Quantum mechanics permit us to know only one component of  $l$ . If  $l_x$  is found  $l_y$  and  $l_z$  completely indetermined. So by convention we choose  $l_z \rightarrow z$ -component of  $l$



$$\langle l_z \rangle = \hbar m_l$$

$m_l = \text{magnetic quantum \#} = 0, \pm 1, \pm 2, \pm 3, \dots, \pm l$

$l = 2 \quad m_l = 0, \pm 1, \pm 2$   
(5 values)

$$\langle l_z \rangle = \langle | \langle l \rangle |$$

$$\langle l \rangle = \hbar \sqrt{l(l+1)}$$

→ Intrinsic quantum # also known as spin quantum no.

\* For the complete description of an electronic state in an atom we require the introduction to new quantum no.

Intrinsic angular momentum  $\downarrow$  spin

Nucleus  $\rightarrow$  Nucleus (spin proton motion  $\frac{1}{2}$ )

$$S = \frac{1}{2} \text{ (constant value)}$$

$$\uparrow \downarrow \pm \frac{1}{2}$$

$$\uparrow \pm \frac{1}{2}$$

$$\downarrow - \frac{1}{2}$$

$$\langle S^2 \rangle = \hbar^2 S(S+1)$$

$$\langle S_z \rangle = \hbar m_s$$

$$m_s = \pm \frac{1}{2}$$

• Total Angular momentum ( $j$ ):-

$$j = l + s$$

\*  $l = 1$  p-state

$$\langle j^2 \rangle = \hbar^2 (j+1)$$

$$\langle j_z \rangle = \hbar m_j$$

$$m_j = m_l + m_s$$

expectation value of  $j$

$$j = 1 \pm \frac{1}{2}, \frac{3}{2}, \frac{1}{2}$$

p state  $\rightarrow$   $p_{3/2}$

$\downarrow$   $p_{1/2}$

• There are two types of coupling b/w spin and angular momentum.

(i) jj coupling.

$$J = j_1 + j_2 + \dots$$

$$j_1 = l_1 + s_1$$

$$j_2 = l_2 + s_2$$

(ii) L-S coupling.

$$L = l_1 + l_2 + \dots = \sum_{i=1}^n l_i$$

$$S = \sum_{i=1}^n s_i$$

## • Nuclear Angular Momentum:

If we have a nucleus in which 8 nucleons exist then vector sum of all the nucleons will give us the angular momentum of nucleus.

$$I = S_p + S_n + L$$

$S_p$  → Isospin  
 $S_n$  → Spin of Protons and neutrons (upward or downward)  
 $L$  → orbital angular momentum of nucleus as they move about the common centre of mass

$S_p$  &  $S_n$  can be  $\pm 1/2$

$$I^2 = \hbar^2 I(I+1)$$

$$I_z = m\hbar \quad (m = -I \dots I)$$

$2I + 1$  value

If we apply ordinary magnetic field the nucleus splits into different states as in case of atomic physics (Zeeman effect) but we need very, very strong B field for such splitting but such high B. field does not exist but if splitting occurs.

Example:-

${}_{9}^{19}\text{F}$   
 9 odd  
 10 even

Proton	Neutron
$\uparrow$	$\uparrow$
$\downarrow$	$\downarrow$
$\downarrow$	$\uparrow$
$\downarrow$	$\downarrow$
$\downarrow$	$\downarrow$

$$I = S_p + S_n$$

Ch #1 Review

Ch #12 → 2.5 Quantum Theory

2.6 Parity. (Study)

Ch #3 → Nucleus Radius, Charge.

## ■ Parity:

Parity operation causes the replacement of co-ordinate with its -ve co-ordinates. It causes the reflection of all the co-ordinate through the centre.

$$\begin{cases} x \rightarrow -x \\ y \rightarrow -y \\ z \rightarrow -z \end{cases}$$

(Cartesian)

$$\begin{cases} r \rightarrow r \\ \theta \rightarrow \pi - \theta \\ \phi \rightarrow \phi + \pi \end{cases} \quad (r, \theta, \phi)$$

(Spherical)

• If a system/wave function on which the parity operator. If it remain unchanged then we expect that none of the observable properties describing the system have changed as a result of reflection.

$$V(x) = V(-x)$$

$$|\psi(x)|^2 = |\psi(-x)|^2$$

$$|\psi(x)| = |\psi(-x)|$$

$$|\psi(x)| = \pm \psi(-x)$$



IF  $\psi(-r) = \psi(r)$  then it is even parity  
- IF  $\psi(r) = -\psi(r)$  odd parity.

Mixed parity doesn't exist. either you have odd or even parity.

In spherical coordinates.

$$Y_{lm}(\pi - \theta, \phi + \pi) = (-1)^l Y_{lm}(\theta, \phi).$$

$$V(r) \neq V(-r).$$

$$|\psi(r)|^2 = |\psi(-r)|^2$$

Parity is conserved in nuclear reaction its conservation.

In 1957  $\beta$ -decay is failed.

\* Parity convert into Beta decay. after Beta experiment

Lecture

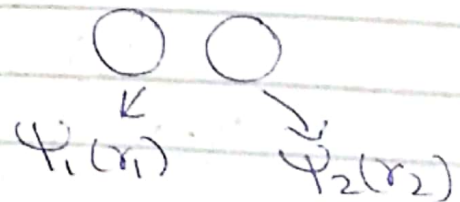
## Quantum Statistics:-

if we have two protons one with wavefunction  $\psi_1$  and another with wavefunction  $\psi_2$ .

Combined effect

$$\psi = \psi_1(r_1) \psi_2(r_2)$$

If these particles are interchanged then how we distinguish them and what



③  $1/2$  → No identical  
fermions

will be their combined wave function.

\* IF Probability density is invariant under this interchange that means we have identical particles.

Combined wavefunction.

$$\Psi_{12} \rightarrow \Psi_{21}$$

$$\Psi_{12} \rightarrow -\Psi_{21} \quad (\text{Not identical particles})$$

\* All combined wavefunctions representing identical particles must be either completely symmetric or completely Antisymmetric. No Mixed symmetry is allowed.

$$\Psi_{12} = \frac{1}{\sqrt{2}} [\Psi_1(x_1)\Psi_2(x_2) \pm \Psi_2(x_1)\Psi_1(x_2)]$$

\* All particles with integral spin (0, 1, 2, ...) have symmetric wave function whereas particles with half integral spins ( $1/2$ ,  $3/2$ ,  $5/2$ , ...) antisymmetric wave function.

2.7 Article from book

2.8 Article (Not included).

→ Problems only from 3<sup>rd</sup> Chapter.

→ Exercise Examples of 1<sup>st</sup> & 2

Chapter. (Mass spectrometers problems).

Properties Mentioned on Page # 80. <sup>imp</sup> Understanding of Deuteron imp in Nuclear Physics

## Ch # 4: The force b/w the nucleons

\* The force with which the nucleons (protons & neutrons) are bound is known as Nucleon.

Nucleon force

\* Nuclear force is the force that results b/c of nucleon-nucleon force.

### Properties:-

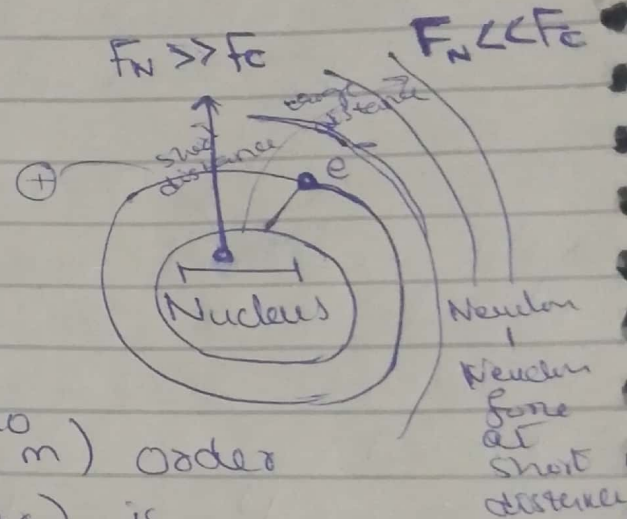
1.  $\rightarrow$  At short distance it is stronger as compared to the Coulomb force (force among protons).

$\rightarrow$  At long distance ( $\sim 10^{-10}$  m) order of nuclei this force (nuclear) is dominating as compared to the Coulomb force of attraction.

2.  $\rightarrow$  At large distance nuclear force become weak and it  $10^{-10}$  m decreases as the distance from the nucleus increases.

3.  $\rightarrow$

- (N-N)
- \* Nucleon - Nucleon/Nuclear force is independent of charge.
  - \* Nucleon - Nucleon force is spin dependent. (Spin are parallel or Antiparallel)





- Up →  $\uparrow$  spin
- Down →  $\downarrow$  spin

④ Nucleon-Nucleon force includes a repulsive term which keeps the nucleus at a certain distance/average separation.

⑤ The nucleon-nucleon force has non-central or tensor component, as it is non-central so

→ Orbital Angular Momentum is R Dependent

Orbital Angular Momentum in case of nucleon-nucleon force/Nucleon force

$$mvr = n\hbar$$



In hydrogen ~~atom~~ we have

1 proton

1 electron.

hydrogen nucleus.

1 proton + 1 neutron

Simplest in periodic table.

Now - this is not hydrogen atom  
this is isotope of hydrogen

${}^2\text{H}$  ~ Deuteron

1 neutron + 1 proton.

• Nuclear Physicist → Deuteron b/c it is simplest nucleus to understand nucleon-nucleon interaction.

\* Deuteron

↳ No excited state  
existence.

### ■ Binding Energy - (For deuteron)

\* The B.E of deuteron is measured precisely which can be determined in three different ways

\* By spectroscopy - we directly measure / determine the mass of deuteron.

$$B.E = \Delta mc^2$$

$$B.E = [m(p) + m(n) - m(^2H)]c^2$$

$$B.E = [m(^1H) + m(n) - m(^2H)]c^2$$

⊕ End of book Table Mass of  
Nuclear Properties.

$$B.E = [1.0078254 + 1.008665 - 2.014102]c^2$$

B.E for deuteron :- 2.223228 MeV.

Spectroscopy :-

$$B.E = 2.2463 \pm 0.00004 \text{ MeV.}$$

(Theoretically  
determined  
value)

Photo

Lec #

## Methods for Finding Binding Energy:-

Three different techniques for finding Binding Energy values.

- (i) Spectroscopy. (previous page)  $\rightarrow 2.22463 \pm 0.0004 \text{ MeV}$
- (ii) Mass doublet Method.
- (iii) Photodissociation.

(i)

$$m(\text{C}^{12}\text{H}^{12}) = m(\text{C}_6\text{D}_6) = (9.289710 \pm 0.000024) \times 10^3 \text{ u}$$

$$m(\text{C}_5\text{-D}_{12}) - m(\text{C}_6\text{D}_6) = (84.616626 \pm 0.000076) \times 10^3 \text{ u}$$

From the 1<sup>st</sup> difference we find by using  $^1\text{H}$  mass equals to  $1.007825037 \text{ u}$ .

$$\rightarrow m(^1\text{H}) = 2.014101789 \pm 0.000000021 \text{ u} \quad \left. \begin{array}{l} \text{D} \rightarrow ^2\text{H} \\ ^1\text{H} \end{array} \right\}$$

$$\rightarrow m(^2\text{H}) = 2.014101771 \pm 0.00000015 \text{ u}$$

(ii)



$$\begin{aligned} \text{B.E.}(^2\text{H}) &= \Delta mc^2 - \gamma \\ &= 2.224 \pm 0.002 \text{ MeV.} \end{aligned}$$

+ Very less error observed in Mass Spectroscopy.  
So Best Method for Determination is Mass Spectroscopy for mass of Deuteron.

## Spin and Parity of Deuteron:

The total angular momentum  $I$  of the Deuteron should have three components.

- \* Spin of Proton  $S_p$
- \* Spin of Neutron  $S_n$
- \* Orbital Angular momentum  $l$

$$I = S_p + S_n + l$$

Measured Spin of Deuteron is 1

$$I = 1$$

$S_n$  and  $S_p$  can be either parallel or antiparallel.

$$S_n = \pm 1/2$$

$$S_p = \pm 1/2$$

$$S_p = +1/2 \quad S_n = +1/2 \quad l = 0$$

$$I = S_p + S_n + l$$

$$I = 1/2 + 1/2 + 0 = 1 \quad \left. \begin{array}{l} \text{spin} \\ \text{parallel} \end{array} \right\}$$

$$I = +1/2 - 1/2 + 1 = 1 \quad \left. \begin{array}{l} \text{spin} \\ \text{antiparallel} \\ l = 1 \end{array} \right\}$$

$$I = -1/2 - 1/2 + 2 = 1 \quad \left. \begin{array}{l} \text{parallel} \\ \text{spin } l = 2 \end{array} \right\}$$

$$I = -1/2 + 1/2 + 1 = 1 \quad \left. \begin{array}{l} \text{Antiparallel} \\ \text{with } l = 1 \end{array} \right\}$$

\* Deuteron  
Measured  
Spin is  
1.  
that when  
 $l$  is 1  
for such  
result is  
1

$\uparrow\uparrow$	$l = 0$	$I = 1$
$\uparrow\downarrow$	$l = 1$	$I = 1$
$\downarrow\downarrow$	$l = 2$	$I = 1$

Note

→ Deuteron consist on Integral number = 1.

Book and Table Atomic Mass is usually Nuclear mass

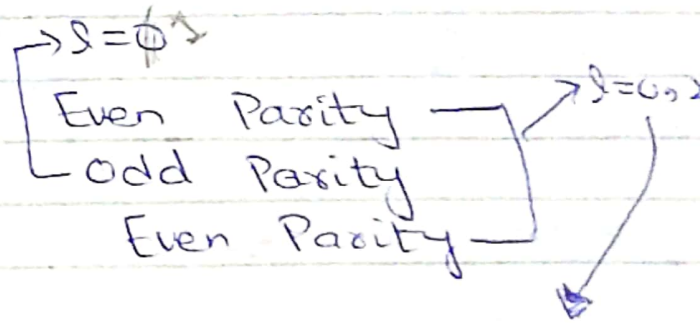
$\gamma \rightarrow -\gamma$  Parity.

Parity is a useful concept both in nuclear physics and Quantum Mechanics. Parity help us to explain the types of function. (either it is symmetric or it is Antisymmetric)

Parity =  $(-1)^l$

$l = 0, 1, 2.$

- If  $l=0$        $P = (-1)^0 = 1$
- $l=1$          $P = (-1)^1 = -1$
- $l=2$          $P = (-1)^2 = 1$



→ It is measured or experimentally determined that deuteron has even parity

→ Should be able to learn book Table (end page) example

Abundance of half life

	Z	A	Atomic Mass (u)	Abundance	half life	$I^{\pi}$
H	1	1	1.007825	99.985%		$1/2^+$
		2	2.014102	0.015%		$1^+$
		3	3.016049	12.3y(18)		$1/2^+$



Conclusion.

Deuteron :-

Simplest, composed of  $1p + 1n$   
even parity,

→ Tunelling effect  
( $h \neq 4$  not included).

- Magnetic Dipole Moment
- Electric Quadrupole Moment

■ Monopole :-

( $q$ ) A single charge moving alone.

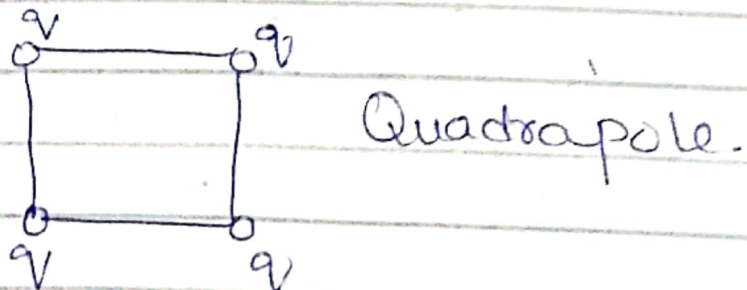
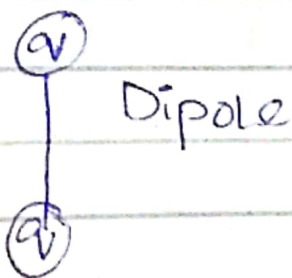
$Ze$

### Angular Momentum

Spin motion of nucleon.

Orbital motion of nucleon.

→ Nucleons present then dipole, monopole can exist



Nucleon spin then

Nucleus is spinning

Both have Electric and Magnetic effects.

→ Magnetic Dipole Moment is represented as  $\mu = iA$

Consider a circular loop current carrying  $i$  and enclosing some area  $A$

$$\mu = iA \rightarrow (1)$$

$$i = q/t = e/t \rightarrow (2)$$

$$S = vt$$

$$t = S/v$$

$$S = 2\pi r$$

$$t = \frac{2\pi r}{v} \rightarrow (3)$$

Using (3) in (2)

$$i = \frac{ev}{2\pi r} \rightarrow (4)$$

Using (4) in (1)

$$\mu = \frac{ev}{2\pi r} A$$

$$A = \pi r^2$$

$$\mu = \frac{ev \pi r^2}{2\pi r}$$

$$\boxed{\mu = \frac{evr}{2}}$$

For circle as proton is moving

X and + by m

$$H = \frac{eVr m}{2m}$$

$$mv r = n h$$

$$V r = \frac{n h}{m}$$

$$L = n h$$

$$H = \frac{eL}{2m}$$

$$n = 1$$

$$H_B = \frac{e h}{2m}$$

Bohr

Magneton.

m = electron.

$H_B$  = Bohr Magneton

$$\text{Nucleon} = H_N = \frac{e h}{2M}$$

(Nuclear Magneton)

$$H_B \gg H_N$$

$$\rightarrow H = 2 H_N$$

H is not non constant b/c it depend on area.

## NUCLEAR ANGULAR MOMENTUM AND SPIN

The total angular momentum of a nucleus containing A nucleons would be a vector sum of the angular momenta of all nucleons.

The total angular momentum is called the nuclear spin represented by I

gahn pa sief charges than  
- vahh ye farada  
- ho ga jahn  
- constant force  
- ho ga vahh  
- tensors hon ge.

$$I^2 = \hbar^2 I(I+1) \quad , \quad I_z = m\hbar \quad , \quad m = -I \dots +I$$

If we have even number of nucleons (so there will be an even number of half integral component that gives total I as an integer.

→ Even A-nuclei I = Integral  
 and for such nuclei which P an odd number of nucleons total I will be half-integral  
 So odd A-nuclei I = half-Integral

Nucleon	
↙	↘
Neutron	Proton
1/2	1/2

→ If we know the wave function of every nucleon we can determine the nuclear parity by multiplying together the particles of each of the A-nucleons.

→ We can end up with a result of  $\bar{\pi}$  that is either positive or negative.

$\bar{\pi}$  Parity:  $\bar{\pi} = \pi_1 \pi_2 \dots \pi_A \rightarrow$  Total parity

Parity can be either positive or negative their is no direct <sup>theoretical</sup> relationship between I and  $\bar{\pi}$ . For any value of I it is possible to have either  $\bar{\pi} = +ve$  or  $\bar{\pi} = -ve$ .

\* Parity is intrinsic property.

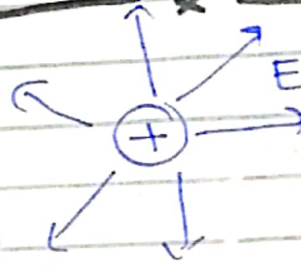
→ Simplest Distribution of charges and current gives only the lowest order of multipole fields. A spherical charge dist. gives only a monopole (Columb) field The

\* Nucleons  $\uparrow$  Size  $\uparrow$  }  $R \propto A^{1/3}$   
 $\downarrow$  } 1-2 fm

higher order fields all vanish

\*  $\rightarrow$  A circular current loop gives only a magnetic dipole field.

As Nucleus is spherized in shape because of this we only have to calculate the lowest order Multipole moments to characterize Electromagnetic properties of the nucleus.



$$E = F/q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

( $E \propto 1/r^3$ )  
 order when dipole observed

For higher order it has larger values. of  $r$

.....  
 Quadropole

$$E \propto 1/r^4$$

Heavy nuclear  $\rightarrow$  Higher Dipoles.

$\rightarrow$  Restriction on multipole moments comes from the spherical symmetry of the nucleus and it is related to the parity of the nucleus state.

Electric Moment Formula. } Parity =  $(-1)^L$  ; \* if  $L=0$   $(-1)^0 = 1$   
 $L=0$  means monopole.

\* if  $L=1$   $(-1)^1 = -1$   
 $L=1$  means Dipole

\* if  $L=2$   $(-1)^2 = 1$  Quadrapole

$\rightarrow$  L. value linked with Multipole.  
 $L = \text{total Orbital Quantum number.}$

→ For Magnetic moment the parity is  $(-1)^{l+1}$  Parity of orbital angular momentum

if $L=0$	$(-1)^{0+1} = -1$	Multipole
$L=1$	$(-1)^{1+1} = 1$	Dipole
$L=2$	$(-1)^{2+1} = -1$	Quadrupole

Form  $M = iA$

$$i = e/vt = \frac{e}{2\pi r/v}$$

$$M = \frac{e m v r}{2m} = \frac{e |L|}{2m} \rightarrow \textcircled{1} \quad \text{Relation of magnetic moment}$$

From eq  $\textcircled{1}$  we conclude that inside nucleus the magnetic dipole moment are produced due to  $L$ .

$$L = m_l \hbar$$

$$m_l = -l, \dots, +l$$

$$M = \frac{e \hbar l}{2m_e} \rightarrow \textcircled{2} \quad \text{Nuclear Magneton} = \mu_N$$

$\mu_B =$   
Bohr  
Magneton

\* if ~~nucleon~~ electron mass is added that Bohr Magneton ( $\mu_B$ )

$$M = e \hbar l / 2m_p \rightarrow \textcircled{2}$$

\* if Proton mass is added than eq  $\textcircled{2}$  will have Nuclear Magneton ( $\mu_N$ )

\* Values

$$\mu_N = 3.1525 \times 10^{-8} \text{ eV/T}$$

$$\mu_B = 5.7884 \times 10^{-5} \text{ eV/T}$$

$$\boxed{\mu_N \ll \mu_B} \rightarrow \text{b/c of difference in masses}$$

$$m_p \gg m_e.$$

In Nuclear and Atomic Physics  $\mu_B$  is dominant so mass of electron is taken while ignoring mass of proton  $\mu_B$  than  $m_e$ ;  $\mu_N$  than  $m_p$ .

→ eq (2) can be written as

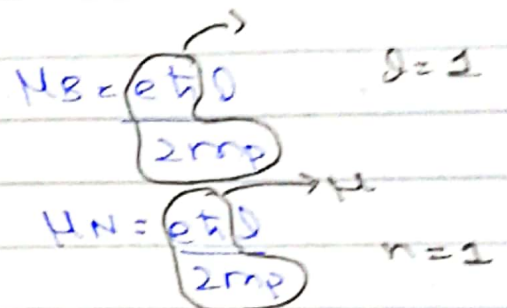
$$\mu = g_L L \mu_N \rightarrow \text{Orbital Angular Momentum}$$

where  $\mu_N$  is Nuclear Magnetron.

$g_L$  is the  $g$ -factor associated with orbital Angular momentum -  $L$

For Proton  $g_L = 1$

\* Neutrons have no. of electric charge  
So  $g_L$  is 0.



→ Spin Magnetic moment

$$\mu = g_S S \mu_N \rightarrow \text{spin magnetic moment}$$

$S = 1/2$  For proton,

Neutron and electron.

→ where  $g_S = 5.5856942 \pm 0.000022$  (for proton)  
 $g_S = 3.8260837 \pm 0.0000018$  (for Neutron)  
 experimental values. ↙ small values.

Nucleus → spin → overall spin → contribution

(neutron + proton spinning together)

So we have minute contribution which is so small.

Theoretically the value of  $g_s = 2$  for proton.  
 $s = 1/2$

Q = Why the difference results in the value of  $g_s = ?$

Protons  $\rightarrow$  quarks (which has own spin)  
Every particle has spin  $1/2$  so we have  
 $g_s = 2$   
Neutron  $\rightarrow$  quark  $\rightarrow$  charge accumulate 0.

\* This difference was first evidence for the existence of quarks.

\* Some other particles are also present (inside) like  $\pi^+$  meson,  $\pi^-$  meson,  $\pi^0$  meson,  $\pi^-$  meson.

Lec:-

Deuteron

$l = 0$  ,  $l = 2$  Even parity.

$l = 1$  odd parity  $\rightarrow$  we didn't take it.

$\rightarrow$  IF  $l = 0$  assumption is correct there should no orbital contribution to the magnetic moment ( $\mu \propto l$ ) and we can assume that the total magnetic moment is simply the combination of the neutron and proton magnetic spin moment.

For Deutrons -

$$\mu = \mu_N + \mu_p$$

$$\mu_s = g_s S \mu_N \quad \{ \text{spin magnetic moment} \}$$



$$M = M_D + M_P$$

$$M = \frac{g_{SN} \mu_N S_N}{\hbar} + \frac{g_{SP} \mu_N S_P}{\hbar} \rightarrow (1)$$

Maximum value of spin is  $+\frac{1}{2}\hbar$

using eq (1)

$$M = \frac{1}{2} g_{SN} \mu_N + \frac{1}{2} g_{SP} \mu_N$$

$$\therefore S_N = \frac{1}{2}\hbar$$

$$M = \frac{1}{2} (g_{SN} + g_{SP}) \mu_N \quad (2)$$

$$S_P = \frac{1}{2}\hbar \text{ max}$$

$$g_{SN} = -3.826084$$

$$S_N = \frac{1}{2}\hbar$$

$$g_{SP} = 5.585691$$

$$S_P = -\frac{1}{2}\hbar \text{ min}$$

$$M = \frac{1}{2} \mu_N (-3.826084 + 5.585691)$$

$$M = 0.879804 \mu_N \rightarrow (3)$$

Experimental value is of magnetic moment for deuteron is  $0.8574376 = 0.0000004 \mu_N \rightarrow (4)$

If we compare eq (3) and (4) we know that the two values are in good agreement with very small discrepancy (which may arise in the present discussion of deuteron because of state  $\{l=2\}$  state).

$\Rightarrow$  wave function for deuteron

$$\psi = a_s \psi(l=0) + a_d \psi(l=2)$$

$a_s$  and  $a_d$  are the constant.

$\Rightarrow$  Calculate its magnetic moment

$$M = a_s^2 M(l=0) + a_d^2 M(l=2)$$

$M(l=0)$  is calculated.

$$M = 0.879804$$

for  $M(l=2) = \frac{1}{4}(3 - g_{sp} - g_{sn})M$ .

gives the calculated value for the state  $(l=2)$  d-state.

$$M(l=2) = \frac{1}{4}(1.240393)M$$

$$M(l=2) = 0.31009825M$$

observed values

$$a_s^2 = 0.96$$

$$a_d^2 = 0.04$$

$$M = 0.96 M(l=0) + 0.04 M(l=2)$$

96%

4%

$l=0$  states contribution = 96%

$l=2$  " " = 4%

In Section 3.5

Lec:

## ■ Electric quadrupole moment:-

It is a parameter which describes the effective shape of the nucleus. The quadrupole moment in C.M. is  $e(3z^2 - r^2) = eQ \rightarrow \textcircled{1}$

\* If a particle moves with spherical symmetry then on the average  $z^2 = x^2 = y^2$  and quadrupole moment vanishes.

⊙ Shape of nucleus is spherical that is why only for spherical symmetry. (Electric quadrupole  $Q=0$ )

\* If a particle moves in a classical orbit say in the xy plane then  $z=0$  so from  $\textcircled{1}$   $eQ = -er^2$

$$\Rightarrow Q = -r^2$$

\* The quadrupole moment in Quantum Mechanics

$$\text{is } eQ = \int \psi^* e(3z^2 - r^2) \psi dV \rightarrow \textcircled{2}$$

For a single proton and for a single neutron  $Q=0$

\* If  $|\psi|^2$  is spherical symmetrical then  $Q=0$

$$\left[ \int z^2 = \frac{r^2}{3} \right] \text{ in eq. } \textcircled{2}$$

\* If  $|\psi|^2$  is concentrated in xy plane. ( $z=0$ ) in eq.  $\textcircled{2}$

$$Q = \int \psi^* (-r^2) \psi dV$$

$$Q = - \int \psi^* r^2 \psi dV$$

$$Q = - \langle r^2 \rangle$$

▶ Here  $\langle r^2 \rangle$  is the mean square radius of the orbit

\* If  $|Y|^2$  is concentrated in the  $z$  plane.

$x=y=0$        $r=z$       then it means

$Q = + \langle 2r^2 \rangle$        $r=z$

\* Nucleons inside the nucleus are distributed in the form of core and valance nucleon

Paired Nucleon  $\rightarrow$  Core  $\rightarrow$  Spherical in Shape.

Paired Nucleon more in spherically symmetric orbits so they do not contribute to  $Q$ .

o Nuclei 9 nucleons.

8 nucleons  $\rightarrow$  core.

1 nucleon  $\rightarrow$  valance nucleon.

► Orbit near the surface of the nucleus.

$r = R_0 A^{1/3}$

$R_0 = 1.2 \text{ fm}$

$A = \text{No. of nucleon.}$

So  $|eQ| \approx eR_0^2 A^{2/3}$

$|Q| \approx R_0^2 A^{2/3}$

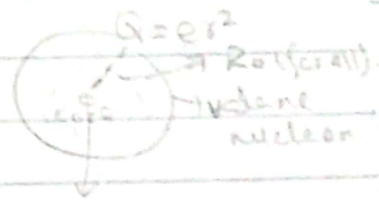
Which ranges  $6 \times 10^{-30} \text{ em}^2$  for light nuclei.  $50 \times 10^{-30} \text{ em}^2$  for heavy nuclei

\* The unit of  $10^{-28} \text{ m}^2$  is used

frequently in nuclear physics/reactions are studied

$1 \text{ barn} = 10^{-28} \text{ m}^2$

$0.06 \text{ eb to } 0.5 \text{ eb}$



no contribution of inner nucleon as it is spherically symmetrical

so their quadrupole moment is different

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

This unit is convenient to measure quadrupole moment.



$$\therefore eQ = r_0^2 A^{2/3}$$

K  
valid for unpaired.

\* The observed Quadrupole moment for Deuteron.

$l=0$  (s-state [spherical shape])  
 $l=2$  (d-state)

$$Q = 0.00288 \pm 0.00002 \text{ b}$$

for  $l=0$  wave function, would have a vanishing quadrupole moment (b/c  $l=0$  spherical symmetry)

\* The mixed wave function for

$$\Psi = a_s \Psi(l=0) + a_d \Psi(l=2) \quad \text{--- (1)}$$

$$eQ = e \int \Psi^* (3z^2 - r^2) \Psi dV \quad \text{--- (2)}$$

gives two contribution one is

## Note

→ Book page # 75  
Table 3.3  
(check)

► Nuclei  
Quadrupole Moments.

Many of the nuclei given in table 3.3 fall within range of 0.06eb to 0.5eb.

but several rare-earth nucleon

\* nucleon  
paired nucleon (which is) → core also contribute in quadrupole moment.

proportional to  $ad^2$  and another term is cross term that is proportional to  $asad$ . After performing the calculation by inserting (1) eq in eq (2) we get:-

$$Q = \frac{\sqrt{2}}{10} asad \langle r^2 \rangle_{sd} - \frac{1}{20} ad^2 \langle r^2 \rangle_{dd}$$

where  $\langle r^2 \rangle_{sd} = \int r^2 R_s(r) R_d(r) r^2 dr$  is the integral of  $r^2$  over the radial wave function. Similarly  $\langle r^2 \rangle_{dd}$  is defined similar to  $\langle r^2 \rangle_{sd}$ . To calculate  $Q$  we must know about the deuteron d-state wave function, and which is not directly measurable d state is consistent with  $4\%$  as deduced from magnetic moment. For the quadrupole moment the poor knowledge of the d-state wave function makes the deduced d-state mixture uncertain

$$(a_0 \psi(l=0) + a_2 \psi(l=2)) (a_0 \psi^*(l=0) + a_2 \psi^*(l=2))$$

## ► Nucleon - Nucleon Scattering:-

The study of deuteron gives some limited information about the nucleon-nucleon interaction because there are no excited states.

We can only study the dynamics

of nucleon-nucleon interactions in the configuration of the deuteron  $J=0$  parallel spins

In order to study nucleon-nucleon scattering interaction in different configuration we can perform nucleon-nucleon scattering experiments. In this an incident beam of nucleons is scattered from a target nucleon.

→ If the target is the nucleus with many nucleons then there will be several target nucleons within the range of nucleus potential for the incident nucleons. So there will be multiple scattering of single nucleon which make complicated effect because of the multiple encounters. So in order to avoid that we select a target of hydrogen nuclei/atom so that the incident nucleon scattered from the individual proton present inside the nucleus of hydrogen.

Article

4.2 :- If the probability of single encounter is small then the probability of multiple encounters will be negligible. Heavy nucleus have large no. of nucleons so there will be multiple encounter, the scattering process will be complex.

Light Nuclei  $\rightarrow$  Small no. of nucleons.  
and scattering is not as complex as in heavy nuclei.

$\Rightarrow$  Nucleon - Nucleon scattering is identical to optical diffraction.

(Pg No 86)

Nucleon - Nucleon scattering is analogous to optical diffraction. How.

Pg # 86 (3 points)

$\rightarrow$  Classical approach.

## Nucleon - Nucleon Scattering:-

Nucleon - Nucleon scattering  $\xrightarrow{\text{CM}}$  Optical diffraction

Nucleon - Nucleon scattering  $\xrightarrow{\text{QM}}$  Square well potential

Interaction of the incident particle with the particle in square well potential.

Not only particle in box is imp but the counter particles are also imp.

$\rightarrow$  So when we are studying the scattering phenomena we are interacted with free incident particles with  $E > 0$

\* Schrödinger eq by assuming  $l=0$



\* By considering that the incident nucleon is striking the target nucleon just at its surface that is the impact parameter is of the order of  $R \approx 1 \text{ fm}$

\* If the incident nucleons are approaching with velocity  $v$ , their angular momentum

$m v R$  as in QM the angular momentum is quantized so,

$$m v R = l \hbar$$

If  $l=0$

$$m v R \ll \hbar$$

$$v \ll \frac{\hbar}{m R}$$

$$\hbar = \frac{6.67 \times 10^{-34}}{2\pi}$$

$$T = \frac{1}{2} m v^2 \ll \frac{1}{2} \frac{\hbar^2}{m R^2} \ll \frac{\hbar^2}{2m R^2} \approx 20 \text{ MeV}$$

(Energy minimum value)

In case if the value  $\ll \frac{\hbar^2 c^2}{2m R^2} \approx 20 \text{ MeV}$  is used.

→ phenomena known as low scattering phenomena and energy is  $20 \text{ MeV}$ . ( $l=0$ )

only that the particle is incident and possible if energy is less than  $20 \text{ MeV}$  than case is not possible scattering

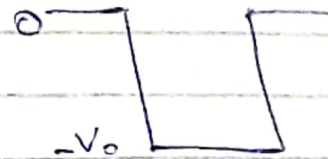
\*  $E > 0$

when Energy is greater than zero to overcome a particle than possible

•  $\delta = 0 \rightarrow$  low scattering phenomena  $\rightarrow$  lighter nuclei  $\rightarrow$  if heavy nuclei system become complicated

$$V(r) = -V_0 \quad r < R$$

$$V(r) = 0 \quad r > R$$



$$\frac{-\hbar^2}{2m} \frac{d^2U}{dr^2} + V(r)U(r) = EU(r)$$

$$U(r) = A \sin k_1 r + B \cos k_1 r$$

$$k_1 = \sqrt{\frac{2m(E + V_0)}{\hbar^2}}$$

$\rightarrow$  for  $r < R$

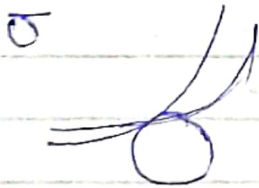
Boundary condition  
B.C. also follow on

$$U(r) = C e^{-k_2 r} + D e^{k_2 r}$$

$$k_2 = \sqrt{\frac{-2mE}{\hbar^2}}$$

$r > R =$

$$U(r) = C' \sin k_2 r + D' \cos k_2 r \quad (\text{For } r > R)$$



When u have different cross sections you have different angles.

(Read paragraph from book)

Nucleon-Nucleon scattering is nuclear force. It is not just  $\delta$  dependent it is also spin dependent.

$\rightarrow$  Contradiction between calculated cross-section (QM) and experimental cross section b/w Nucleon Nucleon interaction. indicate that there is scattering missing

Spin part

$$S = S_p + S_n$$

$$S = \frac{1}{2} + \frac{1}{2}$$

$\swarrow$                        $\searrow$   
 1                              0  
 Triplet                      singlet state.  
 states.

For states

determination:  $2S+1$  (Multiplicity).

So we can conclude that Nucleon-Nucleon force is spin dependent.

### Possible scatterings:-

- |  |   |   |
|--|---|---|
| <ul style="list-style-type: none"> <li>① p-p</li> <li>② n-n</li> </ul> | } | Incident nucleon and target nucleon are same.       |
| <ul style="list-style-type: none"> <li>③ p-n</li> <li>④ n-p</li> </ul> | } | Incident nucleon are different from target nucleon. |

Nature charges

Two charges interact so charges same than coulomb force is also present with Nuclear force.

⑥ Nuclear force is charge dependent

$$F = k \frac{q_1 q_2}{r^2}$$

if  $\neq$  zero then Coulomb force zero and it has no coulomb force with nuclear force.

**Lighter Nuclei**

↓  
Less scattering rate.

↓  
Higher intensity.  
(after process  
(beam scattered beam).  
(beam intensity after scattering))

↓  
Mathematical formulation is simple.

**Heavy Nuclei**

↓  
High scattering rate.

↓  
Low Intensity

↓  
Mathematical formulation is complex.

- \* When two nucleons come together they exert a nucleon-nucleon force.
- \* When energy is less than zero energy is there is no scattering.

## Properties of Nuclear force:- Pg 200

- ① The interaction b/w two nucleon consist to lowest order of an attractive central potential square well potential fairly well fits with the observed data in case of nucleons.

$V_c(r)$

↳ central potential (depend on  $r$ )

$r$  is the inner nucleon distance

This is theoretically.

Now experimentally:- Nucleon-Nucleon Parameter

↳ Scattering parameter → phase shift

↳ Exp data → Assume potential force  
 $V_c(r)$   $V_c$  potential

- ② The nucleon-nucleon interaction is strongly spin dependent. → The observation results b/c of the failure to observe a singlet bound states of the deuteron.

- Measure difference b/w singlet and triplet cross-section.

- Potential modification → spin term include.

If we have two nucleons, <sup>with</sup>  $S_1$  and  $S_2$  and other with  $S_1, S_2$ . All combinations of  $S_1$  and  $S_2$  are not permitted.

(\*) Nuclear force must follow some symmetry which restrict the form of potential.

(\*) Parity ( $x \rightarrow -x$ )  $\rightarrow$  conservation of angular momentum  
 $L = \mathbf{r} \times \mathbf{p}$   
 $\downarrow$   
coordinate reversed.

(\*) The time reversal ( $t \rightarrow -t$ ) reactions are  
IF these conditions are fulfilled so from here we got a values.

Linear and angular momentum if they are reversed.

Combinations:-  $S_1$  and  $S_2$   
 $AS_1 + BS_2$

$$\left. \begin{array}{l} V_c(r) \sim S_1 \\ V_c(r) \sim S_2 \end{array} \right\} \rightarrow \textcircled{1}$$

Linear combination:-  $AS_1 + BS_2$  when we put  $t \rightarrow -t$  so there is negative sign so we take square

Time reversed conservation is imp for the nuclear force and for the central potential.

IF they  $\textcircled{1}$  are present in potential they will violate the time reversed potential.

$S_1 \cdot S_2$  is the simplest term which includes the spin of both nucleon.

$S_1^2, S_2^2$  are allowed in the expression of potential  $V_e(r)$ .

$S_1 \cdot S_2$  for singlet state.

$S_1 \cdot S_2$  for triplet state.

In order to find  $S_1, S_2$  combination of singlet and triplet we have to evaluate the total spin which is  $S = S_1 + S_2$  (total spin)

$$S = S_1 + S_2$$

$$S \cdot S = (S_1 + S_2)(S_1 + S_2)$$

$$S^2 = S_1^2 + S_1 S_2 + S_2 S_1 + S_2^2$$

$$2S_1 \cdot S_2 = S^2 - S_1^2 - S_2^2$$

$$S_1 \cdot S_2 = \frac{1}{2} [S^2 - S_1^2 - S_2^2].$$

As in Q.M  $S^2 = S(S+1)\hbar^2 \rightarrow \textcircled{1}$

$$\langle S_1 \cdot S_2 \rangle = [\langle S^2 \rangle - \langle S_1^2 \rangle - \langle S_2^2 \rangle] \rightarrow \textcircled{2}$$

Use eq  $\textcircled{1}$  in eq  $\textcircled{2}$

$$\langle S_1 \cdot S_2 \rangle = \frac{1}{2} [S(S+1) - S_1(S_1+1) - S_2(S_2+1)] \hbar^2$$

For singlet state  $S=0, S_1=1/2, S_2=1/2$   
(2S+1)

Use in eq  $\textcircled{2}$

$$\langle S_1 \cdot S_2 \rangle = \frac{1}{2} \left[ -\frac{1}{2} \left( \frac{1}{2} + 1 \right) - \frac{1}{2} \left( \frac{1}{2} + 1 \right) \right] \hbar^2$$

$$'' = \frac{1}{2} \left[ -\frac{3}{4} - \frac{3}{4} \right] \hbar^2$$

$$'' = \frac{1}{2} \left[ -\frac{3}{4} \right] \hbar^2$$

isotropic (symmetric)  
 $\uparrow \uparrow$   
 $\downarrow \downarrow$

means  $\odot \rightarrow \ominus$   
 nuclear force  
 tensor force

$$\langle S_1 \cdot S_2 \rangle = -\frac{3}{4} \hbar^2$$

Similarly find the triplet state is usually  
 $(2S+1)$   $S_1 = 1/2, S_2 = 1/2, S=1$

$$\langle S_1 \cdot S_2 \rangle = \frac{1}{4} \hbar^2 \text{ for triplet state.}$$

$S_1 \cdot S_2$   $V_c(r)$  include in central potential term.

$$V(r) = - \left( \frac{S_1 \cdot S_2}{\hbar^2} - \frac{1}{4} \right) V_1(r) + \left( \frac{S_1 \cdot S_2}{\hbar^2} + \frac{3}{4} \right) V_2(r)$$

$\downarrow$  Singlet state potential       $\downarrow$  triplet state potential

② The interaction potential includes a non central term known as a tensor potential. Evidence for the tensor forces comes primarily from the observed quadrupole moment of the ground state of the deuteron. As s-state ( $J=0$ ) wave function is spherically symmetric the electric quadrupole moment vanishes wave function with mixed  $J$ -state must form some non-central potential (tensor potential).

$$V_c(r) \rightarrow V(\vec{r})$$

$\vec{r}$  direction and magnitude.



$S_1^2, S_2^2$  are allowed  
Potential  $V(r)$

$S_1 \cdot S_2$

Lec:

## The Nucleon-Nucleon Interaction/ Properties of nuclear force:-

① The nucleon-nucleon force is  
charge symmetric.

This means that the proton-proton interaction  
is identical to the neutron-neutron interaction.  
After correcting <sup>(mass carrying)</sup> the Coulomb force is proton  
proton interaction.

Here charge refers to the character of the  
nucleon (proton or neutron) and not to the  
electric charge.

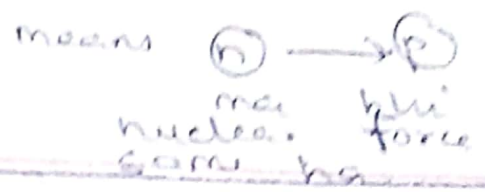
→ Evidence in support to this comes from  
the equality of pp and nn scattering  
length. After applying the correction term  
in pp interaction. Some of its parameters are

$$\left. \begin{aligned} a &= -1.71 \pm 0.2 \text{ fm.} \\ r_0 &= 2.84 \pm 0.03 \text{ fm.} \end{aligned} \right\} \rightarrow \textcircled{1}$$

They are in good agreement with nn  
interaction

$$\left. \begin{aligned} a &= -16.6 \pm 0.5 \text{ fm} \\ r_0 &= 2.66 \pm 0.15 \text{ fm.} \end{aligned} \right\} \textcircled{2}$$

Last point (symmetric)  
Elaborate.  
nn  
pp



① and ② support the charge symmetry.  
⑤ The nucleon-nucleon force is nearly charge  
→ The nuclear forces  $nn, pp, pn$  are identical after correcting for the  $pp$  Coulomb force b/w two  $pp$  interaction  
nn → just nuclear force.  
pp → [Coulomb] and Nuclear both we have to eliminate it  
pn → only nuclear force  
Correcting term.

⑥ The nucleon-nucleon interaction becomes repulsive at short distances.

This conclusion follows from qualitative considerations of the nuclear density. as we add more nucleons the nucleus grows in such a way that its central density remains roughly constant and thus something is keeping the

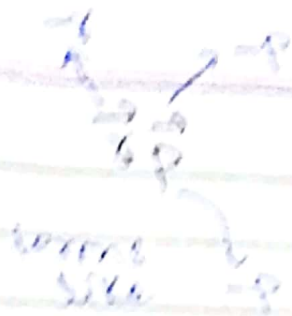
7) The nucleon-nucleon interaction may also depend on the relative velocity or momentum of the nucleon.

Weak force  $\rightarrow V(\vec{x}) \rightarrow$  Parity conservation  $\vec{x} \rightarrow \vec{x}$   
 $\hookrightarrow$  Time reversal  $t \rightarrow -t$

$V(\vec{x}) \sim$  terms with linear power of  $p$   
terms with square power

$p \rightarrow -p$  parity operator  
 $p \rightarrow -p$  time reversal

In  $V(\vec{x})$  terms with linear power of momentum are not included b/c Parity and time reversal invariance is violated



$\vec{p}_1 \times \vec{p}_2 \rightarrow$  Parity conserved

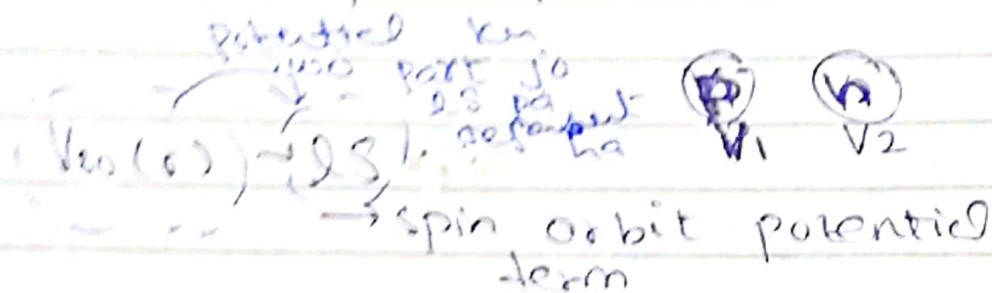
but time reversal is still violated.

Possible structure for this term is

$$V(r) \vec{S} \times \vec{p}^2$$

where  $S = S_1 + S_2$

Total spin of two nucleons.

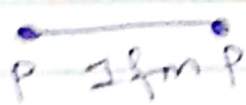


Time reversed  
 ma spin  
 change to  
 jay ge  
 up  $\rightarrow$  down  
 down  $\rightarrow$  up

\* Although high order terms may be present but only first order term in  $P$  satisfies the symmetries of both parity and time reversal.

## Yukawa's theory of Nuclear force:-

- 1) Nuclear force  $\rightarrow$  binding the Nucleus  $\rightarrow$  proton + neutron.
- 2) Nucleus is about  $\sim 1\text{fm}$



If we take two protons at a distance of  $1\text{fm}$

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = 230 \text{ N}$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

So the two protons are repelling each other with a force of 230 N (large force)



→ Then how is it possible that two protons are still packed inside the nucleus. Even in some nucleus we have more than two protons in a tiny region of space.

→ There is also another force other than Coulomb repulsive force inside the nucleus and that is the nuclear force.

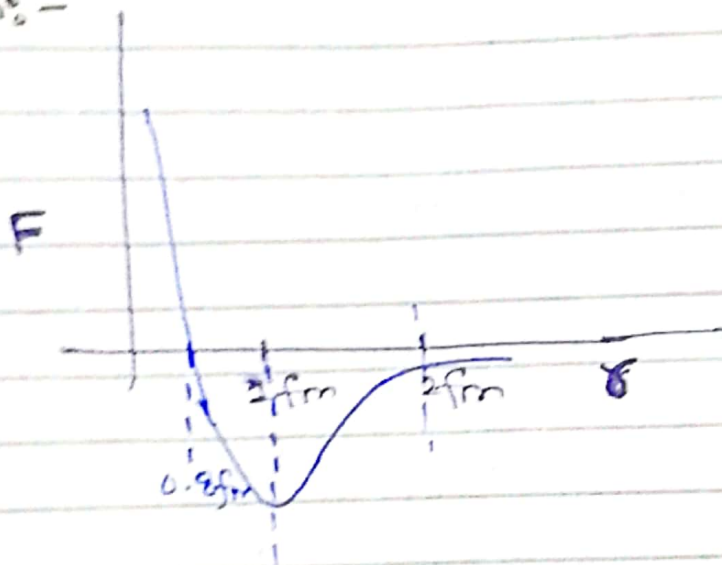
✓ The nuclear force b/w the two protons at a distance of about 1 fm is around 2500 N. So it is 100 times more powerful than the repulsive force.

Q: Is nuclear force only attractive?  
No but at shorter distances.

Question: Nucleus size, two protons repelling. Nuclear force is 100 times greater than the repulsive force.

As nuclear force is binding  $\rightarrow$  Attractive  
but nuclear force is not just attractive  
it can be repulsive at short distances.  
\* So the nature of direction and repulsion  
for the nucleus force depends on the  
distance at which we are observing.

Graph:-



\* At distance less than  $0.8 \text{ fm}$  this  
nuclear force is Repulsive that means  
that it pushes the nucleons apart.

\* At distance - from  $0.8 \text{ fm} - 2 \text{ fm}$  - the  
nuclear force is attractive.

\* For the distance more than  $2 \text{ fm}$  the  
force become negligible.

None of the successful theories describing the behaviour of nuclear force was given by a Japanese scientist Hideki Yukawa in 1935.

This theory is known as Yukawa's theory of nuclear force or meson theory.

According to the theory:-

① Nuclear force exist as a result of exchange of particles.

② The Neutron and Protons are continuously emitting or absorbing another particle that is  $\pi$ -meson.

→ Whenever the near by neutron or Proton emit or absorb a particle this leads to a force which is nuclear force.

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